

Dr Oliver Mathematics
AQA Further Maths Level 2
June 2024 Paper 1
1 hour 45 minutes

The total number of marks available is 80.

You must write down all the stages in your working.

You are **not** permitted to use a scientific or graphical calculator in this paper.

1. Work out the value of

(2)

$$\sqrt{\frac{t}{20}}$$

where $t = 2.42 \times 10^3$.

Solution

Well,

$$\begin{aligned}\sqrt{\frac{2.42 \times 10^3}{20}} &= \sqrt{1.21 \times 10^2} \\ &= \sqrt{121} \\ &= \underline{\underline{11}}.\end{aligned}$$

2. Factorise

(1)

$$x^2 - y^2.$$

Solution

Difference of two squares:

$$x^2 - y^2 = \underline{\underline{(x + y)(x - y)}}.$$

3. The n th term of a sequence is

(1)

$$\frac{3n + 4}{n}.$$

Circle the limiting value of

$$\frac{3n+4}{n}$$

as $n \rightarrow \infty$.

1 3 4 7

Solution

Well,

$$\begin{aligned}\frac{3n+4}{n} &= \frac{3n}{n} + \frac{4}{n} \\ &= 3 + \frac{4}{n} \\ &\rightarrow \underline{\underline{3}},\end{aligned}$$

as $n \rightarrow \infty$.

4. The equations of two straight lines are

(2)

$$y - 3x = 4 \text{ and } 6y = 18x - 5.$$

Show that the lines are parallel.

Solution

Now,

$$y - 3x = 4 \Rightarrow y = 3x + 4$$

and divide the second one by 6:

$$6y = 18x - 5 \Rightarrow y = 3x - \frac{5}{6}.$$

Clearly, the gradients are the same.

Hence, the lines are parallel.

- 5.

(3)

$$y = \frac{4x^3 + x^7}{x^4}.$$

Work out $\frac{dy}{dx}$.

Solution

Well,

$$y = \frac{4x^3 + x^7}{x^4} \Rightarrow 4x^{-1} + x^3$$
$$\Rightarrow \underline{\underline{\frac{dy}{dx} = -4x^{-2} + 3x^2.}}$$

6. Points $A(-12, 1)$ and $B(12, -1)$ lie on a circle.

(3)

AB is a diameter of the circle.

Work out the equation of the circle.

Solution

Well, the centre of the circle is

$$\left(\frac{-12 + 12}{2}, \frac{1 + (-1)}{2} \right) = (0, 0).$$

Now, the radius squared is

$$\begin{aligned} (-12 - 0)^2 + (1 - 0)^2 &= 144 + 1 \\ &= 145. \end{aligned}$$

Hence, the equation of the circle is

$$\underline{\underline{x^2 + y^2 = 145.}}$$

7. A point $P(x, y)$ is transformed using the transformation represented by

(3)

$$\begin{pmatrix} 4 & 0 \\ -2 & 3 \end{pmatrix}.$$

The image of P is $(-8, 7)$.

Work out the values of x and y .

Solution

Let the matrix be $\mathbf{A}$. Then

$$\det \mathbf{A} = 12 - 0 = 0$$

and

$$\mathbf{A}^{-1} = \frac{1}{12} \begin{pmatrix} 3 & 0 \\ 2 & 4 \end{pmatrix}.$$

Now,

$$\begin{aligned} \begin{pmatrix} 4 & 0 \\ -2 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} &= \begin{pmatrix} -8 \\ 7 \end{pmatrix} \Rightarrow \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 & 0 \\ -2 & 3 \end{pmatrix}^{-1} \begin{pmatrix} -8 \\ 7 \end{pmatrix} \\ &\Rightarrow \begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{12} \begin{pmatrix} 3 & 0 \\ 2 & 4 \end{pmatrix} \begin{pmatrix} -8 \\ 7 \end{pmatrix} \\ &\Rightarrow \begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{12} \begin{pmatrix} -24 \\ 12 \end{pmatrix} \\ &\Rightarrow \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -2 \\ 1 \end{pmatrix}; \end{aligned}$$

hence, $x = -2$ and $y = 1$.

