Dr Oliver Mathematics
GCSE Mathematics
2017 November Paper 3H: Calculator
1 hour 30 minutes

The total number of marks available is 80.
You must write down all the stages in your working.

1. The table shows information about the heights of 80 children

Height, (h cm)	Frequency
$130 < h \leqslant 140$	4
$140 < h \leqslant 150$	11
$150 < h \leqslant 160$	24
$160 < h \leqslant 170$	22
$170 < h \leqslant 180$	19

(a) Find the class interval that contains the median. (1)

Solution

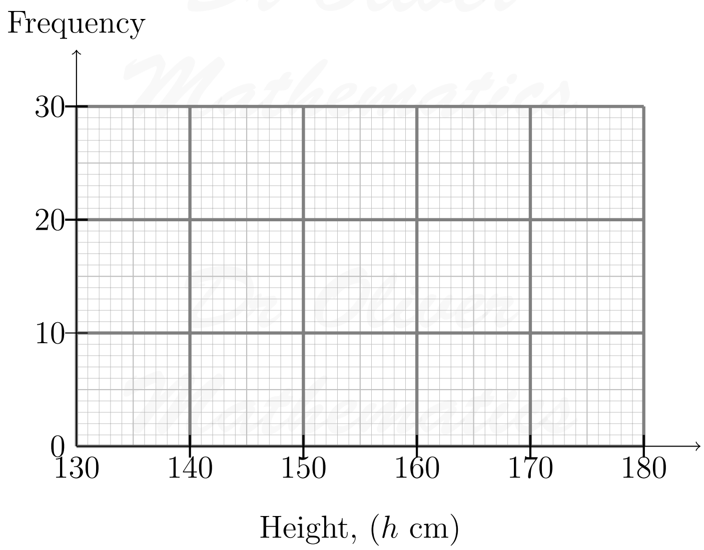
Height, (h cm)	Frequency	Cumulative Frequency
$130 < h \leqslant 140$	4	4
$140 < h \leqslant 150$	11	$11 + 4 = 15$
$150 < h \leqslant 160$	24	$15 + 24 = 39$
$160 < h \leqslant 170$	22	$39 + 22 = 61$
$170 < h \leqslant 180$	19	$61 + 19 = 80$

Now, since the median piece of information is at

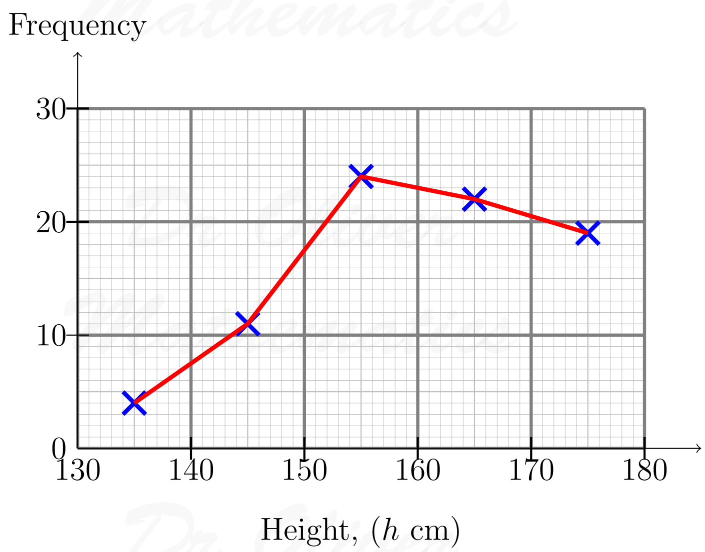
$$\frac{80 + 1}{2} = 40\tfrac{1}{2},$$

we can say the class interval that contains the median is $\underline{\underline{160 < h \leqslant 170}}$.

(b) Draw a frequency polygon for the information in the table. (2)



Solution



2. In London, 1 litre of petrol costs 108.9p. (3)
   In New York, 1 US gallon of petrol costs \$2.83.

$$1 \text{ US gallon} = 3.785 \text{ litres}$$
$$£1 = \$1.46.$$

   In which city is petrol better value for money, London or New York?
   You must show your working.

Solution

Convert the US gallon and dollars to litres and pounds respectively:

$$\begin{aligned}\text{cost} &= 2.83 \times \frac{1}{3.785} \times \frac{1}{1.46} \\ &= 0.512\,115\,235 \text{ (FCD)} \\ &= 51.211\ldots \text{ p;}\end{aligned}$$

hence, the city is petrol better value for money is New York.

