Dr Oliver Mathematics
Further Mathematics: Core Pure Mathematics 1
June 2022: Calculator
1 hour 30 minutes

The total number of marks available is 75.
You must write down all the stages in your working.
Inexact answers should be given to three significant figures unless otherwise stated.

1.
$$f(z) = z^3 + az + 52, \text{ where } a \text{ is a real constant.}$$

Given that $2 - 3\text{i}$ is a root of the equation $f(z) = 0$,

(a) write down the other complex root. (1)

Solution
$\underline{\underline{z = 2 + 3\text{i}}}$.

(b) Hence, (4)

(i) solve completely $f(z) = 0$,

Solution

$\times$	z	-2	-3i
z	z^2	$-2z$	$-3z\text{i}$
-2	$-2z$	$+4$	$+6\text{i}$
$+3\text{i}$	$+3z\text{i}$	-6i	$+9$

So
$$[z - (2 + 3\text{i})][z - (2 - 3\text{i})] = z^2 - 4z + 13.$$

Now,
$$\frac{z^3}{z^2} = z \text{ and } \frac{+52}{+13} = 4$$

and we will see that the final expression is $(x + 4)$. So

$$f(z) = [z - (2 + 3\text{i})][z - (2 - 3\text{i})](x + 4)$$

and the roots are
$$\underline{\underline{z = 2 \pm 3\text{i}, \ -4}}.$$

Next,

(ii) determine the value of a.

Solution

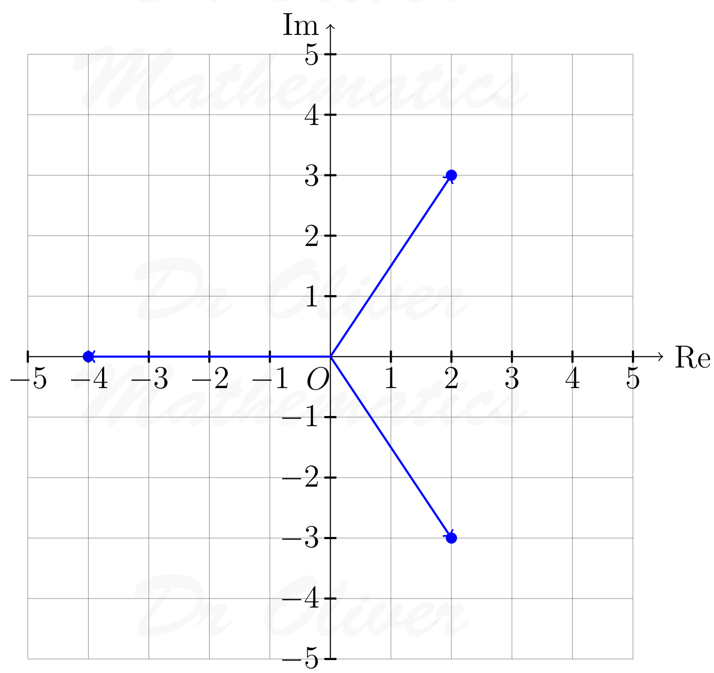
$\times$	z^2	$-4z$	$+13$
z	z^3	$-4z^2$	$+13z$
$+4$	$+4z^2$	$-16z$	$+52$

Finally,

$$a = 13 - 16 = \underline{\underline{-3}}.$$

(c) Show all the roots of the equation $\mathrm{f}(z) = 0$ on a single Argand diagram. (1)

Solution



2. **In this question you must show all stages of your working.** (4)
Solutions relying entirely on calculator technology are not acceptable.

Determine the values of x for which

$$64\cosh^4 x - 64\cosh^2 x - 9 = 0.$$

Give your answers in the form $q\ln 2$ where q is rational and in simplest form.