8. Solve by factorising

(3)

$$2x^3 - 9x^2 - 5x = 0.$$

Solution

Now,

$$2x^3 - 9x^2 - 5x = 0 \Rightarrow x(2x^2 - 9x - 5) = 0$$

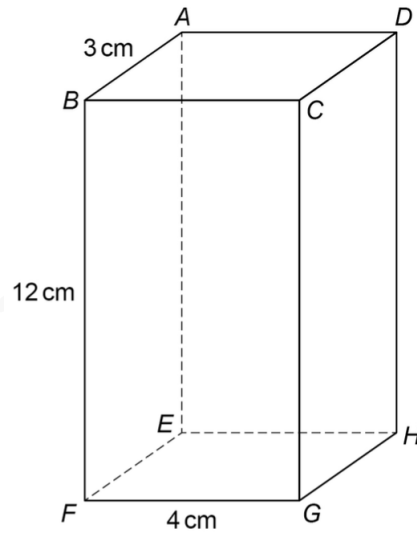
$$\left. \begin{array}{l} \text{add to:} \quad -9 \\ \text{multiply to: } (+2) \times (-5) = -10 \end{array} \right\} +1, -10$$

e.g.,

$$\begin{aligned} &\Rightarrow x[2x^2 + x - 10x - 5] = 0 \\ &\Rightarrow x[x(2x + 1) - 5(2x + 1)] = 0 \\ &\Rightarrow x(x - 5)(2x + 1) = 0 \\ &\Rightarrow x = 0, x - 5 = 0, \text{ or } 2x + 1 = 0 \\ &\Rightarrow \underline{\underline{x = 0, x = 5, \text{ or } x = -\frac{1}{2}}}. \end{aligned}$$

9. $ABCDEFGH$ is a cuboid.

(3)



Work out the length AG .

Solution

Three-dimensional Pythagoras':

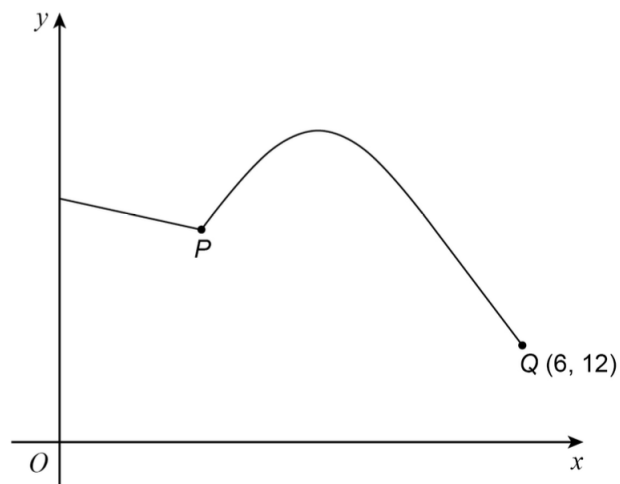
$$\begin{aligned}
 AG &= \sqrt{AB^2 + BF^2 + FG^2} \\
 &= \sqrt{3^2 + 12^2 + 4^2} \\
 &= \sqrt{9 + 144 + 16} \\
 &= \sqrt{169} \\
 &= \underline{\underline{13 \text{ cm.}}}
 \end{aligned}$$

10. A function f is given by

(5)

$$f(x) = \begin{cases} -\frac{1}{2}x + 21 & \text{for } 0 \leq x \leq 2, \\ ax^2 + bx & \text{for } 2 < x \leq 6. \end{cases}$$

A sketch of $y = f(x)$ is shown.



Work out the values of a and b .