3. A gold bar has a mass of 12.5 kg. (3)

The density of gold is 19.3 g/cm^3.

Work out the volume of the gold bar.
Give your answer correct to 3 significant figures.

Solution

Now,

$$\begin{aligned}\text{mass} = \text{density} \times \text{volume} &\Rightarrow \text{volume} = \frac{\text{mass}}{\text{density}} \\ &\Rightarrow \text{volume} = \frac{12.5 \text{ kg}}{19.3 \text{ g/cm}^3} \\ &\Rightarrow \text{volume} = \frac{12.5 \text{ kg}}{0.019\,3 \text{ kg/cm}^3} \\ &\Rightarrow \text{volume} = 647.668\,393\,8 \text{ (FCD)} \\ &\Rightarrow \text{volume} = \underline{\underline{648 \text{ cm}^3 \text{ (3 sf)}}}.\end{aligned}$$

4. There are only blue pens, green pens, and red pens in a box. (3)

The ratio of the number of blue pens to the number of green pens is 2 : 5.
The ratio of the number of green pens to the number of red pens is 4 : 1.

There are less than 100 pens in the box.

What is the greatest number of red pens in the box?

Solution

Well,

$$\text{blue} : \text{green} = 2 : 5 = 8 : 20$$

and

$$\text{green} : \text{red} = 4 : 1 = 20 : 5.$$

So

$$\begin{aligned} \text{blue} : \text{green} : \text{red} &= 8 : 20 : 5 \ (8 + 20 + 5 = 33) \\ &= 16 : 40 : 10 \ (16 + 40 + 10 = 66) \\ &= 24 : 60 : 15 \ (24 + 60 + 15 = 99). \end{aligned}$$

Hence, there are 15 red pens.

5. (a) Find the value of the reciprocal of 1.6. (1)
Give your answer as a decimal.

Solution

$$\frac{1}{1.6} = 0.625.$$

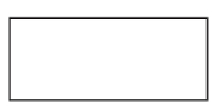
Jess rounds a number, x, to one decimal place.
The result is 9.8.

(b) Write down the error interval for x. (2)

Solution

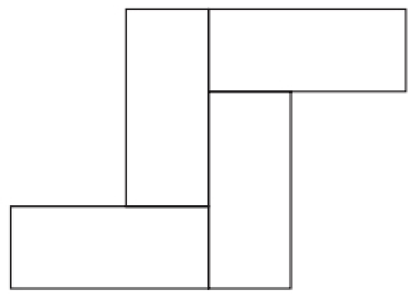
$9.75 \leqslant x < 9.85$.

6. Here is a rectangle. (5)



The length of the rectangle is 7 cm longer than the width of the rectangle.

4 of these rectangles are used to make this 8-sided shape.

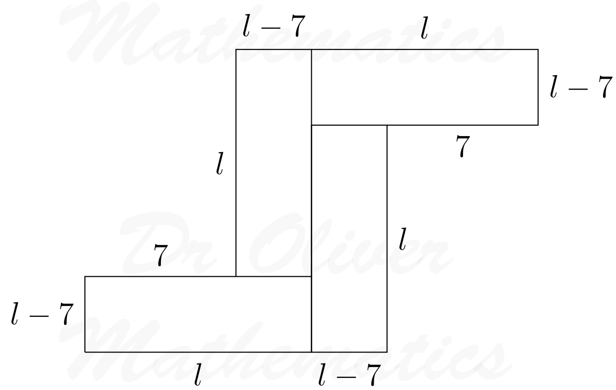


The perimeter of the 8-sided shape is 70 cm.

Work out the area of the 8-sided shape.

Solution

Let l cm be the length of the rectangle. Then $(l-7)$ cm is the width of the rectangle.



Now,

$$\begin{aligned}
\text{perimeter} = 70 &\Rightarrow l + (l-7) + 7 + l + (l-7) + l + (l-7) + 7 + l + (l-7) = 70 \\
&\Rightarrow 8l = 84 \\
&\Rightarrow l = 10\tfrac{1}{2} \text{ cm} \\
&\Rightarrow l - 7 = 3\tfrac{1}{2} \text{ cm.}
\end{aligned}$$

Hence,

$$\begin{aligned}
\text{area} &= 4 \times 10\tfrac{1}{2} \times 3\tfrac{1}{2} \\
&= \underline{\underline{147 \text{ cm}^2}}.
\end{aligned}$$

7. Work out (2)

$$(13.8 \times 10^7) \times (5.4 \times 10^{-12}).$$

Give your answer as an ordinary number.