Solution

$$\left.\begin{array}{ll}\text{add to:} & -64 \\ \text{multiply to:} & (+64)\times(-9) = -576\end{array}\right\} -72,\ +8$$

E.g.,

$$\begin{aligned} 64\cosh^4 x - 64\cosh^2 x - 9 = 0 &\Rightarrow 64\cosh^4 x - 72\cosh^2 x + 8\cosh^2 x - 9 = 0 \\ &\Rightarrow 8\cosh^2 x(8\cosh^2 x - 9)x + 1(8\cosh^2 x - 9) = 0 \\ &\Rightarrow (8\cosh^2 x + 1)(8\cosh^2 x - 9) = 0 \end{aligned}$$

but $\cosh x > 0$!

$$\begin{aligned} &\Rightarrow 8\cosh^2 x - 9 = 0 \\ &\Rightarrow 8\cosh^2 x = 9 \\ &\Rightarrow \cosh^2 x = \tfrac{9}{8} \\ &\Rightarrow \cosh x = \pm\tfrac{3\sqrt{2}}{4} \end{aligned}$$

but $\cosh x > 0$!

$$\begin{aligned} &\Rightarrow \cosh x = \tfrac{3\sqrt{2}}{4} \\ &\Rightarrow x = \pm\ln\left[\tfrac{3\sqrt{2}}{4} + \sqrt{(\tfrac{3\sqrt{2}}{4})^2 - 1}\right] \\ &\Rightarrow x = \pm\ln\left[\tfrac{3\sqrt{2}}{4} + \sqrt{\tfrac{9}{8} - 1}\right] \\ &\Rightarrow x = \pm\ln\left[\tfrac{3\sqrt{2}}{4} + \sqrt{\tfrac{1}{8}}\right] \\ &\Rightarrow x = \pm\ln\left[\tfrac{3\sqrt{2}}{4} + \tfrac{\sqrt{2}}{4}\right] \\ &\Rightarrow x = \pm\ln\sqrt{2} \\ &\Rightarrow \underline{\underline{x = \pm\tfrac{1}{2}\ln 2}}. \end{aligned}$$

3. (a) Determine the general solution of the differential equation (3)

$$\cos x\frac{\mathrm{d}y}{\mathrm{d}x} + y\sin x = e^{2x}\cos^2 x,$$

giving your answer in the form $y = \mathrm{f}(x)$.

Solution

$$\begin{aligned} \cos x \frac{\mathrm{d}y}{\mathrm{d}x} + y \sin x &= \mathrm{e}^{2x} \cos^2 x \\ \Rightarrow \quad \frac{\mathrm{d}y}{\mathrm{d}x} + y \tan x &= e^{2x} \cos x. \end{aligned}$$

Now,

$$\begin{aligned} \mathrm{IF} &= \mathrm{e}^{\int \tan x \, \mathrm{d}x} \\ &= \mathrm{e}^{\ln \sec x} \\ &= \sec x \end{aligned}$$

and

$$\begin{aligned} \frac{\mathrm{d}y}{\mathrm{d}x} + y \tan x = \mathrm{e}^{2x} \cos x &\Rightarrow \sec x \frac{\mathrm{d}y}{\mathrm{d}x} + y \sec x \tan x = \mathrm{e}^{2x} \\ &\Rightarrow \frac{\mathrm{d}}{\mathrm{d}x}(y \sec x) = \mathrm{e}^{2x} \\ &\Rightarrow y \sec x = \int \mathrm{e}^{2x} \, \mathrm{d}x \\ &\Rightarrow y \sec x = \tfrac{1}{2}\mathrm{e}^{2x} + c \\ &\Rightarrow \underline{\underline{y = \cos x(\tfrac{1}{2}\mathrm{e}^{2x} + c)}}. \end{aligned}$$

Given that $y = 3$ when $x = 0$,

(b) determine the smallest positive value of x for which $y = 0$. (3)

