Dr Oliver Mathematics
AQA GCSE Mathematics
2017 June Paper 1: Non-Calculator
1 hour 30 minutes

The total number of marks available is 80.
You must write down all the stages in your working.

1. Simplify   (1)
$$2^5 \times 2^3.$$

   Circle your answer.
$$4^8 \quad 2^8 \quad 2^{15} \quad 4^{15}$$
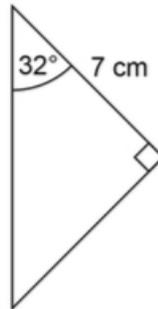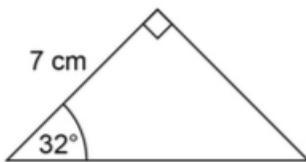

Solution

Well,
$$2^5 \times 2^3 = 2^{5+3} = 2^8$$

so
$$4^8 \quad \underline{\underline{2^8}} \quad 2^{15} \quad 4^{15}$$

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2. Circle the reason why these triangles are congruent.   (1)

Not drawn accurately

$$\text{SSS} \quad \text{SAS} \quad \text{ASA} \quad \text{RHS}$$

> **Solution**
>
> The pair of triangles have a right-angle, one line in common, and one angle in common
> so
> $$\text{SSS} \quad \text{SAS} \quad \underline{\text{ASA}} \quad \text{RHS}$$

3. Which of these is a geometric progression? (1)
   Circle your answer.

   $$2, 4, 6, 8, 10 \qquad\qquad 2, 6, 18, 54, 162$$
   $$2, 3, 5, 8, 12 \qquad\qquad 2, 6, 10, 14, 18$$

> **Solution**
>
> $a = 2$ and
> $$r = \frac{6}{2} = 3$$
> so
> $$2, 4, 6, 8, 10 \qquad\qquad \underline{\underline{2, 6, 18, 54, 162}}$$
> $$2, 3, 5, 8, 12 \qquad\qquad 2, 6, 10, 14, 18$$

4. (1)
   $$a : b = 4 : 3.$$

   Circle the correct statement.

   $$b \text{ is } \tfrac{4}{7} \text{ of } a \quad b \text{ is } \tfrac{3}{7} \text{ of } a \quad b \text{ is } \tfrac{4}{3} \text{ of } a \quad b \text{ is } \tfrac{3}{4} \text{ of } a$$

> **Solution**
>
> Well,
> $$a : b = 4 : 3 \Rightarrow \frac{b}{a} = \tfrac{3}{4}$$
> $$\Rightarrow b = \tfrac{3}{4}a$$

so
$$b \text{ is } \tfrac{4}{7} \text{ of } a \quad b \text{ is } \tfrac{3}{7} \text{ of } a \quad b \text{ is } \tfrac{4}{3} \text{ of } a \quad \underline{\underline{b \text{ is } \tfrac{3}{4} \text{ of } a}}$$

5. Write 36 as a product of prime factors. (3)
   Give your answer in index form.

Solution

$$
\begin{array}{c|c}
  & 36 \\
\hline
2 & 18 \\
\hline
2 & 9 \\
\hline
3 & 3 \\
\hline
3 & 1 \\
\hline
\end{array}
$$

so
$$36 = \underline{\underline{2^2 \times 3^2}}.$$

6. The table shows information about the times for 10 people to complete a task. (4)

Time, t minutes	Frequency
$0 < t \leqslant 20$	1
$20 < t \leqslant 40$	6
$40 < t \leqslant 60$	3

These statements are about the mean and range of the actual times.
Tick the correct box for each statement.

	True	False
The mean could be less than 20 minutes		
The mean could be more than 40 minutes		
The mean could be less than 40 minutes		
The range could be more than 40 minutes		
The range could be less than 40 minutes		
The range could be more than 60 minutes		

3

Solution

	True	False
The mean could be less than 20 minutes		False
The mean could be more than 40 minutes	True	
The mean could be less than 40 minutes	True	
The range could be more than 40 minutes	True	
The range could be less than 40 minutes	True	
The range could be more than 60 minutes		False

7.   (2)

$$\tfrac{3}{5} \text{ of a number is } 162.$$

Work out the number.

Solution

Let the original number be x. Then

$$\tfrac{3}{5}x = 162 \Rightarrow x = \tfrac{5}{3} \times 162$$
$$\Rightarrow x = 5 \times 54$$
$$\Rightarrow \underline{\underline{x = 270}}.$$

8.   (2)

$$x \text{ km/h} = y \text{ mph}.$$

Use

$$8 \text{ km/h} = 5 \text{ mph}$$

to write a formula for y in terms of x.