Solution

$$\begin{aligned}(13.8 \times 10^7) \times (5.4 \times 10^{-12}) &= 7.452 \times 10^{-4} \\ &= \underline{\underline{0.000\,745\,2}}.\end{aligned}$$

8. When a drawing pin is dropped, it can land point down or point up.

Lucy, Mel, and Tom each dropped the drawing pin a number of times.

The table shows the number of times the drawing pin landed point down and the number of times the drawing pin landed point up for each person.

	Lucy	Mel	Tom
Point Down	31	53	16
Point Up	14	27	9

Rachael is going to drop the drawing pin once.

(a) Whose results will give the best estimate for the probability that the drawing pin will land point up? (1)
Give a reason for your answer.

Solution

	Lucy	Mel	Tom	Overall
Point Down	31	53	16	100
Point Up	14	27	9	50
Total	45	80	25	150

Mel because, e.g., he dropped the more times than either Lucy or Tom combined.

Stuart's going to drop the drawing pin twice.

(b) Use all the results in the table to work out an estimate for the probability that the drawing pin will land point up the first time and point down the second time. (2)

Solution

$$\begin{aligned}\text{P(point up, point down)} &= \tfrac{1}{3} \times \tfrac{2}{3} \\ &= \underline{\underline{\tfrac{2}{9}}}.\end{aligned}$$

9. Jack bought a new boat for £12 500.

The value, £V, of Jack's boat at the end of n years is given by the formula

$$V = 12\,500 \times (0.85)^n.$$

(a) At the end of how many years was the value of Jack's boat first less than 50% of the value of the boat when it was new? (2)

Solution

Year	Value
1	$V = 12\,500 \times (0.85)^1 = 10\,200$
2	$V = 12\,500 \times (0.85)^2 = 8\,670$
3	$V = 12\,500 \times (0.85)^3 = 7\,369.50$
4	$V = 12\,500 \times (0.85)^4 = 6\,264.075$
5	$V = 12\,500 \times (0.85)^5 = 5\,324.463\,75$

It takes 5 years.

A savings account pays interest at a rate of R% per year.
Jack invests £5 500 in the account one year year.

At the end of the year, Jack pays tax on the interest at a rate of 40%.
After paying the tax, he gets £79.20.

(b) Work out the value of R. (3)

Solution

$$\text{Amount of interest before tax} = \frac{79.20}{1 - 0.4} = £132.$$

So

$$R = \frac{132}{5\,500} \times 100\% = \underline{\underline{2.4\%}}.$$

10. There are only blue counters, yellow counters, green counters, and red counters in a bag. At counter is taken at random from the bag.

The table shows the probabilities of getting a blue counters or a yellow counters or a green counter.

Colour	Blue	Yellow	Green	Red
Probability	0.2	0.35	0.4	

(a) Work out the probability of getting a red counter. (1)

Solution

$$\begin{aligned} \mathrm{P}(R) &= 1 - (0.2 + 0.35 + 0.4) \\ &= 1 - 0.95 \\ &= \underline{\underline{0.05}}. \end{aligned}$$