Solution

$$\begin{aligned} x = 0, y = 3 \Rightarrow 3 &= \tfrac{1}{2} + c \\ \Rightarrow c &= \tfrac{5}{2} \end{aligned}$$

and so

$$y = \cos x(\tfrac{1}{2}\mathrm{e}^{2x} + \tfrac{5}{2}).$$

Now,

$$\begin{aligned} y = 0 &\Rightarrow \cos x(\tfrac{1}{2}\mathrm{e}^{2x} + \tfrac{5}{2}) = 0 \\ &\Rightarrow \cos x = 0 \end{aligned}$$

as $\frac{1}{2}e^{2x} + \frac{5}{2} > 0$

$$\Rightarrow \underline{\underline{x = \tfrac{1}{2}\pi}}.$$

4. (a) Use the method of differences to prove that for $n > 2$, (4)

$$\sum_{r=2}^{n} \ln\left(\frac{r+1}{r-1}\right) \equiv \ln\left(\frac{n(n+1)}{2}\right).$$

Solution

$$\begin{aligned} & \sum_{r=2}^{n} \ln\left(\frac{r+1}{r-1}\right) \\ \equiv\ & \sum_{r=2}^{n} [\ln(r+1) - \ln(r-1)] \\ \equiv\ & (\ln 3 - \ln 1) + (\ln 4 - \ln 2) + \ldots \\ & \qquad + [\ln n - \ln(n-2)] + [\ln(n+1) - \ln(n-1)] \\ \equiv\ & \ln(n+1) + \ln n - \ln 2 \\ \equiv\ & \underline{\underline{\ln\left(\frac{n(n+1)}{2}\right)}}, \end{aligned}$$

as $\ln 1 = 0$.

(b) Hence find the exact value of (3)

$$\sum_{r=51}^{100} \ln\left(\frac{r+1}{r-1}\right).$$

Give your answer in the form $a \ln\left(\frac{b}{c}\right)$, where a, b, and c are integers to be determined.

Solution

$$\begin{aligned}
\sum_{r=51}^{100} \ln\left(\frac{r+1}{r-1}\right)^{35} &= 35 \sum_{r=51}^{100} \ln\left(\frac{r+1}{r-1}\right) \\
&= 35\left\{\sum_{r=2}^{100} \ln\left(\frac{r+1}{r-1}\right) - \sum_{r=2}^{50} \ln\left(\frac{r+1}{r-1}\right)\right\} \\
&= 35\left\{\ln\left(\frac{100(100+1)}{2}\right) - \ln\left(\frac{50(50+1)}{2}\right)\right\} \\
&= 35\left\{\ln(5\,050) - \ln(1\,275)\right\} \\
&= 35 \ln\left(\frac{5\,050}{1\,275}\right) \\
&= \underline{\underline{35 \ln\left(\tfrac{202}{51}\right)}};
\end{aligned}$$

hence, $\underline{\underline{a = 35}}$, $\underline{\underline{b = 202}}$, and $\underline{\underline{c = 51}}$.

5.

$$\mathbf{M} = \begin{pmatrix} a & 2 & -3 \\ 2 & 3 & 0 \\ 4 & a & 2 \end{pmatrix},$$

where a is a constant.

(a) Show that $\mathbf{M}$ is non-singular for all values of a. (2)

Solution

$$\begin{aligned}
\det \mathbf{M} &= a(6-0) - 2(4-0) - 3(2a-12) \\
&= 6a - 8 - 6a + 36 \\
&= 28 \\
&\neq 0;
\end{aligned}$$

hence, $\mathbf{M}$ is $\underline{\underline{\text{non-singular}}}$ for all values of a.

(b) Determine, in terms of a, $\mathbf{M}^{-1}$. (4)

Solution

Matrix of minors:

$$\begin{pmatrix} 6 & 4 & 2a-12 \\ 3a+4 & 2a+12 & a^2-8 \\ 9 & 6 & 3a-4 \end{pmatrix}$$

Matrix of cofactors:

$$\begin{pmatrix} 6 & -4 & 2a-12 \\ -3a-4 & 2a+12 & -a^2+8 \\ 9 & -6 & 3a-4 \end{pmatrix}$$

Transpose:

$$\begin{pmatrix} 6 & -3a-4 & 9 \\ -4 & 2a+12 & -6 \\ 2a-12 & -a^2+8 & 3a-4 \end{pmatrix}$$

Inverse:

$$\mathbf{M}^{-1} = \underline{\underline{\tfrac{1}{28}\begin{pmatrix} 6 & -3a-4 & 9 \\ -4 & 2a+12 & -6 \\ 2a-12 & -a^2+8 & 3a-4 \end{pmatrix}}}.$$