Solution

Well, at P ,

$$\begin{aligned}x = 2 &\Rightarrow y = -\frac{1}{2}(2) + 21 \\&\Rightarrow y = 20\end{aligned}$$

and

$$\begin{aligned}x = 2, y = 20 &\Rightarrow 20 = 4a + 2b \\&\Rightarrow 2a + b = 10 \quad (1).\end{aligned}$$

At Q ,

$$\begin{aligned}x = 6, y = 12 &\Rightarrow 12 = 36a + 6b \\&\Rightarrow 6a + b = 2 \quad (2).\end{aligned}$$

Do $(2) - (1)$:

$$4a = -8 \Rightarrow \underline{\underline{a = -2}}.$$

insert this into (1):

$$\begin{aligned}&\Rightarrow 2(-2) + b = 10 \\&\Rightarrow -4 + b = 10 \\&\Rightarrow \underline{\underline{b = 14}}.\end{aligned}$$

[Check: $6(-2) + 14 = -12 + 14 = 2 \checkmark$]

11. $(3x - 7)$ is a factor of

$$3x^3 - 4x^2 - 13x + 14.$$

(a) Work out the other two linear factors.

(3)

Solution

We use synthetic division:

$$\begin{array}{r|rrrr} \frac{7}{3} & 3 & -4 & -13 & 14 \\ & \downarrow & 7 & 7 & -14 \\ \hline & 3 & 3 & -6 & 0 \end{array}$$

so

$$\begin{aligned} 3x^3 - 4x^2 - 13x + 14 &= (x - \frac{7}{3})(3x^2 + 3x - 6) \\ &= (3x - 7)(x^2 + x - 2) \end{aligned}$$

$$\left. \begin{array}{l} \text{add to:} \\ \text{multiply to:} \end{array} \right\} \begin{array}{l} +1 \\ -2 \end{array} + 2, -1$$

$$= (3x - 7)(x + 2)(x - 1).$$

Hence, one linear factor is $(x + 2)$ and the other one is $(x - 1)$.

$(x - 2)$ is a factor of

$$ax^4 - 3ax^2 + 5x - 22.$$

(b) Work out the value of a .

(3)

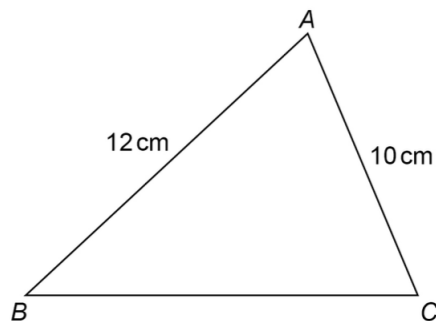
Solution

Factor theorem:

$$\begin{aligned} 16a - 24a + 10 - 22 &= 0 \Rightarrow -8a = 12 \\ &\Rightarrow \underline{\underline{a = -\frac{3}{2}}}. \end{aligned}$$

12. In triangle ABC , $\cos BAC = \frac{3}{4}$.

(3)



Work out the length BC .

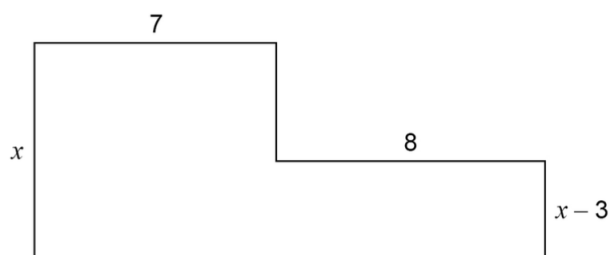
Solution

Well,

$$\begin{aligned}
 BC^2 &= AB^2 + AC^2 - 2 \times AB \times AC \times \cos BAC \\
 \Rightarrow BC^2 &= 12^2 + 10^2 - 2 \times 12 \times 10 \times \frac{3}{4} \\
 \Rightarrow BC^2 &= 144 + 100 - 240 \times \frac{3}{4} \\
 \Rightarrow BC^2 &= 144 + 100 - 180 \\
 \Rightarrow BC^2 &= 244 - 180 \\
 \Rightarrow BC^2 &= 64 \\
 \Rightarrow \underline{\underline{BC = 8 \text{ cm.}}}
 \end{aligned}$$

13. A garden patio is made from two rectangles.
All lengths are in metres.

(4)



The area of the patio is **less than** 51 m^2 .