Solution

Well,

$$\frac{y}{x} = \frac{5}{8} \Rightarrow y = \tfrac{5}{8}x$$

and so

$$\underline{\underline{y = \tfrac{5}{8}x \text{ km/h}}}.$$

9.

$$\text{Density} = \frac{\text{mass}}{\text{volume}}.$$

(a) The mass of solid A is 6 times the mass of solid B.   (1)
The volume of solid A is 3 A times the volume of solid B.

Complete the sentence:

The density of solid A is .... times the density of solid B.

Solution

Well,

$$\text{density} = \frac{6 \times \text{mass}}{3 \times \text{volume}}$$
$$= 2 \times \frac{\text{mass}}{\text{volume}}$$

and so

The density of solid A is $\underline{\underline{2}}$ times the density of solid B.

$$\text{Average speed} = \frac{\text{distance}}{\text{time}}.$$

(b) If the distance is halved and the time is doubled, what happens to the average   (1)
speed?
Circle your answer.

$$\times 2 \qquad \times 4 \qquad \text{no change} \qquad \div 2 \qquad \div 4$$

Solution

Well,

$$\text{average speed} = \frac{\frac{1}{2} \times \text{distance}}{2 \times \text{time}} = \frac{1}{4} \times \frac{\text{distance}}{\text{time}}$$

and so

$$\times 2 \qquad \times 4 \qquad \text{no change} \qquad \div 2 \qquad \underline{\underline{\div 4}}$$

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10. Solve the simultaneous equations. (3)

$$2x + y = 18$$
$$x - y = 6.$$

Solution

$$2x + y = 18 \quad (1)$$
$$x - y = 6 \quad (2)$$

Add $(1) + (2)$:

$$3x = 24 \Rightarrow \underline{\underline{x = 8}}$$

we will use equation (2):

$$\Rightarrow 8 - y = 6$$
$$\Rightarrow \underline{\underline{y = 2}}.$$

11. Billy wants to buy these tickets for a show. (5)

- 4 adult tickets at £15 each and
- 2 child tickets at £10 each

A 10% booking fee is added to the ticket price.
3% is then added for paying by credit card.

Work out the **total** charge for these tickets when paying by credit card.

Solution

The total price of the tickets is

$$(4 \times 15) + (2 \times 10) = 60 + 20$$
$$= 80.$$

We add on the booking fee:
$$\frac{110}{100} \times 80 = 88$$

and, finally, paying by credit card:

$$88 + (0.03 \times 88) = 88 + 2.64$$
$$= \underline{\pounds 90.64}.$$

12. Here is a circle touching a square. (3)
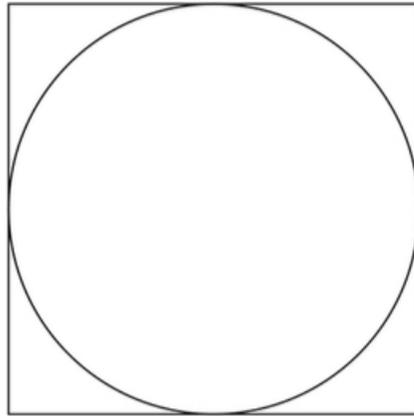
Not drawn accurately

The area of the square is 64 cm².

Work out the area of the circle.
Give your answer in terms of π.

Solution

One side of the square is
$$\sqrt{64} = 8 \text{ cm}$$
and the radius is
$$\frac{8}{2} = 4 \text{ cm}.$$
Hence,
$$\text{area of the circle} = \pi \times 4^2$$
$$= \underline{\underline{16\pi \text{ cm}^2}}.$$

13. Write the number (2)

$$\text{six million five thousand two hundred}$$

in standard form.

Solution

$$6\,005\,200 = \underline{6.005\,2 \times 10^6}.$$

14. Solve (1)

$$-3x > 6.$$

Solution

$$-3x > 6 \Rightarrow \underline{\underline{x < -2}}.$$

15. (1)

$$\tfrac{1}{6}, \ \tfrac{1}{7}, \ \tfrac{1}{8}, \ \text{and} \ \tfrac{1}{9} \ \text{are four fractions.}$$

How many of these fractions convert to a recurring decimal?
Circle your answer.

$$0 \quad 1 \quad 2 \quad 3 \quad 4$$

Solution

Well,

$$\tfrac{1}{6} = 0.1\dot{6}$$
$$\tfrac{1}{7} = 0.\dot{1}42\,85\dot{7}$$
$$\tfrac{1}{8} = 0.125$$
$$\tfrac{1}{9} = 0.\dot{1}$$

and so

$$0 \quad 1 \quad 2 \quad \underline{\underline{3}} \quad 4$$

16. A fair spinner has five equal sections numbered 1, 2, 3, 4, and 5.
A fair six-sided dice has five red faces and one green face.

