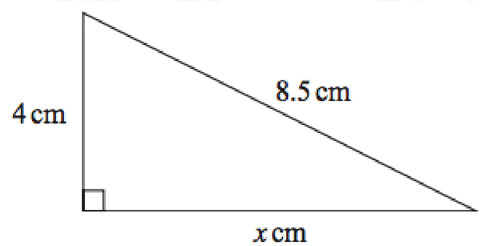
Dr Oliver Mathematics
GCSE Mathematics
2022 June Paper 3H: Calculator
1 hour 30 minutes

The total number of marks available is 80.
You must write down all the stages in your working.

1. Here is a right-angled triangle. (2)



Work out the value of x.

Solution

Pythagoras' theorem:

$$\begin{aligned} 4^2 + x^2 = (8.5)^2 &\Rightarrow 16 + x^2 = 72.25 \\ &\Rightarrow x^2 = 56.25 \\ &\Rightarrow \underline{\underline{x = 7.5}} \end{aligned}$$

2.

$$T = 4m^2 - 11.$$

(a) Work out the value of T when $m = -3$. (2)

Solution

$$\begin{aligned} T &= 4[(-3)^2] - 11 \\ &= 4(9) - 11 \\ &= 36 - 11 \\ &= \underline{\underline{25}}. \end{aligned}$$

(b) Make p the subject of the formula (2)

$$d = 3p + 4.$$

Solution

$$\begin{aligned} d = 3p + 4 &\Rightarrow 3p = d - 4 \\ &\Rightarrow \underline{\underline{p = \frac{d-4}{3}}}. \end{aligned}$$

3. Rick, Selma and Tony are playing a game with counters. (5)

- Rick has some counters.
- Selma has twice as many counters as Rick.
- Tony has 6 counters less than Selma.

In total they have 54 counters.

$$\text{The number of counters Rick has} : \text{the number of counters Tony has} = 1 : p.$$

Work out the value of p.

Solution

Let x be the number of counters that Rick has.
Then Selma has $2x$ counters and Tony has $(2x - 6)$ counters.
Now,

$$\begin{aligned} x + 2x + (2x - 6) = 54 &\Rightarrow 5x - 6 = 54 \\ &\Rightarrow 5x = 60 \\ &\Rightarrow x = 12, \\ &\Rightarrow 2x = 24, \\ &\Rightarrow 2x - 6 = 18. \end{aligned}$$

Finally,

$$\begin{aligned} \text{number of counters Rick has} : \text{number of counters Tony has} &= 2 : 3 \\ &= \underline{\underline{1 : \tfrac{3}{2}}}. \end{aligned}$$

4. Jo is going to buy 15 rolls of wallpaper. (4)

Here is some information about the cost of rolls of wallpaper from each of two shops.

Chic Decor	**Style Papers**
3 rolls for £36	Pack of 5 rolls normal price £70 12% off the normal price

Jo wants to buy the 15 rolls of wallpaper as cheaply as possible.

Should Jo buy the wallpaper from Chic Decor or from Style Papers?
You must show how you get your answer.

Solution

Chic Decor:

$$15 \times \frac{36}{3} = £180.$$

Style Papers:

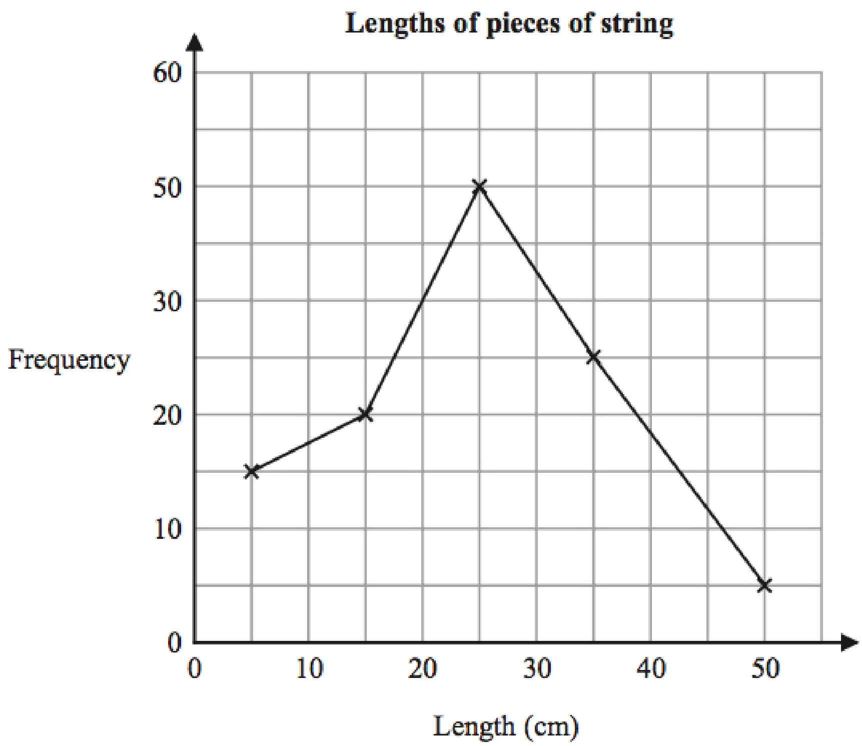
$$\begin{aligned}(1 - 0.12) \times 15 \times \frac{70}{5} &= 0.88 \times 15 \times 14 \\ &= £184.80.\end{aligned}$$