Work out the range of possible values of x .

Give your answer in the form $a < x < b$ where a and b are both integers.
You **must** show your working.

Solution

Well,

$$\begin{aligned}\text{shape} &= \text{big rectangle} - \text{small rectangle} \\ &= (15 \times x) - (8 \times 3) \\ &= 15x - 24.\end{aligned}$$

Now,

$$\begin{aligned}15x - 24 < 51 &\Rightarrow 15x < 75 \\ &\Rightarrow x < 5.\end{aligned}$$

But clearly $x > 3$ (why?) So,

$$\underline{\underline{3 < x < 5.}}$$

14. (a) Write

$$2x^2 - 16x + 7$$

(3)

in the form

$$k(x + m)^2 + n,$$

where k , m , and n are integers.

Solution

Well,

$$2x^2 - 16x + 7 = 2[x^2 - 8x] + 7$$

$$\text{coefficient of } x : \quad -8$$

$$\text{half it:} \quad -4$$

$$\text{square it:} \quad (-4)^2 = +16$$

$$= 2[(x^2 - 8x + 16) - 16] + 7$$

$$= 2[(x - 4)^2 - 16] + 7$$

$$= 2(x - 4)^2 - 32 + 7$$

$$= \underline{\underline{2(x - 4)^2 - 25;}}$$

hence, $\underline{\underline{k = 2}}$, $\underline{\underline{m = -4}}$, and $\underline{\underline{n = -25}}$.

(b) Solve

$$(x - 1)^2 - 5 = 0.$$

(1)

Solution

Now,

$$\begin{aligned}(x - 1)^2 - 5 = 0 &\Rightarrow (x - 1)^2 = 5 \\ &\Rightarrow x - 1 = \pm\sqrt{5} \\ &\Rightarrow \underline{\underline{x = 1 \pm \sqrt{5}}}.\end{aligned}$$

15. (a) Matrix **M** represents a reflection in the line $y = -x$.

(1)

Write down matrix **M**.

Solution

$$\underline{\underline{\mathbf{M} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}}}.$$

(b)

$$\mathbf{N} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}.$$

(2)

Describe geometrically the single transformation represented by $\mathbf{N}^2$.

Solution

Well,

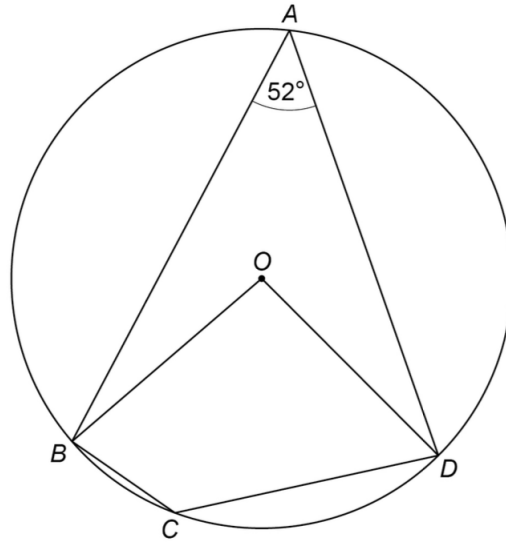
$$\begin{aligned}\mathbf{N}^2 &= \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \\ &= \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix};\end{aligned}$$

hence, it is a rotation, through 180°, about (0,0).

16. A , B , C , and D are points on a circle, centre O .

(4)

$$\text{Angle } OBC : \text{angle } ODC = 5 : 3.$$



Work out the size of angle OBC .
You **must** show your working.