6. (a) Express as partial fractions (3)

$$\frac{2x^2+3x+6}{(x+1)(x^2+4)}.$$

Solution

$$\begin{aligned}\frac{2x^2+3x+6}{(x+1)(x^2+4)} &\equiv \frac{A}{(x+1)} + \frac{Bx+C}{(x^2+4)} \\ &\equiv \frac{A(x^2+4)+(Bx+C)(x+1)}{(x^2+4)}\end{aligned}$$

which means

$$2x^2+3x+6 \equiv A(x^2+4)+(Bx+C)(x+1).$$

$\underline{x=-1}$: $5 = 5A \Rightarrow A = 1$.
$\underline{x=0}$: $6 = 4 + C \Rightarrow C = 2$.
$\underline{x=1}$: $11 = 5 + 2B + 4 \Rightarrow B = 1$.
Hence,

$$\frac{2x^2+3x+6}{(x+1)(x^2+4)} = \underline{\underline{\frac{1}{(x+1)} + \frac{x+2}{(x^2+4)}}}.$$

(b) Hence, show that (4)

$$\int_0^2 \frac{2x^2+3x+6}{(x+1)(x^2+4)}\,\mathrm{d}x = \ln(a\sqrt{2}) + b\pi,$$

where a and b are constants to be determined.

Solution

$$\begin{aligned}
\int_0^2 \frac{2x^2+3x+6}{(x+1)(x^2+4)}\,\mathrm{d}x &= \int_0^2 \left(\frac{1}{x+1} + \frac{x}{x^2+4} + \frac{2}{x^2+4}\right)\mathrm{d}x \\
&= \left[\ln|x+1| + \tfrac{1}{2}\ln|x^2+4| + \arctan(\tfrac{1}{2}x)\right]_{x=0}^{2} \\
&= \left(\ln 3 + \tfrac{1}{2}\ln 8 + \arctan(\tfrac{1}{2})\right) - \left(0 + \tfrac{1}{2}\ln 4 + 0\right) \\
&= \ln 3 + \tfrac{1}{2}\ln 2^3 + \tfrac{1}{4}\pi - \tfrac{1}{2}\ln 4 \\
&= \ln 3 + \tfrac{3}{2}\ln 2 + \tfrac{1}{4}\pi - \tfrac{1}{2}\ln 2^2 \\
&= \ln 3 + \tfrac{3}{2}\ln 2 + \tfrac{1}{4}\pi - \ln 2 \\
&= \ln 3 + \tfrac{1}{2}\ln 2 + \tfrac{1}{4}\pi \\
&= \ln 3 + \ln\sqrt{2} + \tfrac{1}{4}\pi \\
&= \underline{\underline{\ln(3\sqrt{2}) + \tfrac{1}{4}\pi}},
\end{aligned}$$

hence, $\underline{\underline{a = 3}}$ and $\underline{\underline{b = \frac{1}{4}}}$.

7. Given that $z = a + b\mathrm{i}$ is a complex number where a and b are real constants,

(a) show that $zz^\star$ is a real number. (2)

Solution

Well, $z^\star = a - b\mathrm{i}$ and we have

$\times$	a	$+b\mathrm{i}$
a	a^2	$+ab\mathrm{i}$
$-b\mathrm{i}$	$-ab\mathrm{i}$	$+b^2$

Finally,

$$zz^\star = a^2 + b^2$$

is a <u>real number</u>.

