

Dr Oliver Mathematics
Applied Mathematics: Mechanics or Statistics
Section B
2009 Paper
1 hour

The total number of marks available is 32.

You must write down all the stages in your working.

1. Obtain the binomial expansion of

$$\left(b - \frac{2}{b}\right)^5$$

(4)

and simplify the expression.

2. Obtain

$$\int_0^{\frac{1}{3}\pi} \cos^5 x \sin x \, dx$$

(4)

by using the substitution $u = \cos x$ or otherwise.

3. A particle moves along a curve in the x - y plane. The curve is defined by the parametric equations

$$x = t^2 + 1, \quad y = 1 - 3t^3,$$

where t is the time elapsed since the start.

- (a) Find $\frac{dy}{dx}$ in terms of t .

(3)

- (b) Hence obtain an equation of the tangent to the curve when $t = 2$.

(2)

4. Determine k such that the matrix

$$\begin{pmatrix} 1 & 1 & 0 \\ 0 & k-2 & -1 \\ 1 & 2 & k \end{pmatrix}$$

(4)

does not have an inverse.

5. An industrial scientist finds that the differential equation

$$t \frac{dx}{dt} - 2x = 3t^2$$

models a production process.

(a) Find the general solution of the differential equation. (5)

(b) Hence find the particular solution given $x = 1$ when $t = 1$. (1)

6. (a) Given (2)

$$f(x) = x \tan 2x$$

for $-\frac{1}{4}\pi < x < \frac{1}{4}\pi$, obtain an expression for $f'(x)$.

(b) Show that (3)

$$f''(x) = 4 \sec^2 2x(1 + 2x \tan 2x).$$

(c) Hence find the exact value of (4)

$$\int_0^{\frac{1}{6}\pi} \frac{1 + 2x \tan 2x}{\cos^2 2x} dx.$$

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