- The spinner is spun.
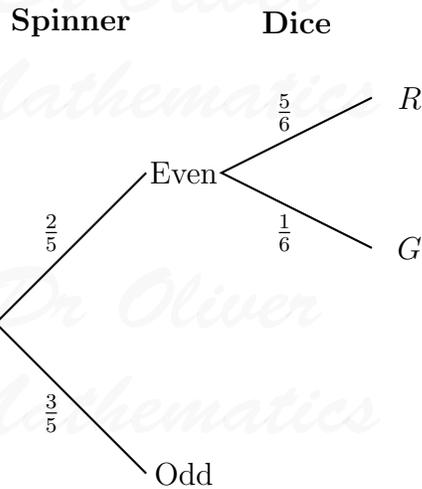- If the spinner shows an even number, the dice is thrown.

(a) Complete the tree diagram for the spinner and the dice. (2)

Spinner **Dice**

```
        Even
       /
      /
     /
     \
      \
       \
        Odd
```

Solution

Spinner **Dice**

```
                     5/6      R
                    /
           Even <
          /        \
      2/5           1/6    G
        /
       /
       \
     3/5
         \
          Odd
```
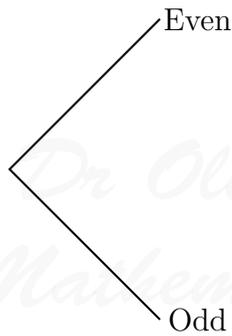
(b) Work out the probability of getting an even number and the colour green. (2)

Solution

$$P(\text{even number}, G) = \tfrac{2}{5} \times \tfrac{1}{6}$$
$$= \tfrac{1}{5} \times \tfrac{1}{3}$$
$$= \underline{\underline{\tfrac{1}{15}}}.$$

17. A is the point $(2, -5)$.
   B is the point $(4, -9)$.

   (a) Show that the gradient of the straight line passing through A and B is -2.　(2)

   Solution

   $$\text{Gradient} = \frac{-9 - (-5)}{4 - 2}$$
   $$= \frac{-4}{2}$$
   $$= \underline{\underline{-2}},$$

   as required.

   (b) C is the point $(-301, 601)$.　(2)

   Does C lie on the straight line passing through A and B?
   You **must** show your working.

   Solution
   The equation of the line is

   $$y - (-5) = -2(x - 2) \Rightarrow y + 5 = -2x + 4$$
   $$\Rightarrow y = -2x - 1.$$

   Now,

   $$x = -301 \Rightarrow y = -2(-301) - 1$$
   $$\Rightarrow y = 602 - 1$$
   $$\Rightarrow y = 601;$$

   hence, C <u>does</u> lie on the straight line passing through A and B.

18. Bottles of drink are for sale at three shops.　(3)
   The normal price of a bottle is the same at each shop.

Shop A	Shop B
Buy 1 bottle	Buy 2 bottles
Get 2 more bottles at half price	Get 3 more bottles at half price

Shop C
30% off a bottle

What is the cheapest way to buy **exactly** 8 bottles?
You can buy from more than one shop.
You **must** show your working.

Solution

Let x be the cost of each drink.

Shop A: Buy 3:
$$x + (2 \times \tfrac{1}{2}x) = x + x = 2x$$
in which case they cost
$$\frac{2x}{3} = \tfrac{2}{3}x \text{ each.}$$

Shop B: Buy 5:
$$2x + (3 \times \tfrac{1}{2}x) = 2x + \tfrac{3}{2}x = \tfrac{7}{2}x$$
in which case they cost
$$\frac{\tfrac{7}{2}x}{5} = \tfrac{7}{10}x \text{ each.}$$

Shop C: Buy 1:
$$(1 - 0.3) \times x = \tfrac{7}{10}x$$
in which case they cost
$$\frac{\tfrac{7}{10}x}{1} = \tfrac{7}{10}x \text{ each.}$$

The cheapest is to buy 6 bottles (2 full price, 4 half prices) from Shop A and 2 bottles (30% off a bottle) from Shop C.

