Dr Oliver Mathematics
OCR FMSQ Additional Mathematics
2007 Paper
2 hours

The total number of marks available is 100.
You must write down all the stages in your working.
You are permitted to use a scientific or graphical calculator in this paper.
Final answers should be given correct to three significant figures where appropriate.

Section A

1. Solve the inequality (3)

$$3(x+2) > 2 - x.$$

Solution

$$\begin{aligned} 3(x+2) > 2 - x &\Rightarrow 3x + 6 > 2 - x \\ &\Rightarrow 4x > -4 \\ &\Rightarrow \underline{\underline{x > -1}}. \end{aligned}$$

2. A particle moves in a straight line. Its velocity, v ms^{-1}, t seconds after passing a point O is given by the equation (4)

$$v = 6 + 3t^2.$$

Find the distance travelled between the times $t = 1$ and $t = 3$.

Solution

$$v = 6 + 3t^2 \Rightarrow s = 6t + t^3 + c,$$

where c is an arbitrary constant. Hence

$$\begin{aligned} \text{distance travelled} &= (6 \times 3 + 3^3 + c) - (6 \times 1 + 1^3 + c) \\ &= (45 + c) - (7 + c) \\ &= \underline{\underline{38 \text{ m}}}. \end{aligned}$$

3. A circle has equation (3)

$$x^2 + y^2 - 4x - 6y + 3 = 0.$$

Find the coordinates of the centre and the radius of the circle.

Solution

$$\begin{aligned} & x^2 + y^2 - 4x - 6y + 3 = 0 \\ \Rightarrow\ & x^2 - 4x + y^2 - 6y = -3 \\ \Rightarrow\ & (x^2 - 4x + 4) + (y^2 - 6y + 9) = -3 + 4 + 9 \\ \Rightarrow\ & (x-2)^2 + (y-3)^2 = 10; \end{aligned}$$

hence, the centre of the circle is $\underline{\underline{(2, 3)}}$ and the radius is $\underline{\underline{\sqrt{10}}}$.

4. Find all the values of x in the range $0° < x < 360°$ that satisfy (5)

$$\sin x = -4\cos x.$$

Solution

$$\begin{aligned} \sin x = -4\cos x \Rightarrow & \frac{\sin x}{\cos x} = -4 \\ \Rightarrow & \tan x = -4 \\ \Rightarrow & x = 104.036\,243\,5,\ 284.036\,243\,5 \text{ (FCD)} \\ \Rightarrow & \underline{\underline{x = 104,\ 284 \text{ (3 sf)}}}. \end{aligned}$$

5. A car is travelling along a motorway at 30 ms^{-1}. At the moment that it passes a point A the brakes are applied so that the car decelerates with constant deceleration. When it reaches a point B, where $AB = 300$ m, the speed of the car is 10 ms^{-1}.

Calculate

(a) the constant deceleration, (3)

Solution

$s = 300$, $u = 30$, $v = 10$, $a = ?$, $t = ?$: we use $v^2 = u^2 + 2as$

$$\begin{aligned} 10^2 = 30^2 + 2 \times a \times 300 &\Rightarrow 100 = 900 + 600a \\ &\Rightarrow 600a = -800 \\ &\Rightarrow a = -1\tfrac{1}{3}; \end{aligned}$$

hence, the constant deceleration is $\underline{\underline{1\frac{1}{3}\ \mathrm{ms}^{-2}}}$.

(b) the time taken to travel from A to B. (2)

Solution

We use $v = u + at$:

$$\begin{aligned} 10 = 30 + (-1\tfrac{1}{3})t &\Rightarrow -\tfrac{4}{3}t = -20 \\ &\Rightarrow t = \underline{\underline{15\ \mathrm{s}}}. \end{aligned}$$

6. Find the equation of the tangent to the curve (4)

$$y = x^3 - 3x + 4$$

at the point $(2, 6)$.

