

Dr Oliver Mathematics
Mathematics: Higher
2009 Paper 1: Non-Calculator
1 hour 30 minutes

The total number of marks available is 70.
You must write down all the stages in your working.

Section A

1. A sequence is defined by

$$u_{n+1} = 3u_n + 4$$

(2)

with $u_1 = 2$.

What is the value of u_3 ?

- A. 34
- B. 21
- C. 18
- D. 13

2. A circle has equation

$$x^2 + y^2 + 8x + 6y - 75 = 0.$$

(2)

What is the radius of this circle?

- A. 5
- B. 10
- C. $\sqrt{75}$
- D. $\sqrt{175}$

3. Triangle PQR has vertices at $P(-3, -2)$, $Q(-1, 4)$, and $R(3, 6)$.
 PS is a median.

(2)

What is the gradient of PS ?

- A. -2
- B. $-\frac{7}{4}$
- C. 1
- D. $\frac{7}{4}$

4. A curve has equation

$$y = 5x^3 - 12x.$$

(2)

What is the gradient of the tangent at the point $(1, -7)$?

- A. -7
- B. -5
- C. 3
- D. 5

5. Here are two statements about the points $S(2, 3)$ and $T(5, -1)$:

(2)

- (1) The length of $ST = 5$ units;
- (2) The gradient of $ST = \frac{4}{3}$.

Which of the following is true?

- A. Neither statement is correct.
- B. Only statement (1) is correct.
- C. Only statement (2) is correct.
- D. Both statements are correct.

6. A sequence is generated by the recurrence relation

(2)

$$u_{n+1} = 0.7u_n + 10.$$

What is the limit of this sequence as $n \rightarrow \infty$?

- A. $\frac{100}{3}$
- B. $\frac{100}{7}$
- C. $\frac{17}{100}$
- D. $\frac{3}{10}$

7. If the exact value of $\cos x$ is $\frac{1}{\sqrt{5}}$, find the exact value of $\cos 2x$.

(2)

- A. $-\frac{3}{5}$
- B. $-\frac{2}{\sqrt{5}}$
- C. $-\frac{2}{\sqrt{5}}$
- D. $\frac{3}{5}$

8. What is the derivative of

$$\frac{1}{4x^3}, x \neq 0?$$

(2)

A. $\frac{1}{12x^2}$

B. $-\frac{1}{12x^2}$

C. $\frac{4}{x^4}$

D. $-\frac{3}{4x^4}$

9. The line with equation

$$y = 2x$$

(2)

intersects the circle with equation

$$x^2 + y^2 = 5$$

at the points J and K .

What are the x -coordinates of J and K ?

A. $x_J = 1, x_K = -1$

B. $x_J = 2, x_K = -2$

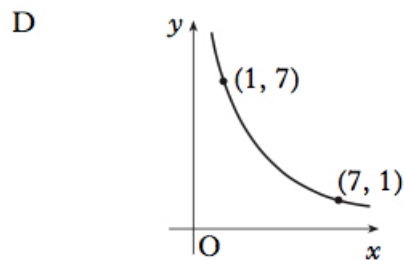
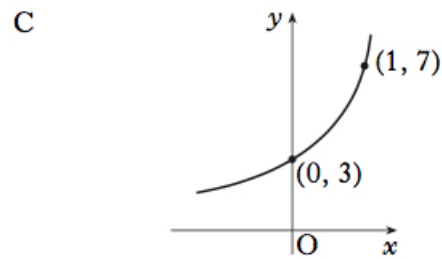
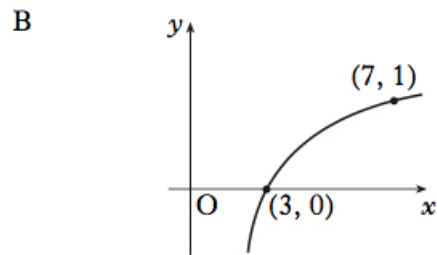
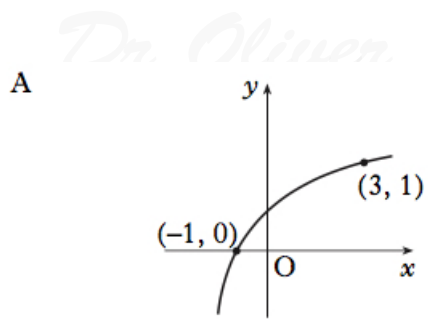
C. $x_J = 1, x_K = -2$

