Dr Oliver Mathematics
Applied Mathematics: Parametric Equations

The total number of marks available is 28.
You must write down all the stages in your working.

1. A curve is defined by the parametric equations
$$x = 5t^2 - 5 \text{ and } y = 3t^3.$$
   (a) Find the value of t corresponding to the point $(0, -3)$. (2)

   (b) Calculate the gradient of the curve at this point. (3)

2. A curve is defined parametrically by (5)
$$x = \frac{t}{t^2+1} \text{ and } y = \frac{t-1}{t^2+1}.$$
   Obtain $\frac{\mathrm{d}y}{\mathrm{d}x}$ as a function of t.

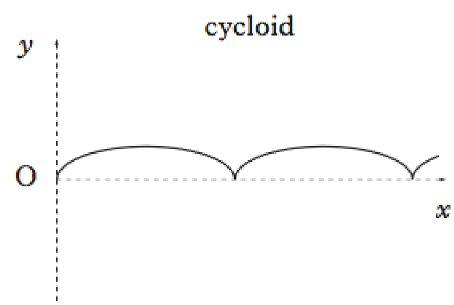
3. A particle moves along a curve in the x-y plane. The curve is defined by the parametric equations
$$x = t^2 + 1,\ y = 1 - 3t^3,$$
   where t is the time elapsed since the start.

   (a) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of t. (3)

   (b) Hence obtain an equation of the tangent to the curve when $t = 2$. (2)

4. A curve is defined by the equations (4)
$$x = 5\cos t \text{ and } y = 3\sin t,\ 0 \leqslant t < 2\pi.$$
   Find the gradient of the curve when $t = \frac{1}{6}\pi$.

5. The cycloid curve below is defined by the parametric equations
$$x = t - \sin t,\ y = 1 - \cos t.$$



(a) Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ in terms of t. (2)

(b) Show that the value of $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}$ is always negative, in the case where $0 < t < 2\pi$. (5)

A particle follows the path of the cycloid where t is the time elapsed since the particle's motion commenced.

(c) Calculate the speed of the particle when $t = \frac{1}{3}\pi$. (2)