Solution

$$y = x^3 - 3x + 4 \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = 3x^2 - 3$$

and

$$x = 2 \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = 9.$$

Hence, the equation of the tangent is

$$\begin{aligned} y - 6 = 9(x - 2) &\Rightarrow y - 6 = 9x - 18 \\ &\Rightarrow \underline{\underline{y = 9x - 12}}. \end{aligned}$$

7. Use calculus to find the x-coordinate of the minimum point on the curve (7)

$$y = x^3 - 2x^2 - 15x + 30.$$

Show your working clearly, giving the reasons for your answer.

Solution

$$y = x^3 - 2x^2 - 15x + 30 \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = 3x^2 - 4x - 15.$$

Now,

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 0 \Rightarrow 3x^2 - 4x - 15 = 0$$

$$\left.\begin{array}{ll}\text{add to:} & -4 \\ \text{multiply to:} & (+3) \times (-15) = -45\end{array}\right\} -9,\ +5$$

$$\begin{aligned}&\Rightarrow 3x^2 - 9x + 5x - 15 = 0\\ &\Rightarrow 3x(x-3) + 5(x-3) = 0\\ &\Rightarrow (3x+5)(x-3) = 0\\ &\Rightarrow 3x + 5 = 0 \text{ or } x - 3 = 0\\ &\Rightarrow x = -\tfrac{5}{3} \text{ or } x = 3.\end{aligned}$$

Now,

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 3x^2 - 4x - 15 \Rightarrow \frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 6x - 4.$$

Next,

$$x = -\tfrac{5}{3} \Rightarrow \frac{\mathrm{d}^2y}{\mathrm{d}x^2} = -14$$

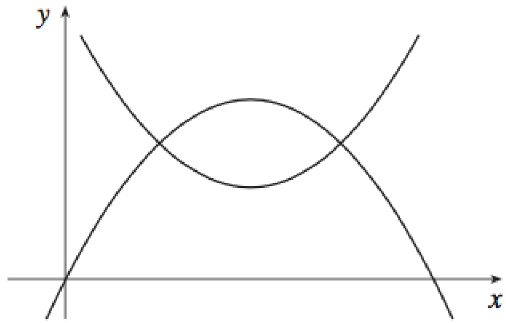
and this is a local maximum but

$$x = 3 \Rightarrow \frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 14$$

and this is a local minimum.

8. The figure shows the graphs of

$$y = 4x - x^2 \text{ and } y = x^2 - 4x + 6.$$



(a) Use an algebraic method to find the x-coordinates of the points where the curves intersect. (3)

Solution

$$4x - x^2 = x^2 - 4x + 6 \Rightarrow 2x^2 - 8x + 6 = 0$$
$$\Rightarrow 2(x^2 - 4x + 3) = 0$$

$$\left.\begin{array}{ll}\text{add to:} & -4 \\ \text{multiply to:} & +3\end{array}\right\} -3,\ -1$$

$$\Rightarrow 2(x-1)(x-3) = 0$$
$$\Rightarrow \underline{\underline{x = 1 \text{ or } x = 3}}.$$

(b) Calculate the area enclosed by the two curves. (4)

Solution

$$\begin{aligned}\text{Area} &= (\text{area beneath } y = 4x - x^2) - (\text{area beneath } x^2 - 4x + 6) \\ &= \int_1^3 [(4x - x^2) - (x^2 - 4x + 6)]\,\mathrm{d}x \\ &= \int_1^3 (-6 + 8x - 2x^2)\,\mathrm{d}x \\ &= \left[-6x + 4x^2 - \tfrac{2}{3}x^3\right]_{x=1}^3 \\ &= (-18 + 36 + 5\tfrac{1}{3}) - (-6 + 4 - \tfrac{2}{3}) \\ &= \underline{\underline{2\tfrac{2}{3}}}.\end{aligned}$$

9. The points A, B, and C have coordinates $(-1, 1)$, $(5, 8)$, and $(8, 3)$ respectively.

(a) Show that $AC = AB$. (2)

Solution

$$\begin{aligned} AC &= \sqrt{[8-(-1)]^2 + (3-1)^2} \\ &= \sqrt{81+4} \\ &= \sqrt{85} \end{aligned}$$

and

$$\begin{aligned} AB &= \sqrt{[(5-(-1)]^2 + (8-1)^2} \\ &= \sqrt{36+49} \\ &= \sqrt{85}; \end{aligned}$$

thus, $\underline{\underline{AC = AB}}$.