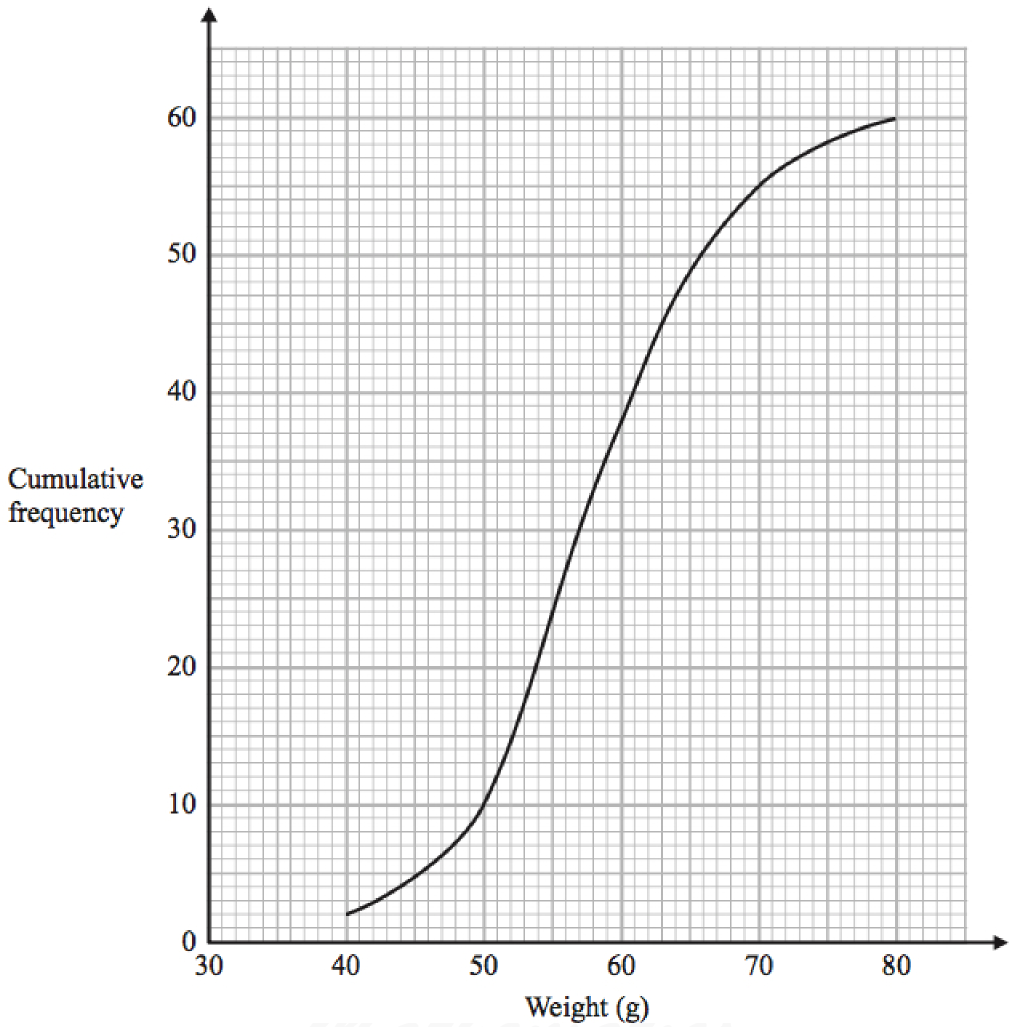
(b) What is the least possible number of counters in the bag? You must give a reason for your answer. (2)

Solution

There must be at least one red counter and the least possible number is

$$\frac{1}{0.05} = \underline{\underline{20}}.$$

11. The cumulative frequency graph shows information about the weight of 60 potatoes.



(a) Use the graph to find an estimate for the median weight. (1)

Solution

Draw a horizontal line across from 30 and read-off: 57 g.

Jamil says, "80 − 40 = 40 so the range of the weights is 40 g."

(b) Is Jamil correct? (1)

You must give a reason for your answer.

Solution

No: e.g.,the maximum weight could be less than 80; the minimum weight could be less than 40.

(c) Show that less than 25% of the potatoes have a weight greater than 65 g. (2)

Solution

Well,

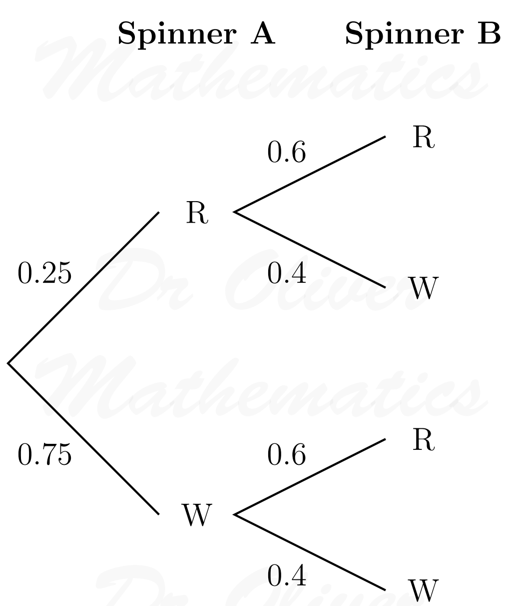
$$\tfrac{3}{4} \times 60 = 45$$

so draw a horizontal line across from 45 and read-off: 63 g. Hence, less than 25% of the potatoes have a weight greater than 65 g

12. Alan has two spinners, spinner **A** and spinner **B**. (3)
Each spinner can land only red or white.

The probability that spinner **A** will land on red is 0.25.
The probability that spinner **B** will land on red is 0.6.