Given that

- $zz^\star = 18$ and
- $\dfrac{z}{z^\star} = \frac{7}{9} + \frac{4\sqrt{2}}{9}\mathrm{i}$,

(b) determine the possible complex numbers z. (5)

Solution

If $z = a + b\mathrm{i}$, then

$$a^2 + b^2 = 18 \quad (1)$$

and

$$\begin{aligned}\tfrac{7}{9} + \tfrac{4\sqrt{2}}{9}\mathrm{i} &= \frac{z}{z^\star} \\ &= \frac{a+b\mathrm{i}}{a-b\mathrm{i}} \\ &= \frac{a+b\mathrm{i}}{a-b\mathrm{i}} \times \frac{a+b\mathrm{i}}{a+b\mathrm{i}} \\ &= \frac{(a^2-b^2)+2ab\mathrm{i}}{a^2+b^2} \\ &= \frac{(a^2-b^2)+2ab\mathrm{i}}{18};\end{aligned}$$

so

$$\begin{aligned}a^2 - b^2 &= 14 \quad (1) \\ ab &= 4\sqrt{2} \quad (2).\end{aligned}$$

Now,

$$ab = 4\sqrt{2} \Rightarrow b = \frac{4\sqrt{2}}{a} \quad (3)$$

and insert into (1):

$$\begin{aligned}a^2 - \left(\frac{4\sqrt{2}}{a}\right)^2 = 14 &\Rightarrow a^2 - \frac{32}{a^2} = 14 \\ &\Rightarrow a^4 - 32 = 14a^2 \\ &\Rightarrow a^4 - 14a^2 - 32 = 0\end{aligned}$$

$$\left.\begin{array}{ll}\text{add to:} & -14 \\ \text{multiply to:} & -32\end{array}\right\} -16,\ +2$$

$$\Rightarrow (a^2-16)(a^2+2) = 0$$

$a^2 + 2 > 0$:

$$\Rightarrow a^2 - 16 = 0$$
$$\Rightarrow a^2 = 16$$
$$\Rightarrow a = 4 \text{ or } a = -4$$
$$\Rightarrow b = \sqrt{2} \text{ or } b = -\sqrt{2};$$

hence, the possible complex numbers z are

$$\underline{\underline{z = 4 + \sqrt{2}\mathrm{i}}} \text{ or } \underline{\underline{z = -4 - \sqrt{2}\mathrm{i}}}.$$

8. (a) Given show that (5)

$$z^n + \frac{1}{z^n} = 2\cos n\theta,\ n \in \mathbb{N},$$

show that

$$32\cos^6\theta \equiv \cos 6\theta + 6\cos 4\theta + 15\cos 2\theta + 10.$$

Solution

Now,

$$\begin{aligned} 64\cos^6\theta &\equiv (2\cos\theta)^6 \\ &\equiv \left(z^n + \frac{1}{z^n}\right)^6 \\ &\equiv z^6 + 6z^4 + 15z^2 + 20 + \frac{15}{z^2} + \frac{6}{z^4} + \frac{1}{z^6} \\ &\equiv \left(z^6 + \frac{1}{z^6}\right) + 6\left(z^4 + \frac{1}{z^4}\right) + 15\left(z^2 + \frac{1}{z^2}\right) + 20 \\ &\equiv 2\cos 6\theta + 12\cos 4\theta + 30\cos 2\theta + 20 \end{aligned}$$

and

$$32\cos^6\theta = \underline{\underline{\cos 6\theta + 6\cos 4\theta + 15\cos 2\theta + 10}},$$

as required.

Figure 1 shows a solid paperweight with a flat base.

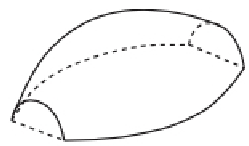


Figure 1: solid paperweight with a flat base

Figure 2 shows the curve with equation

$$y = H\cos^3\left(\tfrac{x}{4}\right),$$

where H is a positive constant and x is in radians.

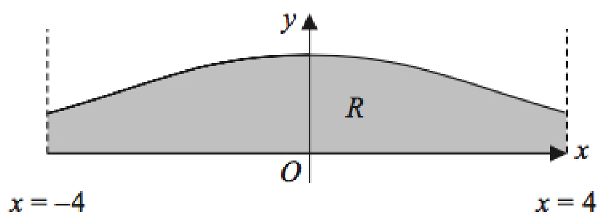


Figure 2: $y = H\cos^3\left(\tfrac{x}{4}\right)$

The region R, shown shaded in Figure 2, is bounded by the curve, the line with equation $x = -4$, the line with equation $x = 4$, and the x-axis.