D. $x_J = -1, x_K = 2$

10. Which of the following graphs has equation

$$y = \log_5(x - 2)?$$

(2)



11. How many solutions does the equation

$$(4 \sin x - \sqrt{5})(\sin x + 1) = 0$$

have in the interval $0 \leq x < 2\pi$?

- A. 4
- B. 3
- C. 2

(2)

D. 1

12. A function f is given by

$$f(x) = 2x^2 - x - 9.$$

(2)

Which of the following describes the nature of the roots of $f(x) = 0$?

- A. No real roots
- B. Equal roots
- C. Real distinct roots
- D. Rational distinct roots

13. k and a are given by

$$\begin{aligned}k \sin a^\circ &= 1, \\k \cos a^\circ &= \sqrt{3},\end{aligned}$$

(2)

where $k > 0$ and $0 \leq a < 90$.

What are the values of k and a ?

- A. $k = 2$ and $a = 60$
- B. $k = 2$ and $a = 30$
- C. $k = \sqrt{10}$ and $a = 60$
- D. $k = \sqrt{10}$ and $a = 30$

14. If

$$f(x) = 2 \sin \left(3x - \frac{\pi}{2} \right) + 5,$$

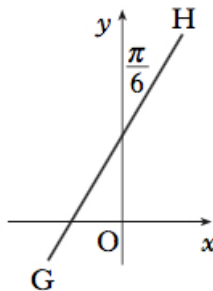
(2)

what is the range of values of $f(x)$?

- A. $-1 \leq f(x) \leq 11$
- B. $2 \leq f(x) \leq 8$
- C. $3 \leq f(x) \leq 7$
- D. $-3 \leq f(x) \leq 7$

15. The line GH makes an angle of $\frac{\pi}{6}$ radians with the y -axis, as shown in the diagram.

(2)



What is the gradient of GH ?

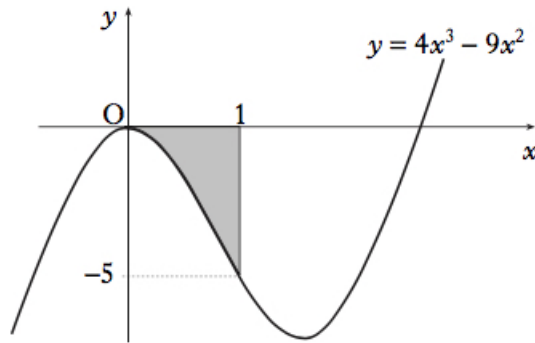
- A. $\sqrt{3}$
- B. $\frac{1}{2}$
- C. $\frac{1}{\sqrt{2}}$
- D. $\frac{\sqrt{3}}{2}$

16. The graph of

$$y = 4x^3 - 9x^2$$

(2)

is shown in the diagram.



Which of the following gives the area of the shaded section?

- A. $[x^4 - 3x^3]_{x=-5}^0$
- B. $-[x^4 - 3x^3]_{x=0}^1$
- C. $[12x^2 - 18x]_{x=-5}^0$
- D. $-[12x^2 - 18x]_{x=0}^1$

17. The vector \mathbf{u} has components

$$\begin{pmatrix} -3 \\ 0 \\ 4 \end{pmatrix}.$$

(2)

Which of the following is a unit vector parallel to \mathbf{u} ?

