Dr Oliver Mathematics
Disproof

In this note, we will examine disproof.

Normally, the question will tell you about a so-called "proof" and we simply come up with *one* counter-example.

Example 1
For all positive integer values of n, $(n^3 - n + 7)$ is prime.

Solution
E.g., we take $n = 7$:

$$7^3 - 7 + 7 = 343 = 7 \times 49,$$

and we have a counter-example. ■

This is easy: we take the constant and susbtitute it in (unless it is 1 and then you have to be sure).

Example 2
If x and y are irrational and $x \neq y$, then xy is irrational.

Solution
E.g., we take $x = \sqrt{2}$ and $y = 2\sqrt{2}$. Then $x \neq y$ but

$$xy = \sqrt{2} \times 2\sqrt{2} = 4,$$

and we have a counter-example. ■

Here are some examples for you to try.

1. $(3^n + 2)$ is prime for all positive integer values of n.
2. For every $n \in \mathbb{N}$, the integer $n^2 - n + 11$ is prime.
3. For all real values of x,
$$\cos(90 - |x|)^\circ = \sin x^\circ.$$
4. If a and b are positive integers and $a \neq b$, then $\log_a b$ is irrational.
5. $(n^2 + 3n + 13)$ is prime for all positive integer values of n.
6. There exist positive integers, a and b, to $a^2 - b^2 = 6$.
7. if a is rational and b is irrational then $\log_a b$ is irrational.
8. If $x, y \in \mathbb{R}$, then $|x + y| = |x| + |y|$.

9. For all $a, b, c \in \mathbb{N}$, if $a|bc$, then $a|b$ or $a|c$.
10. If $x, y \in \mathbb{R}$, and $|x + y| = |x - y|$, then $y = 0$.
11. Samantha says that "all primes are odd". Is she correct?
12. The sum of two distinct square numbers is a square number.
13. All positive cube numbers are either even or one less than a multiple of 3.
14. If the sum of two integers is even, then one of the summands is even.
15. All natural numbers are either prime or have more than one factor.
16. If a and b are natural numbers, then so is their difference.