19. Here is some information about the marks of 60 students in a test.

Mark, m	Frequency
$40 < m \leqslant 50$	9
$50 < m \leqslant 60$	16
$60 < m \leqslant 70$	20
$70 < m \leqslant 80$	8
$80 < m \leqslant 90$	7

(a) Draw a cumulative frequency graph. (3)

Solution

We need the totals:

Mark, m	Frequency	Total Frequency
$40 < m \leqslant 50$	9	9
$50 < m \leqslant 60$	16	$9 + 16 = 25$
$60 < m \leqslant 70$	20	$25 + 20 = 45$
$70 < m \leqslant 80$	8	$45 + 8 = 53$
$80 < m \leqslant 90$	7	$53 + 7 = 60$
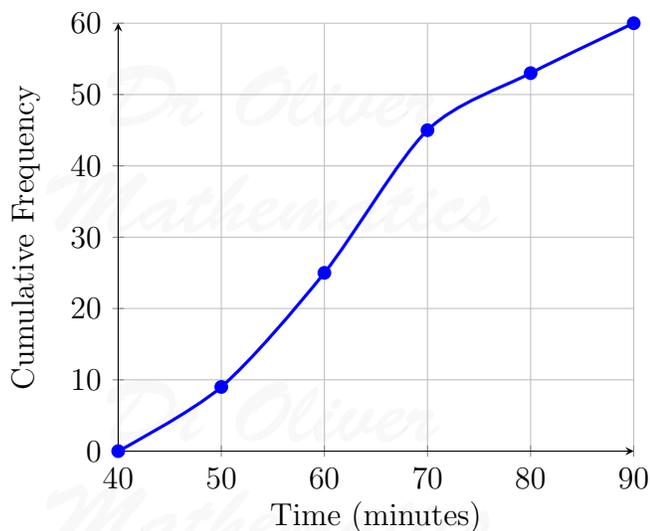
(b) Use your graph to estimate the lowest mark of the top 20% of students. (2)

Solution

Well,

$$\frac{20}{100} \times 60 = 12$$

12

and
$$60 - 12 = 48.$$
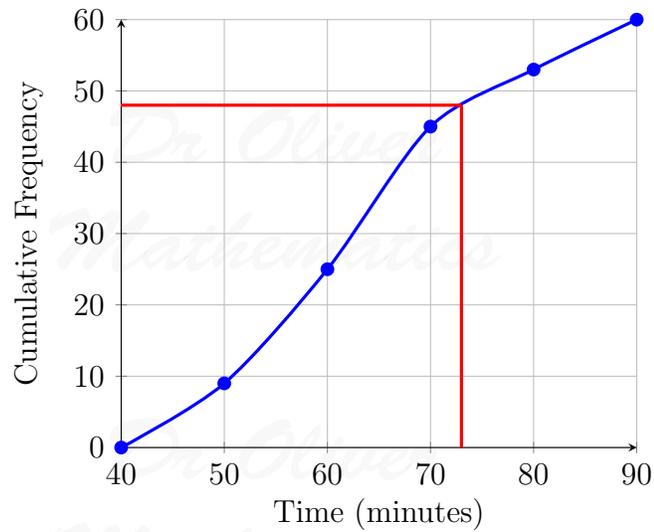
Correct read-off: approximately <u>73 marks</u>.

20. Work out the diameter of the circle (1)

$$x^2 + y^2 = 64.$$

Circle your answer.

$$8 \quad 16 \quad 32 \quad 128$$

Solution

Well,
$$\sqrt{64} = 8$$
and
$$2 \times 8 = 16$$
and so
$$8 \quad \underline{16} \quad 32 \quad 128$$

21. (a) The diagram shows rectangles A and B. (1)
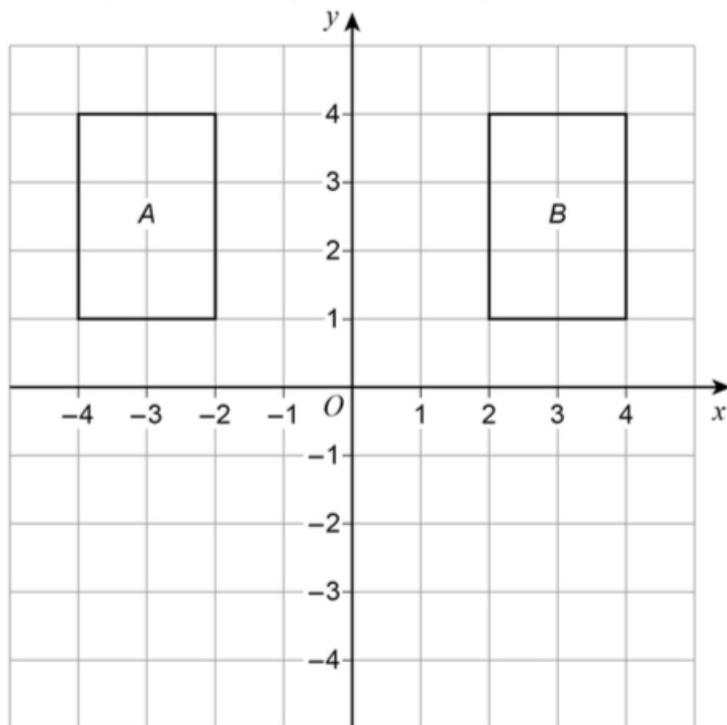
Rectangle A can be mapped to rectangle B by a single transformation.