Solution

The angle of a circle subtends at the centre is twice the angle at the circumference:

$$\begin{aligned}\text{angle } BOC &= 2 \times \text{angle } BAC \\ &= 2 \times 52 \\ &= 104^\circ.\end{aligned}$$

Now, $ABCD$ is a cyclic quadrilateral so that means

$$\begin{aligned}\text{angle } BCD &= 180 - \text{angle } BAC \\ &= 180 - 52 \\ &= 128^\circ.\end{aligned}$$

Next,

$$\begin{aligned}\text{angle } BCD + \text{angle } BOC &= 104 + 128 \\ &= 232^\circ\end{aligned}$$

and that means

$$\begin{aligned}\text{angle } OBC + \text{angle } ODC &= 360 - 232 \\ &= 128^\circ.\end{aligned}$$

Now,

$$5 + 3 = 8.$$

So,

$$\begin{aligned}\text{angle } OBC &= \frac{5}{8} \times 128 \\ &= 5 \times 16 \\ &= \underline{\underline{80^\circ}}.\end{aligned}$$

17. Show that

$$\frac{21x}{3x^2 - 2x - 8} - \frac{7}{x - 2}$$

simplifies to

$$\frac{k}{3x^2 - 2x - 8},$$

where k is an integer.

Solution

Well,

$$\left. \begin{array}{l} \text{add to:} \\ \text{multiply to: } (+3) \times (-8) = -24 \end{array} \right\} +4, -6$$

and

$$\begin{aligned}3x^2 - 2x - 8 &= 3x^2 + 4x - 6x - 8 \\ &= x(3x + 4) - 2(3x + 4) \\ &= (x - 2)(3x + 4).\end{aligned}$$

Now,

$$\begin{aligned}\frac{21x}{3x^2 - 2x - 8} - \frac{7}{x - 2} &= \frac{21x}{3x^2 - 2x - 8} - \frac{7(3x + 4)}{(x - 2)(3x + 4)} \\ &= \frac{21x - 7(3x + 4)}{3x^2 - 2x - 8} \\ &= \frac{21x - 21x - 28}{3x^2 - 2x - 8} \\ &= \frac{-28}{\underline{\underline{3x^2 - 2x - 8}}};\end{aligned}$$

hence, $\underline{\underline{k = -28}}$.

(3)

18. Rationalise the denominator and simplify fully

(4)

$$\frac{3\sqrt{5} + \sqrt{3}}{\sqrt{5} - \sqrt{3}}.$$

Solution

Well,

×	$\sqrt{5}$	$+\sqrt{3}$
$\sqrt{5}$	5	$+\sqrt{15}$
$-\sqrt{3}$	$-\sqrt{15}$	-3

and

×	$\sqrt{5}$	$+\sqrt{3}$
$3\sqrt{5}$	15	$+3\sqrt{15}$
$+\sqrt{3}$	$+\sqrt{15}$	+3

So,

$$\begin{aligned} \frac{3\sqrt{5} + \sqrt{3}}{\sqrt{5} - \sqrt{3}} &= \frac{3\sqrt{5} + \sqrt{3}}{\sqrt{5} - \sqrt{3}} \times \frac{\sqrt{5} + \sqrt{3}}{\sqrt{5} + \sqrt{3}} \\ &= \frac{18 + 4\sqrt{15}}{2} \\ &= \frac{2(9 + 2\sqrt{15})}{2} \\ &= \underline{\underline{9 + 2\sqrt{15}}}. \end{aligned}$$

19. The equation of a curve is

(5)

$$y = x^3 - 3x^2 + 5.$$

Work out the stationary points of the curve and determine their nature.

You **must** show your working.