(b) Write down the coordinates of M, the midpoint of BC. (1)

Solution

$$M = \left(\frac{5+8}{2}, \frac{8+3}{2}\right) = \underline{\underline{(6\tfrac{1}{2}, 5\tfrac{1}{2})}}.$$

(c) Show that the lines BC and AM are perpendicular. (2)

Solution

$$\begin{aligned} \text{Gradient of } BC &= \frac{3-8}{8-5} \\ &= -\tfrac{5}{3} \end{aligned}$$

and

$$\begin{aligned} \text{gradient of } AM &= \frac{5\frac{1}{2}-1}{6\frac{1}{2}-(-1)} \\ &= \frac{4\frac{1}{2}}{7\frac{1}{2}} \\ &= \tfrac{3}{5}. \end{aligned}$$

As

$$\text{grad}_{BC} \times \text{grad}_{AM} = -1,$$

BC and AM are $\underline{\underline{\text{perpendicular}}}$.

(d) Find the equation of the line AM. (2)

Solution

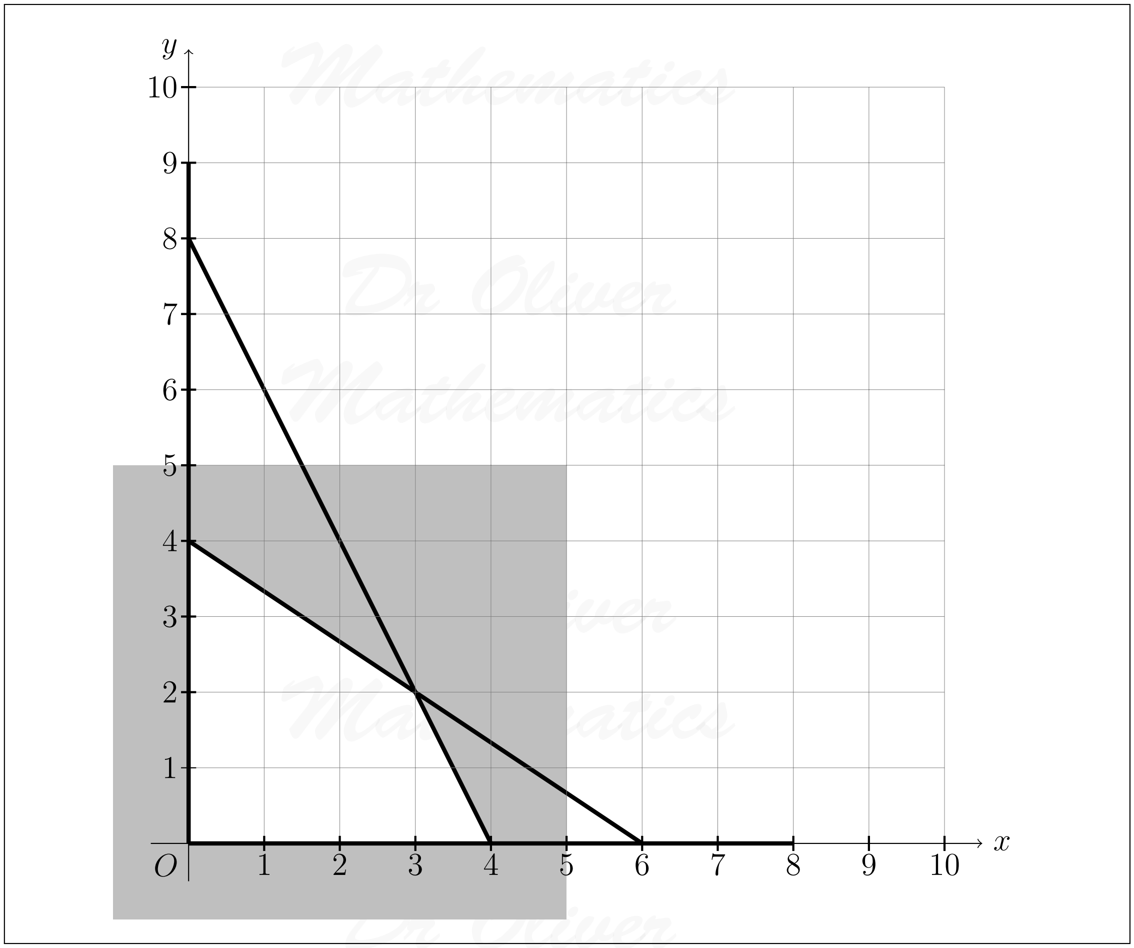
The equation of the line AM is

$$y - 1 = \tfrac{3}{5}(x+1) \Rightarrow y - 1 = \tfrac{3}{5}x + \tfrac{3}{5}$$