Javed says, "The only single transformation is a reflection in the y-axis because the rectangles are on opposite sides of the y-axis."

Is he correct?
Tick a box.

Yes ☐   No ☐

Give a reason for your answer.

Solution
No, Javed is <u>not correct</u>: it could be an enlargement, scale factor -1, about the point $(0, 2.5)$, transformation by $\begin{pmatrix} 6 \\ 0 \end{pmatrix}$, etc.

(b) This diagram shows triangles CDE and PQR.   (3)

14
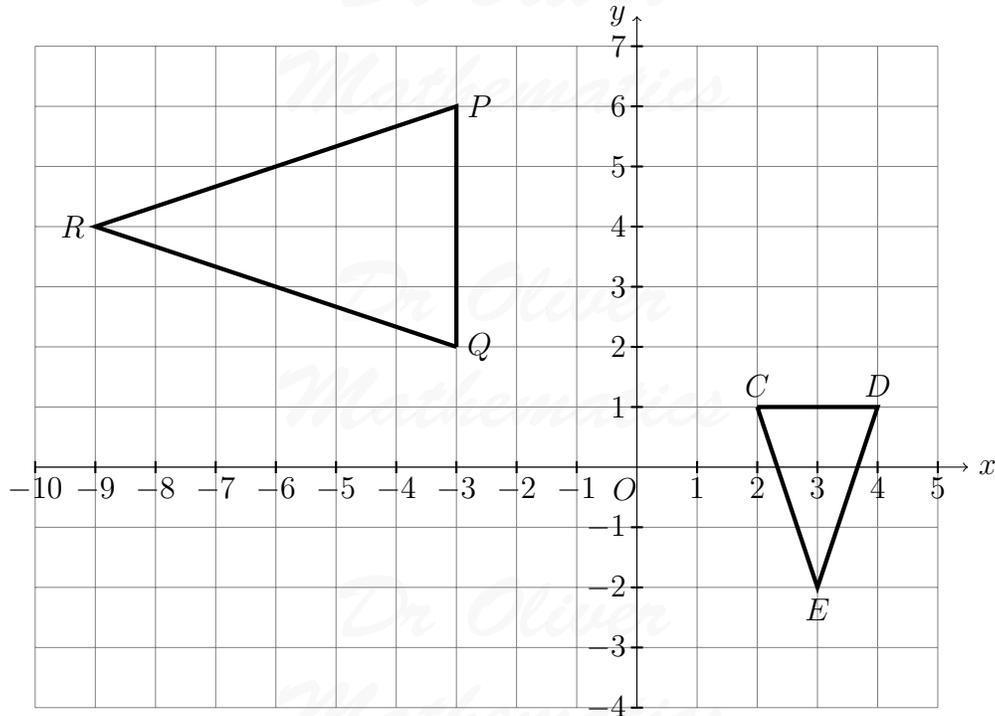
CDE is mapped to PQR by combining two single transformations.
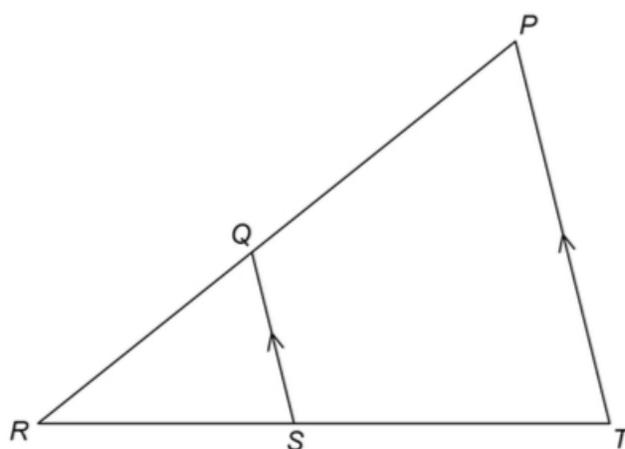The first is a rotation of $90°$ anticlockwise about E.

