## Dr Oliver Mathematics Mathematics: Advanced Higher 2024 Paper 1: Non-Calculator 1 hour

The total number of marks available is 35. You must write down all the stages in your working.

- 1. Differentiate the following with respect to x:
  - (a)  $y = \cot 3x$ , (2)
  - (b)  $f(x) = 5x(4x 7)^{\frac{1}{2}}$ . (2)
- 2. A complex number is defined by
- z = 1 + i.
- (a) Express z in polar form.
- (b) Use de Moivre's theorem to evaluate  $z^8$ .
- 3. A geometric sequence of positive terms has third term 36 and fifth term 16.
  - (a) Calculate the value of the common ratio.
  - (b) Calculate the value of the first term.
    - (c) State why the associated geometric series has a sum to infinity.
    - (d) Find the value of this sum to infinity.
- 4. Matrix  $\mathbf{A}$  is defined by

$$\mathbf{A} = \left(\begin{array}{cc} 6 & 1\\ 11 & 3 \end{array}\right).$$

(a) Find  $\mathbf{A}^{-1}$ , the inverse of matrix  $\mathbf{A}$ .

Matrix  ${\bf B}$  is defined by

$$\mathbf{B} = \left(\begin{array}{cc} -4 & 3\\ -5 & 2 \end{array}\right).$$

- (b) Find the matrix **M** such that
- $\mathbf{AM} = \mathbf{B}.$
- 5. The function f is defined by

$$\mathbf{f}(x) = x^3 - x, \, x \in \mathbb{R}.$$

(a) Determine whether f(x) is even, odd, or neither.

Mathematics

(2)

(2)

(2)

(2)

(2)

(1)

(1)

(1)

(2)

	(b) Show that the graph of $f(x)$ has a point of inflection.	(2)
6.	(a) Find the $2 \times 2$ matrix, <b>A</b> , associated with a reflection in the <i>x</i> -axis.	(1)
	(b) Describe the transformation associated with the matrix	(1)
	$\mathbf{B} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}.$ (c) Find the 2 × 2 matrix, <b>C</b> , associated with a reflection in the <i>x</i> -axis followed by the transformation associated with $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ .	(2)
7.	A curve is defined by the equation	
	$x^2y + 4xy^2 = -32, \ y > 0.$	
	(a) Use implicit differentiation to find an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ .	(3)
	<ul><li>The curve has only one stationary point.</li><li>(b) Find the coordinates of the stationary point.</li></ul>	(3)
8.	Use the substitution $u = \tan 2x$	(4)
	to evaluate $\int_0^{\frac{1}{8}\pi} \frac{\sqrt{\tan 2x}}{\cos^2 2x}  \mathrm{d}x.$	