$$\Rightarrow \underline{\underline{y = \tfrac{3}{5}x + 1\tfrac{3}{5}}}.$$

10. (a) By drawing suitable graphs on the same axes, indicate the region for which the following inequalities hold. You should shade the region which is **not** required. (5)

$$2x + 3y \leqslant 12$$
$$2x + y \leqslant 8$$
$$y \geqslant 0$$
$$x \geqslant 0.$$

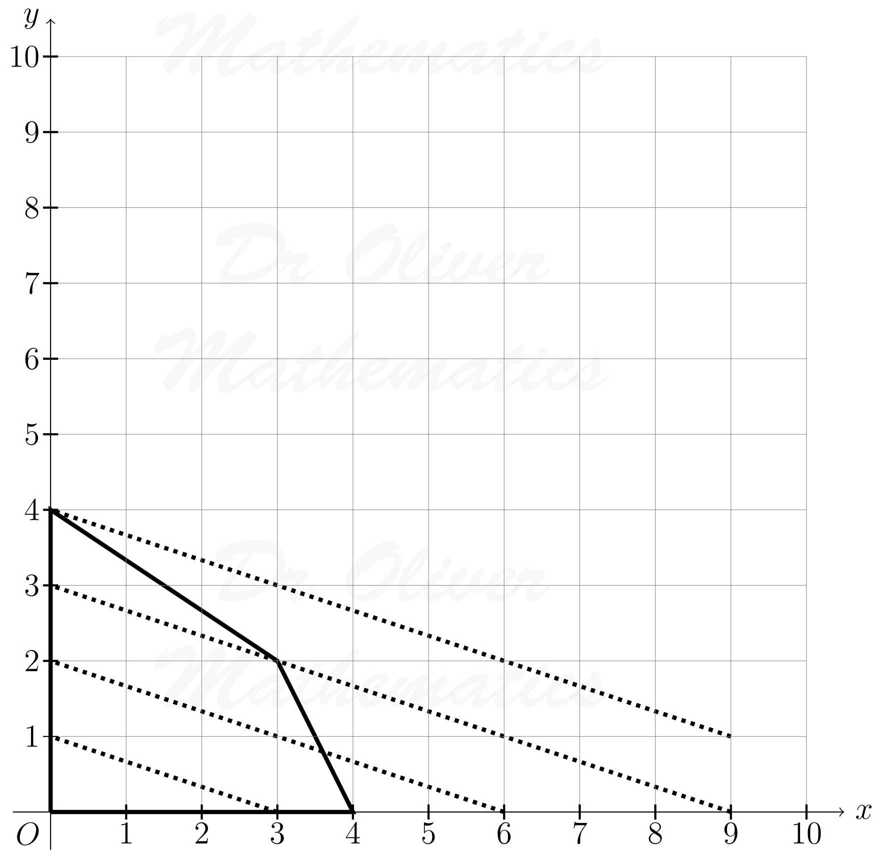
Solution



(b) Find the maximum value of $x + 3y$ subject to these conditions. (2)

Solution

Two obvious methods. The first is to go around the polygon and determine $x + 3y$ at its vertices. The second is to go $x + 3y = ?$ and translate this line out and upwards: $x + 3y = 0$, $x + 3y = 0.5$, $x + 3y = 1$, $\ldots$, until we reach the very last point on its vertices.



$x = 0, y = 4 \Rightarrow x + 3y = \underline{\underline{12}}$.