Solution

Well,

$$y = x^3 - 3x^2 + 5 \Rightarrow \frac{dy}{dx} = 3x^2 - 6x \\ \Rightarrow \frac{d^2y}{dx^2} = 6x - 6.$$

Now,

$$\frac{dy}{dx} = 0 \Rightarrow 3x^2 - 6x = 0 \\ \Rightarrow 3x(x - 2) = 0 \\ \Rightarrow x = 0 \text{ or } x = 2.$$

If

$$x = 0 \Rightarrow y = 0 - 0 + 5 = 5, \\ x = 2 \Rightarrow y = 8 - 12 + 5 = 1.$$

Next,

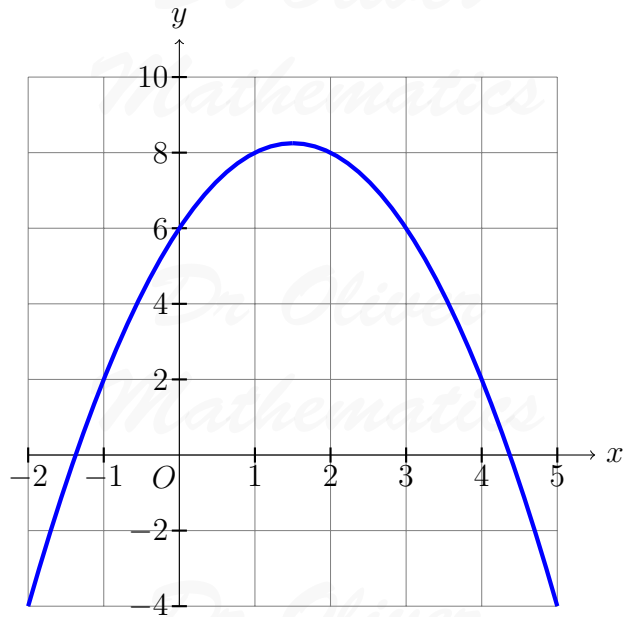
$$x = 0 \Rightarrow \frac{dy}{dx} = -6 < 0, \\ x = 2 \Rightarrow \frac{dy}{dx} = 6 > 0;$$

hence, (0, 5) is a maximum turning point and (2, 1) is a minimum turning point.

20. Here is the graph of

$$y = -x^2 + 3x + 6, \text{ for } -2 \leq x \leq 5.$$

(4)



By drawing a suitable **straight line**, work out approximate solutions to

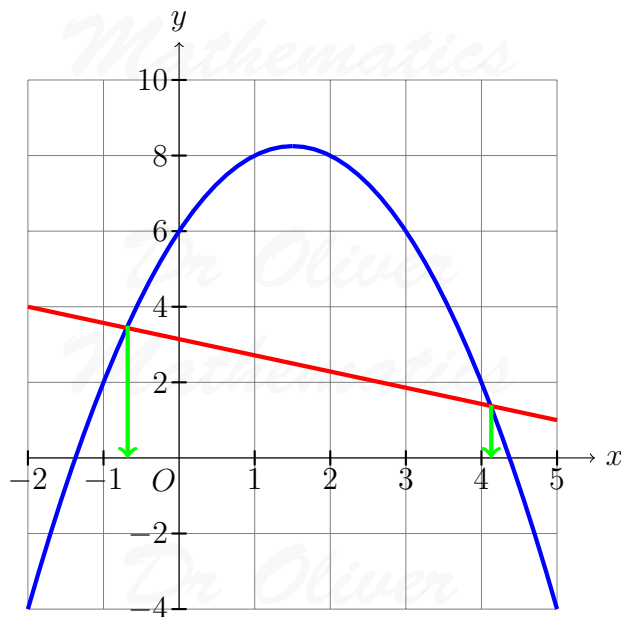
$$x^2 - \frac{7}{2}x - 3 = 0.$$

Solution

Well,

$$x^2 - \frac{7}{2}x - 3 = 0 \Rightarrow -\frac{1}{2}x + 3 = -x^2 + 3x + 6$$

and so we need $y = -x^2 + 3x + 6$ and $y = -\frac{1}{2}x + 3$ to answer this question.