The probability tree diagram shows this information.

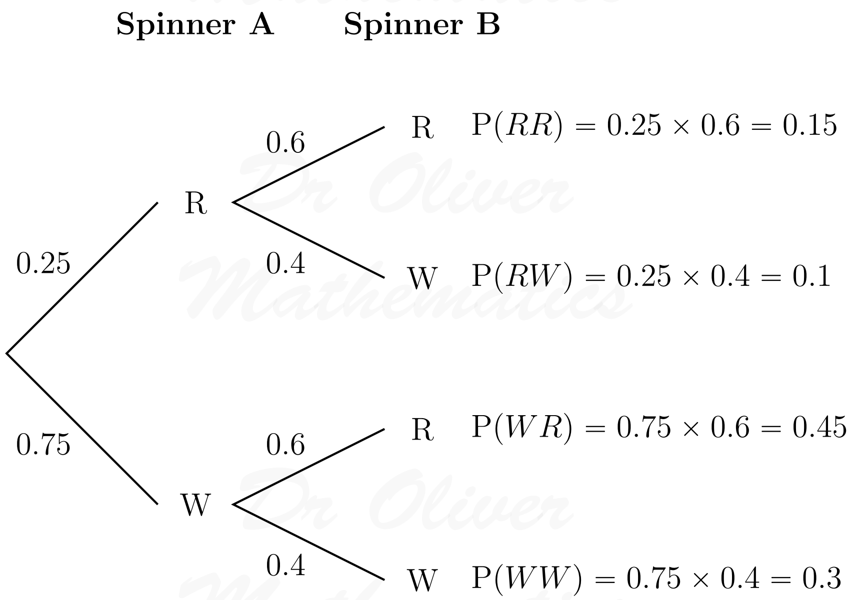


Alan spins spinner **A** once and he spins spinner **B** once.
He does this a number of times.

The number of times **both** spinners land on red is 24.

Work out an estimate for the number of times **both** spinners land on white.

Solution



It is double! Hence, an estimate for the number of times both spinners land on white is 48.

13. Write (2)

$$x^2 + 6x - 7$$

in the form

$$(x + a)^2 + b,$$

where a and b are integers.

Solution

$$\begin{aligned}(x^2 + 6x) - 7 &= (x^2 + 6x + 9) - 7 - 9 \\ &= \underline{\underline{(x + 3)^2 - 16}};\end{aligned}$$

hence, $\underline{\underline{a = 3}}$ and $\underline{\underline{b = -16}}$.

14. Cone **A** and cone **B** are mathematically similar. (3)
The ratio of the volume of cone **A** to the volume of cone **B** is 27 : 8.

The surface area of cone **A** is 297 cm^2.

Show that the surface area of cone **B** is 132 cm^2.

Solution

Well, the volume scale factor (VSF) is

$$27 : 8 = 3^3 : 2^3,$$

the length scale factor (LSF) is

$$3 : 2,$$

and the area scale factor (ASF) is

$$3^2 : 2^2 = 9 : 4.$$

Hence, the surface area of cone **B** is

$$297 \times \tfrac{4}{9} = \underline{\underline{132 \text{ cm}^2}},$$

as required.

15. (a) Show that the equation (2)

$$x^3 + 7x - 5 = 0$$

has a solution between $x = 0$ and $x = 1$.

Solution

Let

$$\text{f}(x) = x^3 + 7x - 5.$$

Then

$$\begin{aligned} \text{f}(0) &= (0)^3 + 7(0) - 5 \\ &= -5 \\ \text{f}(1) &= 1^3 + 7(1) - 5 \\ &= 3. \end{aligned}$$

Hence, as $\text{f}(x)$ is continuous, $\text{f}(x)$ has a $\underline{\underline{\text{solution}}}$ between $x = 0$ and $x = 1$.

(b) Show that the equation (2)

$$x^3 + 7x - 5 = 0$$

can be rearranged to give

$$x = \frac{5}{x^2 + 7}.$$

Solution

$$\begin{aligned} x^3 + 7x - 5 = 0 &\Rightarrow x^3 + 7x = 5 \\ &\Rightarrow x(x^2 + 7) = 5 \\ &\Rightarrow \underline{\underline{x = \frac{5}{x^2 + 7}}}, \end{aligned}$$

as required.