Describe fully the second transformation.

Solution

An <u>enlargement</u>, <u>scale factor −2</u>, about the point <u>(−1, 0)</u>.
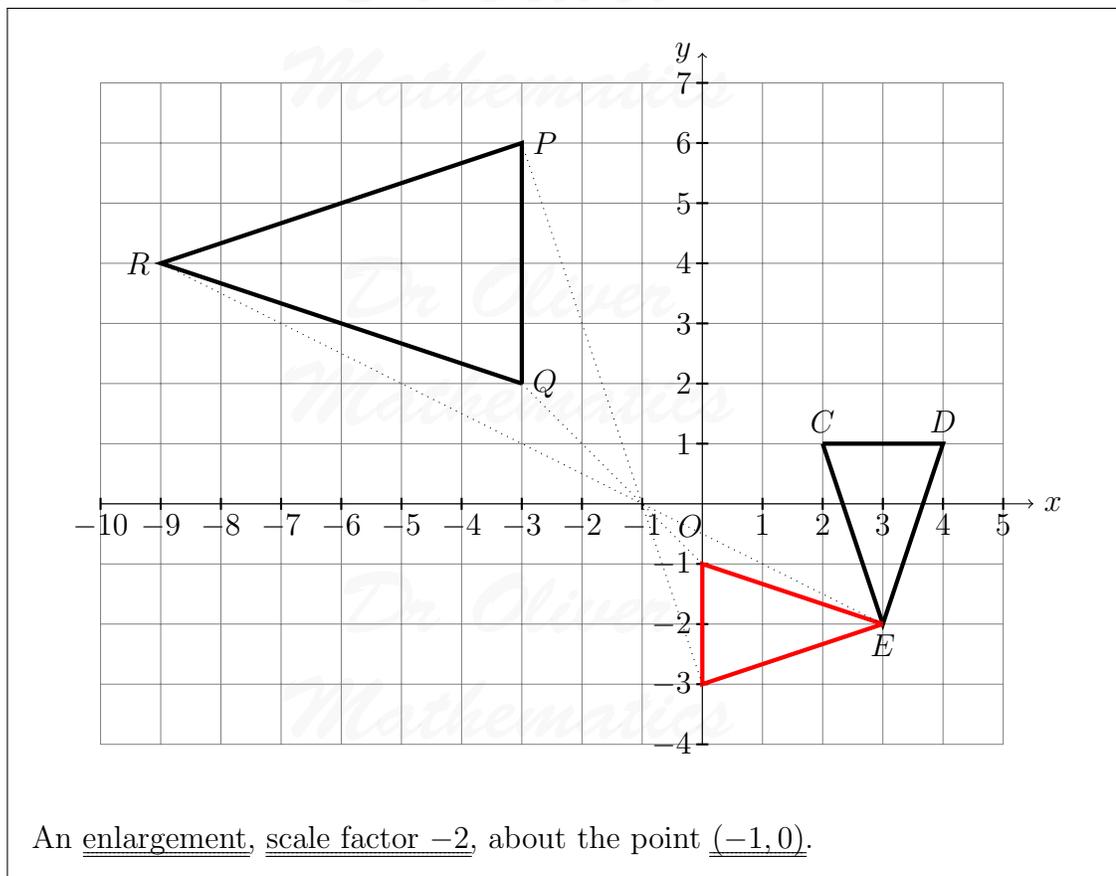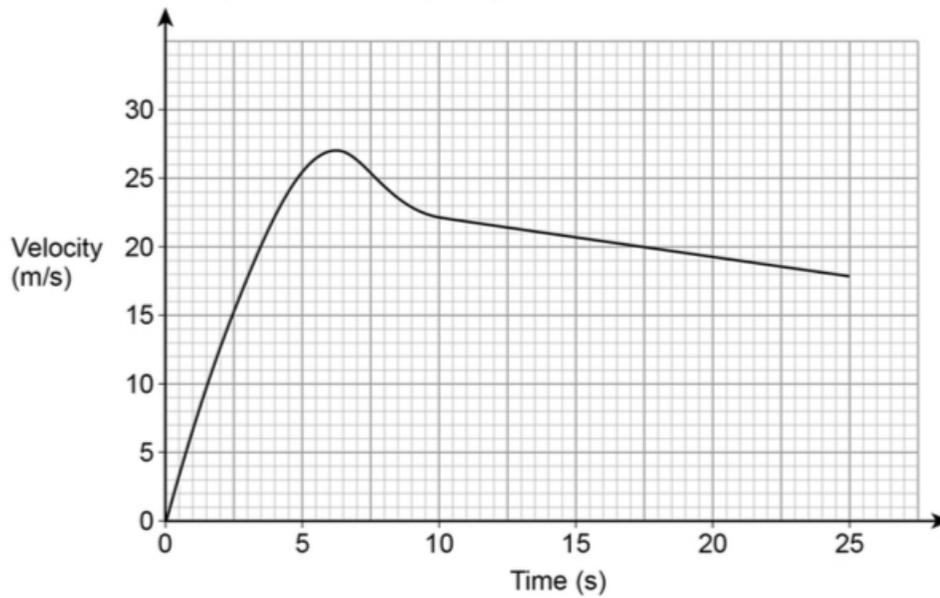
22. PRT and QRS are similar triangles.　(1)