- A. $-\frac{3}{5}\mathbf{i} + \frac{4}{5}\mathbf{k}$
- B. $-3\mathbf{i} + 4\mathbf{k}$
- C. $-\frac{3}{\sqrt{7}}\mathbf{i} + \frac{4}{\sqrt{7}}\mathbf{k}$
- D. $-\frac{1}{3}\mathbf{i} + \frac{1}{4}\mathbf{k}$

18. Given that

(2)

$$f(x) = (4 - 3x^2)^{-\frac{1}{2}}$$

on a suitable domain, find $f'(x)$.

A. $-3x(4 - 3x^2)^{-\frac{1}{2}}$

B. $-\frac{1}{2}(4 - 3x^2)^{-\frac{3}{2}}$

C. $2(4 - 3x^3)^{\frac{1}{2}}$

D. $3x(4 - 3x^2)^{-\frac{3}{2}}$

19. For what values of x is

(2)

$$6 + x - x^2 < 0?$$

A. $x > 3$ only

B. $x < -2$ only

C. $x < -2$ or $x > 3$

D. $-3 < x < 2$

20.

(2)

$$A = 2\pi r^2 + 6\pi r.$$

What is the rate of change of A with respect to r when $r = 2$?

A. 10π

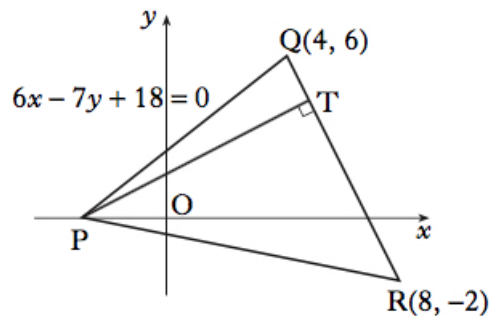
B. 12π

C. 14π

D. 20π

Section B

21. Triangle PQR has vertex P on the x -axis, as shown in the diagram.



Q and R are the points $(4, 6)$ and $(8, -2)$ respectively.
The equation of PQ is

$$6x - 7y + 18 = 0.$$

- (a) State the coordinates of P . (1)
 (b) Find the equation of the altitude of the triangle from P . (3)

The altitude from P meets the line QR at T .

- (c) Find the coordinates of T . (4)

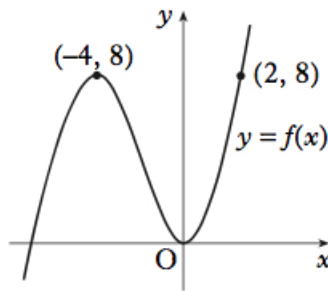
22. D , E , and F have coordinates $(10, -8, -15)$, $(1, -2, -3)$, and $(-2, 0, 1)$. respectively.

- (a) (i) Show that D , E , and F are collinear. (4)
 (ii) Find the ratio in which E divides DF .

G has coordinates $(k, 1, 0)$.

- (b) Given that DE is perpendicular to GE , find the value of k . (4)

23. The diagram shows a sketch of the function $y = f(x)$.



- (a) Copy the diagram and on it sketch the graph of $y = f(2x)$. (2)
 (b) On a separate diagram, sketch the graph of $y = 1 - f(2x)$. (3)

24. (a) Using the fact that (3)

$$\frac{7\pi}{12} = \frac{\pi}{3} + \frac{\pi}{4},$$

find the exact value of $\sin\left(\frac{7\pi}{12}\right)$.

(b) Show that (2)

$$\sin(A + B) + \sin(A - B) = 2 \sin A \cos B.$$

- (c) (i) Express $\frac{\pi}{12}$ in terms of $\frac{\pi}{3}$ and $\frac{\pi}{4}$. (4)
 (ii) Hence or otherwise find the exact value of

$$\sin\left(\frac{7\pi}{12}\right) + \sin\left(\frac{\pi}{12}\right).$$