(c) Starting with $x_0 = 1$, use the iteration formula (3)

$$x_{n+1} = \frac{5}{x_n^2 + 7}$$

three times to find an estimate for the solution of

$$x^3 + 7x - 5 = 0.$$

Solution

$$\begin{aligned} x_1 &= \frac{5}{1^2 + 7} = \underline{\underline{0.625}} \\ x_2 &= \frac{5}{(0.625)^2 + 7} = \underline{\underline{0.676\,532\,769\,6 \text{ (FCD)}}} \\ x_3 &= \frac{5}{(0.676\ldots)^2 + 7} = \underline{\underline{0.670\,448\,300\,1 \text{ (FCD)}}}. \end{aligned}$$

(d) By substituting your answer to part (c) into (2)

$$x^3 + 7x - 5,$$

comment on the accuracy of your estimate for the solution to

$$x^3 + 7x - 5 = 0.$$

Solution

$$(0.670\ldots)^3 + 7(0.670\ldots) - 5 = -5.494\ldots \times 10^{-3}$$

Yes, it is accurate because $-5.494\ldots \times 10^{-3}$ is close to 0.

16. The petrol consumption of a car, in litres per 100 kilometres, is given by the formula (3)

$$\text{petrol consumption} = \frac{100 \times \text{Number of litres of petrol used}}{\text{Number of kilometres travelled}}.$$

Nathan's car travelled 148 kilometres, correct to 3 significant figures.
The car used 11.8 litres of petrol, correct to 3 significant figures.

Nathan says, "My car used less than 8 litres of petrol per 100 kilometres."

Could Nathan be wrong?
You must show how you get your answer.

Solution

The car travelled

$$147.5 \leqslant \text{kilometres} < 148.5$$

and used

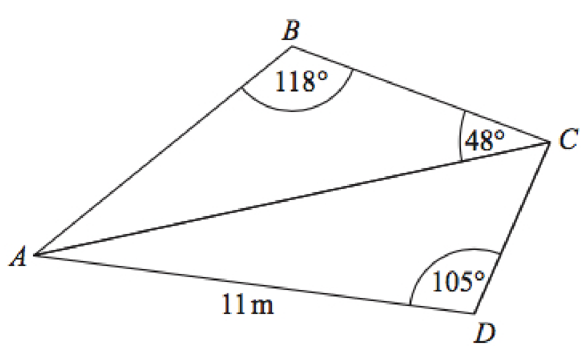
$$11.75 \leqslant \text{litres} < 11.85.$$

The upper limit is

$$\frac{100 \times 11.85}{147.5} = 8.033\,898\,305 \text{ (FCD)};$$

so, yes, Nathan could be wrong.

17. ABC and ADC are triangles. (5)



The area of triangle ADC is 56 m^2.

Work out the length of AB.
Give your answer correct to 1 decimal place.

Solution

This is a very good question!

$$\begin{aligned} \text{Area of } \triangle ADC = 56 \Rightarrow \tfrac{1}{2} \times 11 \times CD \times \sin 105^\circ &= 56 \\ \Rightarrow CD &= \frac{56}{5.5 \sin 105^\circ} \\ \Rightarrow CD &= 10.540\,993\,84 \text{ (FCD)}. \end{aligned}$$

