Dr Oliver Mathematics
Probability: Part 1

1. Suppose Alice and Bob are equally strong squash players.

   Is it more probable that Alice will beat Bob in 3 games out of 4 or in 5 games out of 8?

Solution

3 games out of 4:

$$\begin{aligned} \mathrm{P}(\text{3 games out of 4}) &= \binom{4}{3}(\tfrac{1}{2})^3(\tfrac{1}{2}) \\ &= 4 \times \tfrac{1}{16} \\ &= \tfrac{1}{4}. \end{aligned}$$

5 games out of 8:

$$\begin{aligned} \mathrm{P}(\text{5 games out of 8}) &= \binom{8}{5}(\tfrac{1}{2})^5(\tfrac{1}{2})^3 \\ &= 56 \times \tfrac{1}{256} \\ &= \tfrac{7}{32} \\ &< \tfrac{8}{32} \\ &< \tfrac{1}{4}. \end{aligned}$$

Hence, it is probable that Alice will beat Bob in 3 games out of 4.