The paperweight is modelled by the solid of revolution formed when R is rotated 180° about the x-axis.

Given that the maximum height of the paperweight is 2 cm,

(b) write down the value of H. (1)

> **Solution**
> $\underline{\underline{H = 2}}$.

(c) Using algebraic integration and the result in part (a), determine, in cm^3, the volume of the paperweight, according to the model. (5)
Give your answer to 2 decimal places.
(Solutions based entirely on calculator technology are not acceptable.)

Solution

Now,

$$\begin{aligned}
\text{volume} &= \int_{-4}^{4} \tfrac{1}{2}\pi \left[2\cos^3\left(\tfrac{x}{4}\right)\right]^2 \, \mathrm{d}x \\
&= \int_{0}^{4} \pi \left[2\cos^3\left(\tfrac{x}{4}\right)\right]^2 \, \mathrm{d}x \\
&= \pi \int_{0}^{4} 4\cos^6\left(\tfrac{x}{4}\right) \, \mathrm{d}x \\
&= \tfrac{1}{8}\pi \int_{0}^{4} 32\cos^6\left(\tfrac{x}{4}\right) \, \mathrm{d}x \\
&= \tfrac{1}{8}\pi \int_{0}^{4} \left(\cos\left(\tfrac{3}{2}x\right) + 6\cos(x) + 15\cos\left(\tfrac{1}{2}x\right) + 10\right) \mathrm{d}x \\
&= \tfrac{1}{8}\pi \left[\tfrac{2}{3}\sin\left(\tfrac{3}{2}x\right) + 6\sin(x) + 30\sin\left(\tfrac{1}{2}x\right) + 10x\right]_{x=0}^{4} \\
&= \tfrac{1}{8}\pi \left\{\left(\tfrac{2}{3}\sin(6) + 6\sin(4) + 30\sin(2) + 40\right) - (0+0+0+0)\right\} \\
&= 24.564\,046\,53 \text{ (FCD)} \\
&= \underline{\underline{24.56 \text{ cm}^3 \text{ (2 dp)}}}.
\end{aligned}$$

(d) State a limitation of the model. (1)

Solution

E.g., the equation of the curve may not be suitable, the measurements may not be accurate, the paperweight may not be smooth, etc.

9. (a) Explain why (1)

$$\int_0^\infty \cosh x \, \mathrm{d}x$$

is an improper integral.

Solution

E.g., $\cosh x$ is <u>undefined</u> at the limit of infinity.

(b) Show that (3)

$$\int_0^\infty \cosh x \, \mathrm{d}x$$

is divergent.

Solution

E.g.,

$$\begin{aligned}\int_0^\infty \cosh x \,\mathrm{d}x &= \lim_{t\to\infty}\int_0^t \cosh x\,\mathrm{d}x \\ &= \lim_{t\to\infty}\left[\sinh x\right]_{x=0}^t \\ &= \lim_{t\to\infty}(\sinh t - 0) \\ &= \lim_{t\to\infty}\sinh t;\end{aligned}$$

well, when $t \to \infty$, $\sinh t \to \infty$.

Hence the integral is divergent.

$$4\sinh x = p\cosh x,$$

where p is a real constant.

(c) Given that this equation has real solutions, determine the range of possible values for p. (2)

Solution

Well,

$$\begin{aligned}4\sinh x = p\cosh x &\Rightarrow \tanh x = \tfrac{1}{4}p \\ &\Rightarrow -1 < \tfrac{1}{4}p < 1\end{aligned}$$

because $-1 < \frac{1}{4}p < 1$

$$\Rightarrow \underline{\underline{-4 < p < 4}}.$$

10. The motion of a pendulum, shown in Figure 3, is modelled by the differential equation

$$\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} + 9\theta = \tfrac{1}{2}\cos 3t,$$

where θ is the angle, in radians, that the pendulum makes with the downward vertical, t seconds after it begins to move.