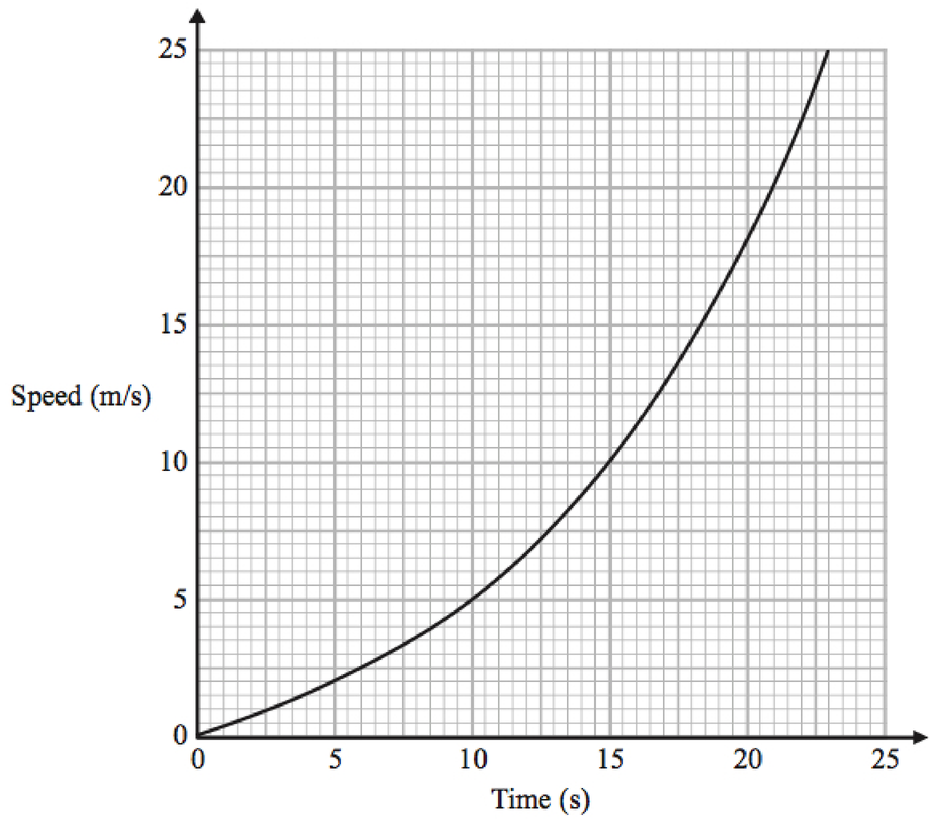
Now, we apply the cosine rule:

$$\begin{aligned} & AC^2 = AD^2 + CD^2 - 2 \times AD \times CD \times \cos ADC \\ \Rightarrow \quad & AC^2 = 11^2 + 10.540\ldots^2 - 2 \times 11 \times 10.540\ldots \times \cos 105^\circ \\ \Rightarrow \quad & AC^2 = 292.133\,170\,2 \text{ (FCD)} \\ \Rightarrow \quad & AC = 17.091\,903\,64 \text{ (FCD)}. \end{aligned}$$

Finally, we apply the sine rule:

$$\begin{aligned} \frac{AB}{\sin ACB} = \frac{AC}{\sin ABC} \Rightarrow \frac{AB}{\sin 48^\circ} &= \frac{17.091\ldots}{\sin 118^\circ} \\ \Rightarrow AB &= \frac{17.091\ldots \sin 48^\circ}{\sin 118^\circ} \\ \Rightarrow AB &= 14.385\,632\,68 \text{ (FCD)} \\ \Rightarrow \underline{\underline{AB}} &\underline{\underline{= 14.4 \text{ m (1 dp)}}}. \end{aligned}$$

18. Here is a speed-time graph.



(a) Work out an estimate for the distance the train travelled in the first 20 seconds. Use 4 strips of equal width. (3)

Solution

Well,

$$h = 20 - 04 = 5.$$

Now,

$$\begin{aligned} \text{estimate} &\approx \tfrac{1}{2} \times 5 \times [0 + 2(2 + 5 + 10) + 18] \\ &= \underline{\underline{130 \text{ m}}}. \end{aligned}$$

(b) Is your answer to (a) an underestimate or an overestimate of the actual distance the train travelled? Give a reason for your answer. (1)

Solution

It is an overestimate: the trapeziums that we use are larger than the area we are required to find.

19. Prove algebraically that the straight line with equation (5)

$$x - 2y = 10$$

is a tangent to the circle with equation

$$x^2 + y^2 = 20.$$

Solution

$$x - 2y = 10 \Rightarrow x = 2y + 10$$

and

$$x^2 + y^2 = 20 \Rightarrow (2y + 10)^2 + y^2 = 20$$

$\times$	$2y$	$+10$
$2y$	$4y^2$	$+20y$
$+10$	$+20y$	$+100$

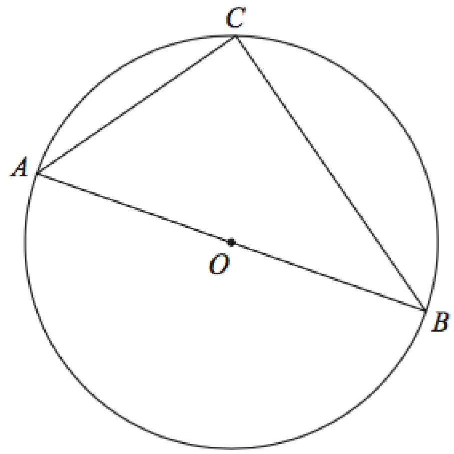
$$\begin{aligned}&\Rightarrow (4y^2 + 40y + 100) + y^2 = 20\\&\Rightarrow 5y^2 + 40y + 80 = 0\\&\Rightarrow 5(y^2 + 8y + 16) = 0\end{aligned}$$