Section B

11. (a) You are given that

$$f(x) = x^3 - 3x^2 - 4x.$$

(i) Find the three points where the curve $y = f(x)$ cuts the x-axis. (4)

Solution

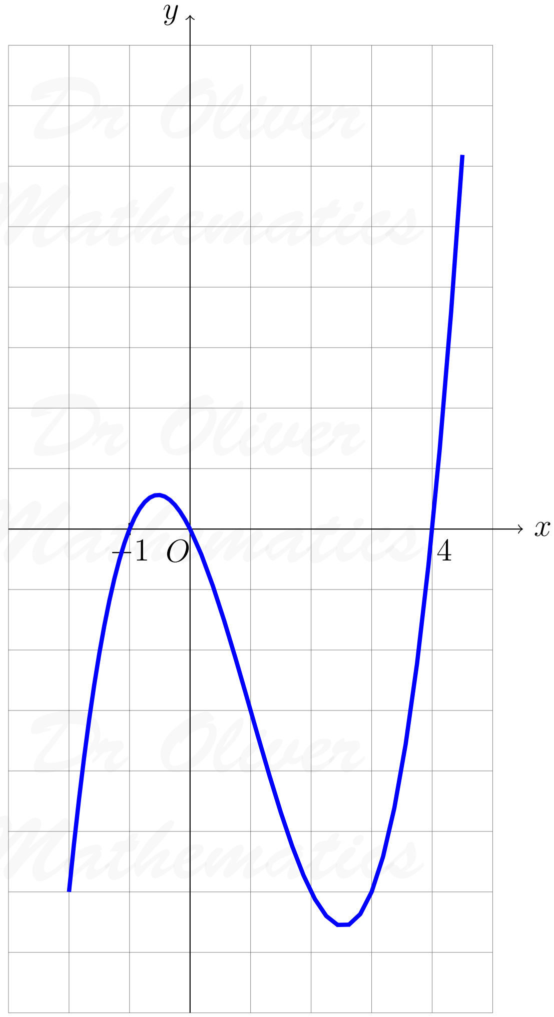
$$x^3 - 3x^2 - 4x = 0 \Rightarrow x(x^2 - 3x - 4) = 0$$

$$\left.\begin{array}{ll}\text{add to:} & -4 \\ \text{multiply to:} & -3\end{array}\right\} -4,\ +1$$

$$\Rightarrow x(x-4)(x+1) = 0$$
$$\Rightarrow \underline{\underline{x = -1,\ x = 0,\ \text{ or } x = 4}}.$$

(ii) Sketch the graph of $y = \mathrm{f}(x)$. (1)

Solution



(b) You are given that

$$\mathrm{g}(x) = x^3 - 3x^2 - 4x + 12.$$

(i) Find the remainder when $g(x)$ is divided by $(x+1)$ (2)

Solution
We use synthetic division:

-1	1	-3	-4	12
	$\downarrow$	-1	-3	0
	1	-4	0	12

Hence, there is a remainder of 12.

(ii) Show that $(x-2)$ is a factor of $g(x)$. (1)

Solution

2	1	-3	-4	12
	$\downarrow$	2	-2	-12
	1	-1	-6	0

Hence, because there is no remainder, $(x-2)$ is a factor of $x^3-3x^2-4x+12$.

(iii) Hence solve the equation $g(x)=0$. (4)

Solution

$$x^3-3x^2-4x+12=0 \Rightarrow (x-2)(x^2-x-6)=0$$

$$\left.\begin{array}{ll}\text{add to:} & -1 \\ \text{multiply to:} & -6\end{array}\right\} -3,\ +2$$

$$\Rightarrow (x-2)(x-3)(x+2)=0$$
$$\Rightarrow x=-2,\ x=2,\ \text{or } x=3.$$

12. The work-force of a large company is made up of males and females in the ratio 9 : 11. One third of the male employees work part-time and one half of the female employees work part-time.

8 employees are chosen at random.