Not drawn accurately

Which of these is equivalent to $\dfrac{QR}{PR}$?

$$\frac{RS}{ST} \qquad \frac{QS}{PT} \qquad \frac{PT}{QS} \qquad \frac{RT}{RS}$$

23. Here is a velocity-time graph of a motorbike for 25 seconds.

(a) After how many seconds was the acceleration zero? (1)

Solution

6 seconds.

(b) Work out the distance travelled in the last 15 seconds. (2)

Solution

$$\text{Distance} = \tfrac{1}{2} \times (22 + 18) \times 15$$
$$= \tfrac{1}{2} \times 40 \times 15$$
$$= 20 \times 15$$
$$= \underline{300 \text{ m}}.$$

24. (a) Work out (1)

$$\sqrt{12\tfrac{1}{4}}$$

as an improper fraction.

Solution

$$\sqrt{12\tfrac{1}{4}} = \sqrt{\frac{49}{4}}$$
$$= \frac{\sqrt{49}}{\sqrt{4}}$$
$$= \frac{7}{2}$$
$$= \underline{\underline{3\tfrac{1}{2}}}.$$

(b) Work out (2)

$$\sqrt[3]{16}$$

as a power of 2.

Solution

$$\sqrt[3]{16} = (2^4)^{\frac{1}{3}}$$
$$= \underline{\underline{2^{\frac{4}{3}}}}.$$

25. In an office (4)

- there are twice as many females as males,
- $\frac{1}{4}$ of the females wear glasses,
- $\frac{3}{8}$ of the males wear glasses, and
- 84 people in the office wear glasses.

Work out the number of people in the office.

Solution

We will call the total of females $2x$: in that case, the number of males is x and the total number is $3x$.

Now, 84 people in the office wear glasses,

$$\left(\tfrac{1}{4} \times 2x\right) + \left(\tfrac{3}{8} \times x\right) = 84 \Rightarrow \tfrac{1}{2}x + \tfrac{3}{8}x = 84$$
$$\Rightarrow \tfrac{7}{8}x = 84$$
$$\Rightarrow \tfrac{1}{8}x = 12$$
$$\Rightarrow x = 96$$
$$\Rightarrow 3x = 288.$$

Hence, there are <u>288 people</u> in the office.

In fact, we will make up a table:

	Glasses	Do Not Wear Glasses	Total
Female	56	168	224
Male	28	36	64
Total	84	204	288

26. Expand and simplify (4)
$$(x - 4)(2x + 3y)^2.$$

Solution

$\times$	$2x$	$+3y$
$2x$	$4x^2$	$+6xy$
$+3y$	$+6xy$	$+9y^2$

so
$$(x - 4)(2x + 3y)^2 = (x - 4)(4x^2 + 12xy + 9y^2).$$

Now,

$\times$	$4x^2$	$+12xy$	$+9y^2$
x	$4x^3$	$+12x^2y$	$+9xy^2$
-4	$-16x^2$	$-48xy$	$-36y^2$

so

$$(x-4)(2x+3y)^2 = 4x^3 + 12x^2y + 9xy^2 - 16x^2 - 48xy - 36y^2.$$

27. $P(-1,4)$ is a point on a circle, centre O. (4)