$$\left.\begin{array}{ll}\text{add to:} & +8\\ \text{multiply to:} & +16\end{array}\right\} +4,\ +4$$

$$\begin{aligned}&\Rightarrow 5(y + 4)^2 = 0\\&\Rightarrow y + 4 = 0\\&\Rightarrow y = -4 \text{ (twice)}\\&\Rightarrow x = 2.\end{aligned}$$

So, we have a repeated root. Hence, the straight line is a tangent.

20. A, B, and C are points on the circumference of a circle, centre O. (4)
AOB is a diameter of the circle.



Prove that angle ACB equals $90°$.
You must **not** use any circle theorems in your proof.

Solution

Let $\angle OAC = x$. Then $\angle OCA = x$ (base angles) and $\angle AOC = 180 - 2x$ (completing the triangle). $\angle BOC = 2x$ (supplementary angles) and

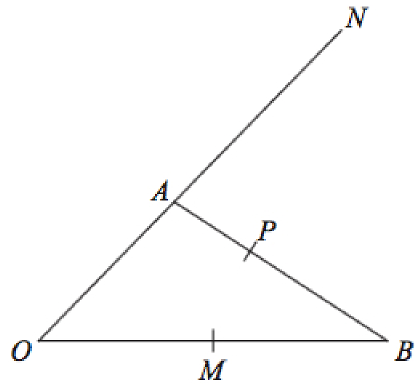
$$\begin{aligned}\angle OBC = \angle OCB &= \tfrac{1}{2}(180 - 2x) \\ &= 90 - x.\end{aligned}$$

Hence,

$$\angle ACB = \angle ACO + \angle OCB = x + (90 - x) = \underline{\underline{90^\circ}},$$

as required.

21. OAN, OMB, and APB are straight lines. (5)



$AN = 2OA$.
M is the midpoint of OB.
$\overrightarrow{OA} = \mathbf{a}$.
$\overrightarrow{OB} = \mathbf{b}$.
$\overrightarrow{AP} = k\overrightarrow{AB}$, where k is a scalar quantity.

Given that MPN is a straight line, find the value of k.

Solution

$$\begin{aligned}\overrightarrow{MN} &= \overrightarrow{MO} + \overrightarrow{ON} \\ &= -\tfrac{1}{2}\mathbf{b} + (\overrightarrow{OA} + \overrightarrow{AN}) \\ &= -\tfrac{1}{2}\mathbf{b} + (\mathbf{a} + 2\mathbf{a}) \\ &= 3\mathbf{a} - \tfrac{1}{2}\mathbf{b}.\end{aligned}$$

and

$$\begin{aligned}\overrightarrow{MP} &= \overrightarrow{MO} + \overrightarrow{OA} + \overrightarrow{AP} \\ &= -\tfrac{1}{2}\mathbf{b} + \mathbf{a} + k\overrightarrow{AB} \\ &= -\tfrac{1}{2}\mathbf{b} + \mathbf{a} + k(\mathbf{b} - \mathbf{a}) \\ &= (1-k)\mathbf{a} + (k - \tfrac{1}{2})\mathbf{b} \\ &= (1-k)\mathbf{a} - (\tfrac{1}{2} - k)\mathbf{b}.\end{aligned}$$

So, divide the ratios:

$$3 : \tfrac{1}{2} = (1-k) : (\tfrac{1}{2} - k) \Rightarrow \frac{3}{1-k} = \frac{\frac{1}{2}}{\frac{1}{2} - k}$$
$$\Rightarrow \tfrac{3}{2} - 3k = \tfrac{1}{2} - \tfrac{1}{2}k$$
$$\Rightarrow 1 = \tfrac{5}{2}k$$
$$\Rightarrow \underline{\underline{k = \tfrac{2}{5}}}.$$