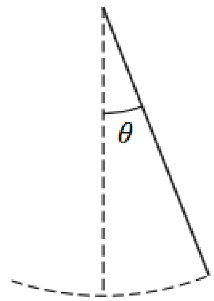


Figure 3: the motion of a pendulum

(a) (i) Show that a particular solution of the differential equation is (4)

$$\theta = \tfrac{1}{12} t \sin 3t.$$

Solution
Let

$$\theta = at \sin 3t.$$

Then

$$\begin{aligned}\frac{\mathrm{d}\theta}{\mathrm{d}t} &= a \sin 3t + 3at \cos 3t \\ \frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} &= 3a \cos 3t + 3a \cos 3t - 9at \sin 3t \\ &= 6a \cos 3t - 9at \sin 3t.\end{aligned}$$

Now,

$$\begin{aligned}\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} + 9\theta = \tfrac{1}{2}\cos 3t &\Rightarrow (6a \cos 3t - 9at \sin 3t) + 9(at \sin 3t) = \tfrac{1}{2}\cos 3t \\ &\Rightarrow 6a \cos 3t = \tfrac{1}{2}\cos 3t \\ &\Rightarrow 6a = \tfrac{1}{2} \\ &\Rightarrow a = \tfrac{1}{12};\end{aligned}$$

hence, the particular solution is

$$\underline{\underline{\theta = \tfrac{1}{12} t \sin 3t}},$$

as required.

(ii) Hence, find the general solution of the differential equation. (4)

Solution

The complementary function:

$$m^2 + 9 = 0 \Rightarrow m = \pm 3\text{i}$$

which means the complementary function is

$$\theta = A \sin 3t + B \cos 3t.$$

Putting them both together:

$$\underline{\underline{\theta = A \sin 3t + B \cos 3t + \tfrac{1}{12} t \sin 3t.}}$$

Initially, the pendulum

- makes an angle of $\frac{1}{3}\pi$ radians with the downward vertical and
- is at rest.

Given that, 10 seconds after it begins to move, the pendulum makes an angle of α radians with the downward vertical,

(b) determine, according to the model, the value of α to 3 significant figures. (4)

Solution

For the first line,

$$\begin{aligned} t = 0, \theta = \tfrac{1}{3}\pi &\Rightarrow 0 + B \cos 0 + 0 = \tfrac{1}{3}\pi \\ &\Rightarrow B = \tfrac{1}{3}\pi. \end{aligned}$$

Now,

$$\begin{aligned} \theta &= A \sin 3t + \tfrac{1}{3}\pi \cos 3t + \tfrac{1}{12} t \sin 3t \\ \Rightarrow \quad \frac{\mathrm{d}\theta}{\mathrm{d}t} &= 3A \cos 3t - \pi \sin 3t + \tfrac{1}{12} \sin 3t + \tfrac{1}{4} t \cos 3t \end{aligned}$$

and

$$\begin{aligned} t = 0, \frac{\mathrm{d}\theta}{\mathrm{d}t} = 0 &\Rightarrow 3A \cos 0 - 0 + 0 = 0 \\ &\Rightarrow A = 0. \end{aligned}$$

Hence,

$$\theta = \tfrac{1}{3}\pi \cos 3t + \tfrac{1}{12} t \sin 3t.$$

Finally,

$$\begin{aligned} t = 10 &\Rightarrow \theta = \tfrac{1}{3}\pi \cos 30 + \tfrac{10}{12} \sin 30 \\ &\Rightarrow \theta = -0.661\,827\,946\,2 \text{ (FCD)}; \end{aligned}$$

hence, the angle made is

$$\underline{\underline{\theta = 0.662 \text{ (3 sf)}}}.$$

Given that the true value of α is 0.62,

(c) evaluate the model. (1)

Solution

Well, 0.662 is reasonably close to 0.62 so it is a good model at $t = 10$.

The differential equation

$$\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} + 9\theta = \tfrac{1}{2} \cos 3t$$

models the motion of the pendulum as moving with forced harmonic motion.

(d) Refine the differential equation so that the motion of the pendulum is simple harmonic motion. (1)

Solution

E.g.,

$$\underline{\underline{\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} + 9\theta = 0}}.$$