Dr Oliver Mathematics
Cambridge O Level Additional Mathematics
2005 November Paper 2: Calculator
2 hours

The total number of marks available is 80.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You must write down all the stages in your working.

1. Variables V and t are related by the equation

$$V = 1\,000\mathrm{e}^{-kt},$$

where k is a constant.

Given that $V = 500$ when $t = 21$, find

(a) the value of k, (2)

(b) the value of V when $t = 30$. (2)

2. The line (5)

$$x + y = 10$$

meets the curve

$$y^2 = 2x + 4$$

at the points A and B.

Find the coordinates of the mid-point of AB.

3. (a) Given that (2)

$$y = 1 + \ln(2x - 3),$$

obtain an expression for $\dfrac{\mathrm{d}y}{\mathrm{d}x}$.

(b) Hence find, in terms of p, the approximate value of y when $x = 2 + p$, where p is small. (3)

4. The function f is given by

$$\mathrm{f} : x \mapsto 2 + 5\sin 3x, \text{ for } 0^\circ \leqslant x \leqslant 180^\circ.$$

(a) State the amplitude and period of f. (2)

(b) Sketch the graph of $y = \mathrm{f}(x)$. (3)

5. The binomial expansion of (6)

$$(1 + px)^n,$$

where $n > 0$, in ascending powers of x is

$$1 - 12x + 28p^2x^2 + qx^3 + \ldots.$$

Find the value of n, of p, and of q.

6. It is given that (6)

$$\mathbf{A} = \begin{pmatrix} 3 & 1 \\ 5 & p \end{pmatrix}$$

and that

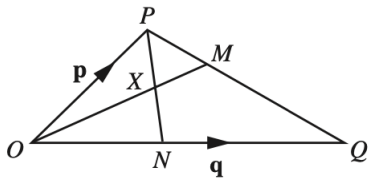
$$\mathbf{A} + \mathbf{A}^{-1} = k\mathbf{I},$$

where p and k are constants and $\mathbf{I}$ is the identity matrix.

Evaluate p and k.

7. In the diagram,

- $\overrightarrow{OP} = \mathbf{p}$,
- $\overrightarrow{OQ} = \mathbf{q}$,
- $\overrightarrow{PM} = \frac{1}{3}\overrightarrow{PQ}$, and
- $\overrightarrow{ON} = \frac{2}{5}\overrightarrow{OQ}$.



(a) Given that $\overrightarrow{OX} = m\overrightarrow{OM}$, express $\overrightarrow{OX}$ in terms of m, $\mathbf{p}$, and $\mathbf{q}$. (2)

(b) Given that $\overrightarrow{PX} = n\overrightarrow{PN}$, express $\overrightarrow{OX}$ in terms of n, $\mathbf{p}$, and $\mathbf{q}$. (3)

(c) Hence evaluate m and n. (2)

8. (a) Find the value of each of the integers p and q for which (2)

$$\left(\frac{25}{16}\right)^{-\frac{3}{2}} = 2^p \times 5^q.$$

(b) (i) Express the equation (2)

$$4^x - 2^{x+1} = 3,$$

as a quadratic equation in 2^x.

(ii) Hence find the value of x, correct to 2 decimal places. (3)

9. The function

$$\mathrm{f}(x) = x^3 - 6x^2 + ax + b,$$

where a and b are constants, is exactly divisible by $(x - 3)$ and leaves a remainder of -55 when divided by $(x + 2)$.

(a) Find the value of a and of b. (4)

(b) Solve the equation $\mathrm{f}(x) = 0$. (4)

10. A curve is such that

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 6x - 2.$$

The gradient of the curve at the point $(2, -9)$ is 3.

(a) Express y in terms of x. (5)

(b) Show that the gradient of the curve is never less than $-\frac{16}{3}$. (3)

11. Each day a newsagent sells copies of 10 different newspapers, one of which is *The Times*.

A customer buys 3 different newspapers.

Calculate the number of ways the customer can select his newspapers

(a) (i) if there is no restriction, (1)

(ii) if 1 of the 3 newspapers is *The Times*. (1)

(b) (i) Calculate the number of different 5-digit numbers which can be formed using the digits 0, 1, 2, 3, and 4 without repetition and assuming that a number cannot begin with 0. (2)

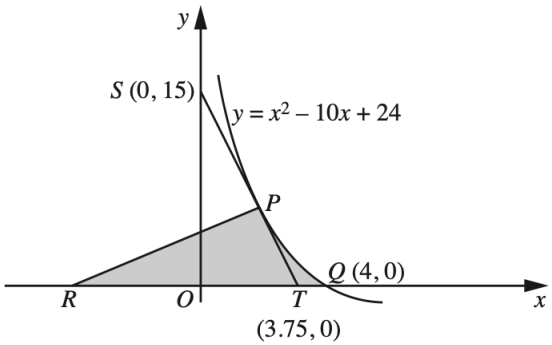
(ii) How many of these 5-digit numbers are even? (4)

EITHER

12. The diagram, which is not drawn to scale, shows part of the curve

$$y = x^2 - 10x + 24$$

cutting the x-axis at $Q(4, 0)$.



The tangent to the curve at the point P on the curve meets the coordinate axes at $S(0, 15)$ and at $T(3.75, 0)$.

(a) Find the coordinates of P. (4)

The normal to the curve at P meets the x-axis at R.

(b) Find the coordinates of R. (2)

(c) Calculate the area of the shaded region bounded by the x-axis, the line PR, and the curve PQ. (5)

OR

13. A curve has the equation

$$y = 2\cos x - \cos 2x,$$

where $0 < x \leqslant \frac{1}{2}\pi$.

(a) Obtain expressions for $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ and $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}$. (4)

(b) Given that $\sin 2x$ may be expressed as $2\sin x \cos x$, find the x-coordinate of the stationary point of the curve and determine the nature of this stationary point. (4)

(c) Evaluate (3)

$$\int_{\frac{1}{3}\pi}^{\frac{1}{2}\pi} y \,\mathrm{d}x.$$