Correct read-off: approximately $x = -0.7$ and $x = 4.1$.

21.

$$(2^{x+3})^{4x+1} = 8^{x-1}.$$

(5)

Work out the possible values of x .

Solution

Well,

$\times$	$4x$	$+1$
x	$4x^2$	$+x$
$+3$	$+12x$	$+3$

so

$$(x + 3)(4x + 1) = 4x^2 + 13x + 3.$$

Now,

$$\begin{aligned} (2^{x+3})^{4x+1} &= 8^{x-1} \Rightarrow 2^{(x+3)(4x+1)} = (2^3)^{x-1} \\ &\Rightarrow 2^{4x^2+13x+3} = 2^{3(x-1)} \end{aligned}$$

they are the same powers of 2:

$$\Rightarrow 4x^2 + 13x + 3 = 3x - 3$$

$$\Rightarrow 4x^2 + 10x + 6 = 0$$

$$\Rightarrow 2(2x^2 + 5x + 3) = 0$$

$$\left. \begin{array}{l} \text{add to:} \\ \text{multiply to:} \end{array} \right\} \begin{array}{l} +5 \\ (+2) \times (+3) = +6 \end{array} + 2, +3$$

e.g.,

$$\Rightarrow 2[2x^2 + 2x + 3x + 3] = 0$$

$$\Rightarrow 2[2x(x + 1) + 3(x + 1)] = 0$$

$$\Rightarrow 2(2x + 3)(x + 1) = 0$$

$$\Rightarrow 2x + 3 = 0 \text{ or } x + 1 = 0$$

$$\Rightarrow \underline{\underline{x = -\frac{3}{2} \text{ or } x = -1.}}$$

22.

(4)

$$f(x) = \frac{3 \sin x \cos x + \sin^2 x}{12 \cos^2 x + 4 \sin x \cos x}.$$

Simplify $f(x)$ and hence solve

$$f(x) = -\frac{\sqrt{3}}{4}, \text{ for } 90^\circ < x < 180^\circ.$$

You **must** show your working.

Solution

Well,

$$\begin{aligned} \frac{3 \sin x \cos x + \sin^2 x}{12 \cos^2 x + 4 \sin x \cos x} &\equiv \frac{\sin x(3 \cos x + \sin x)}{4 \cos x(3 \cos x + \sin x)} \\ &\equiv \frac{\sin x}{4 \cos x} \\ &\equiv \underline{\underline{\frac{1}{4} \tan x}}, \end{aligned}$$

as required.

Now,

$$\begin{aligned}f(x) = -\frac{\sqrt{3}}{4} &\Rightarrow \frac{1}{4} \tan x = -\frac{\sqrt{3}}{4} \\&\Rightarrow \tan x = -\sqrt{3} \\&\Rightarrow \underline{x = 120}.\end{aligned}$$

23. The n th term of a quadratic sequence is

(5)

$$an^2 - 5n + c,$$

where a and c are integers.

The first four terms of the sequence are

$$2 \quad x \quad 16 \quad y.$$

Work out the values of x and y .

Solution

The first term is

$$a(1^2) - 5(1) + c = 2 \Rightarrow a + c = 7 \quad (1)$$

and the third term is

$$a(3^2) - 5(3) + c = 16 \Rightarrow 9a + c = 31 \quad (2).$$

Do (2) - (1):

$$\begin{aligned}8a &= 24 \Rightarrow a = 3 \\&\Rightarrow c = 4.\end{aligned}$$

[Check: $9(3) + 4 = 31$ ✓]

Finally, the second term is

$$\begin{aligned}3(2^2) - 5(2) + 4 &= 12 - 10 + 4 \\&= \underline{6}\end{aligned}$$

and the fourth term is

$$\begin{aligned}3(4^2) - 5(4) + 4 &= 48 - 20 + 4 \\&= \underline{32}.\end{aligned}$$