Find the probability that

(a) all are males, (2)

Solution

$$\begin{aligned}\text{P(all are males)} &= (\tfrac{9}{20})^8 \\ &= 1.681\,512\,539 \times 10^{-3} \text{ (FCD)} \\ &= \underline{\underline{1.68 \times 10^{-3} \text{ (3 sf)}}}.\end{aligned}$$

(b) exactly 5 are females, (4)

Solution

$$\begin{aligned}\text{P(exactly 5 are females)} &= \binom{8}{5}(\tfrac{11}{20})^5(\tfrac{9}{20})^3 \\ &= 0.256\,826\,016\,6 \text{ (FCD)} \\ &= \underline{\underline{0.257 \text{ (3 sf)}}}.\end{aligned}$$

(c) at least 2 work part-time. (6)

Solution

Well,

$$\begin{aligned}M_{PT} : M_{FT} : W_{PT} : W_{FT} &= 3 : 6 : 5.5 : 5.5 \\ &= 6 : 12 : 11 : 11\end{aligned}$$

and so

$$PT : FT = 17 : 23.$$

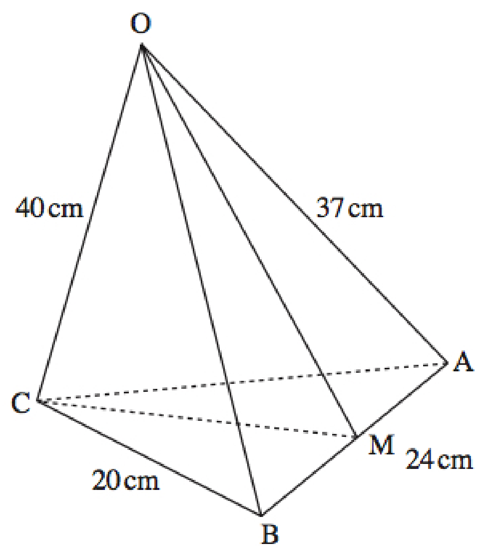
Hence,

$$\begin{aligned}&\text{P(at least 2 work part-time)} \\ = \;& 1 - \text{P(0 work part-time)} - \text{P(1 works part-time)} \\ = \;& 1 - (\tfrac{23}{40})^8 - \binom{8}{1}(\tfrac{23}{40})^7(\tfrac{17}{40}) \\ = \;& 0.917\,393\,913\,9 \text{ (FCD)} \\ = \;& \underline{\underline{0.917 \text{ (3 sf)}}}.\end{aligned}$$

13. In the pyramid $OABC$,

- $OA = OB = 37$ cm,
- $OC = 40$ cm,
- $CA = CB = 20$ cm, and
- $AB = 24$ cm

M is the midpoint of AB.



Calculate

(a) the lengths OM and CM, (3)

Solution

$$\begin{aligned} OM &= \sqrt{37^2 - 12^2} \\ &= \sqrt{1\,225} \\ &= \underline{\underline{35 \text{ cm}}} \end{aligned}$$

and

$$\begin{aligned} CM &= \sqrt{20^2 - 12^2} \\ &= \sqrt{256} \\ &= \underline{\underline{16 \text{ cm}}}. \end{aligned}$$

(b) the angle between the line OC and the plane ABC, (4)

Solution

$$\cos OCM = \frac{40^2 + 16^2 - 35^2}{2 \times 40 \times 16} \Rightarrow \cos OCM = \tfrac{631}{1\,280}$$
$$\Rightarrow \angle OCM = 60.464\,103\,62 \text{ (FCD)}$$
$$\Rightarrow \underline{\underline{\angle OCM = 60.5^\circ \text{ (3 sf)}}}.$$

(c) the volume of the pyramid. (5)