Not drawn accurately
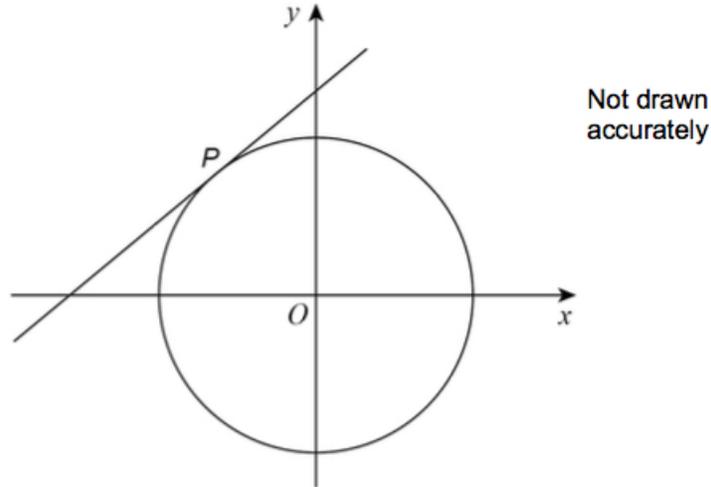
Work out the equation of the tangent to the circle at P.
Give your answer in the form $y = mx + c$.

Solution

Well,

$$m_{OP} = \frac{4-0}{-1-0}$$
$$= -4$$

and

$$m_{\text{normal}} = -\frac{1}{-4} = \tfrac{1}{4}.$$

Hence, the equation of the tangent is

$$y - 4 = \tfrac{1}{4}[x - (-1)] \Rightarrow y - 4 = \tfrac{1}{4}(x+1)$$
$$\Rightarrow y - 4 = \tfrac{1}{4}x + \tfrac{1}{4}$$
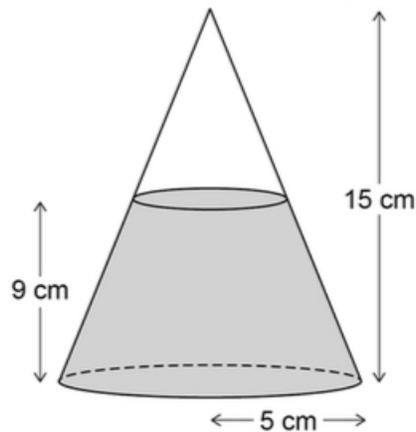$$\Rightarrow y = \tfrac{1}{4}x + \tfrac{17}{4}.$$

20

28. (4)

Volume of cone $= \frac{1}{3}\pi r^2 h$ where r is the radius and h is the perpendicular height.

A cone has a

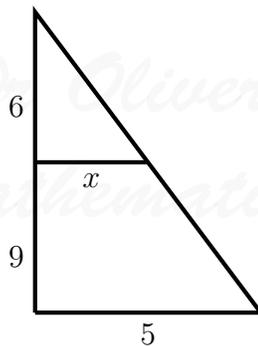- horizontal base of radius 5 cm and
- height of 15 cm.

The cone contains water to a depth of 9 cm.

Work out the volume of the water, in cm^3.
Give your answer in terms of π.

Solution

Similar triangles:

21

Now,

$$\frac{x}{6} = \frac{5}{15} \Rightarrow x = 6 \times \tfrac{1}{3}$$
$$\Rightarrow x = 2.$$

Finally,

$$\text{volume of the water} = \text{whole cone} - \text{truncated cone}$$
$$= (\tfrac{1}{3} \times \pi \times 5^2 \times 15) - (\tfrac{1}{3} \times \pi \times 2^2 \times 6)$$
$$= (\pi \times 25 \times 5) - (\pi \times 4 \times 2)$$
$$= 125\pi - 8\pi$$
$$= \underline{\underline{117\pi \text{ cm}^3}}.$$

29. Simplify (4)

$$\frac{2\sin 45^\circ - \tan 45^\circ}{4\tan 60^\circ}.$$

Give your answer in the form

$$\frac{\sqrt{a} - \sqrt{b}}{c},$$

where a, b, and c are integers.

Solution

$$\frac{2\sin 45^\circ - \tan 45^\circ}{4\tan 60^\circ} = \frac{2\frac{\sqrt{2}}{2} - 1}{4\sqrt{3}}$$
$$= \frac{\sqrt{2} - 1}{4\sqrt{3}}$$
$$= \frac{\sqrt{2} - 1}{4\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$
$$= \frac{\sqrt{2}\sqrt{3} - \sqrt{3}}{4 \times 3}$$
$$= \underline{\underline{\frac{\sqrt{6} - \sqrt{3}}{12}}};$$

hence, $\underline{\underline{a = 6}}$, $\underline{\underline{b = 3}}$, and $\underline{\underline{c = 12}}$.