Solution

$$\text{Height of the pyramid} = 40 \sin 60.464\ldots^\circ.$$

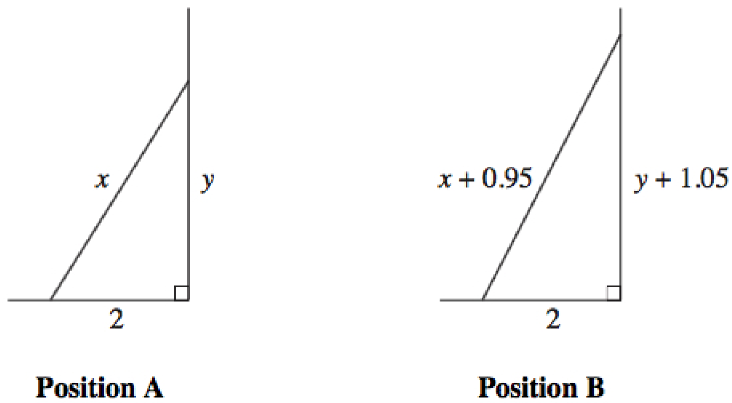
Now,

$$\begin{aligned} \text{area of the base} &= \tfrac{1}{2} \times 16 \times 24 \\ &= 192 \text{ cm}^2 \end{aligned}$$

and, finally,

$$\begin{aligned} \text{volume of the pyramid} &= \tfrac{1}{3} Ah \\ &= \tfrac{1}{3} \times 192 \times 40 \sin 60.464\ldots \\ &= 2\,227.320\,363 \text{ (FCD)} \\ &= \underline{\underline{2\,230 \text{ cm}^3 \text{ (3 sf)}}}. \end{aligned}$$

14. An extending ladder has two positions. In position A, the length of the ladder is x metres and, when the foot of the ladder is placed 2 metres from the base of a vertical wall, the ladder reaches y metres up the wall.



In position B, the ladder is extended by 0.95 metres and it reaches an extra 1.05 metres up the wall.

The foot of the ladder remains 2 m from the base of the wall.

(a) Use Pythagoras' theorem for position A and position B to write down two equations in x and y. (2)

Solution

For A,

$$\underline{\underline{2^2 + y^2 = x^2}}$$

and, for B,

$$\underline{\underline{2^2 + (y + 1.05)^2 = (x + 0.95)^2}}.$$

(b) Hence show that (3)

$$2.1y = 1.9x - 0.2.$$

Solution

$$\begin{aligned} & 2^2 + (y + 1.05)^2 = (x + 0.95)^2 \\ \Rightarrow\ & 2^2 + (y^2 + 2.1y + 1.1025) = (x^2 + 1.9x + 0.9025) \\ \Rightarrow\ & 2.1y + 1.1025 = 1.9x + 0.9025 \\ \Rightarrow\ & \underline{\underline{2.1y = 1.9x - 0.2}}, \end{aligned}$$

as required.

(c) Using these equations, form a quadratic equation in x. (7)
Hence find the values of x and y.

Solution

$$\begin{aligned}
2^2 + y^2 = x^2 &\Rightarrow 4 + [\tfrac{1}{2.1}(1.9x - 0.2)]^2 = x^2 \\
&\Rightarrow 4 + \tfrac{1}{4.41}(3.61x^2 - 0.76x + 0.04) = x^2 \\
&\Rightarrow 4 + (\tfrac{361}{441}x^2 - \tfrac{76}{441}x + \tfrac{4}{441}) = x^2 \\
&\Rightarrow \tfrac{80}{441}x^2 + \tfrac{76}{441}x - 4\tfrac{4}{441} = 0 \\
&\Rightarrow 80x^2 + 76x - 1\,768 = 0 \\
&\Rightarrow 4(20x^2 + 19x - 442) = 0
\end{aligned}$$

$$\left.\begin{array}{ll}
\text{add to:} & +19 \\
\text{multiply to:} & (+20) \times (-442) = -8\,840
\end{array}\right\} -85,\ +104$$

$$\begin{aligned}
&\Rightarrow 4[20x^2 - 85x + 104x - 442] = 0 \\
&\Rightarrow 4[5x(4x - 17) + 26(4x - 17)] = 0 \\
&\Rightarrow 4(5x + 26)(4x - 17) = 0 \\
&\Rightarrow 5x + 26 = 0 \text{ or } 4x - 17 = 0 \\
&\Rightarrow x = -5\tfrac{1}{5} \text{ or } x = 4\tfrac{1}{4};
\end{aligned}$$

as $x > 0$, $\underline{\underline{x = 4\tfrac{1}{4} \text{ and } y = 3\tfrac{3}{4}}}$.