Hence, Jo must buy the wallpaper from Chic Decor, saving £4.80.

5. The table gives information about the lengths, in cm, of some pieces of string. (2)

Length (t cm)	Frequency
$0 < t \leqslant 10$	15
$10 < t \leqslant 20$	20
$20 < t \leqslant 30$	50
$30 < t \leqslant 40$	25
$40 < t \leqslant 50$	5

Amos draws a frequency polygon for the information in the table.



Write down **two** mistakes that Amos has made.

Solution

E.g., Amos has forgotten '40' on the vertical axis,
Amos has plotted the point for $40 < t \leqslant 50$ at the right end and not in the middle (as has for rest of them).

6. Jessica runs for 15 minutes at an average speed of 6 miles per hour. (4)
She then runs for 40 minutes at an average speed of 9 miles per hour.

It takes Amy 45 minutes to run the same total distance that Jessica runs.

Work out Amy's average speed.
Give your answer in miles per hour.

Solution

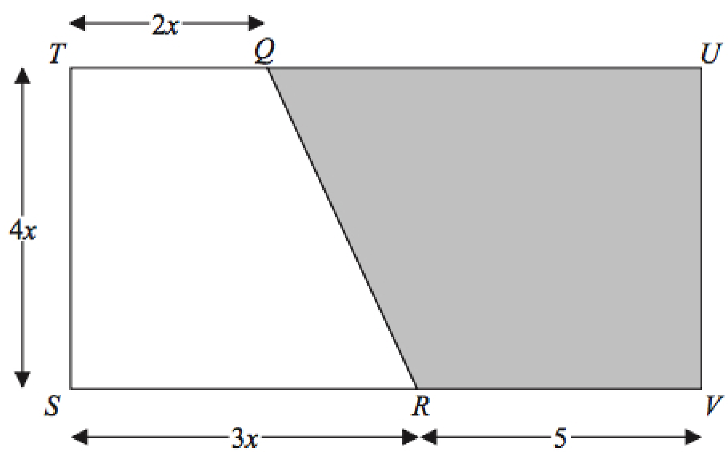
Jessica runs a distance of

$$\begin{aligned}(\tfrac{15}{60} \times 6) + (\tfrac{40}{60} \times 9) &= 1.5 + 6 \\ &= 7.5 \text{ miles.}\end{aligned}$$

Finally,

$$\begin{aligned}\text{Amy's average speed} &= \frac{7.5 \text{ miles}}{45 \text{ mins}} \\ &= \frac{7.5 \text{ miles}}{\frac{3}{4} \text{ hour}} \\ &= \underline{\underline{10 \text{ miles per hour}}}.\end{aligned}$$

7. The diagram shows rectangle $STUV$. (3)
TQU and SRV are straight lines.
All measurements are in cm.



The area of trapezium $QUVR$ is A cm^2.

Show that
$$A = 2x^2 + 20x.$$

Solution

Well,

$$\begin{aligned}\text{area of trapezium } RSTQ &= \tfrac{1}{2} \times 4x \times (2x + 3x)\\ &= 2x \times 5x\\ &= 10x^2\end{aligned}$$

and

$$\begin{aligned}\text{area of rectangle } STUV &= 4x \times (3x + 5)\\ &= 12x^2 + 20x.\end{aligned}$$

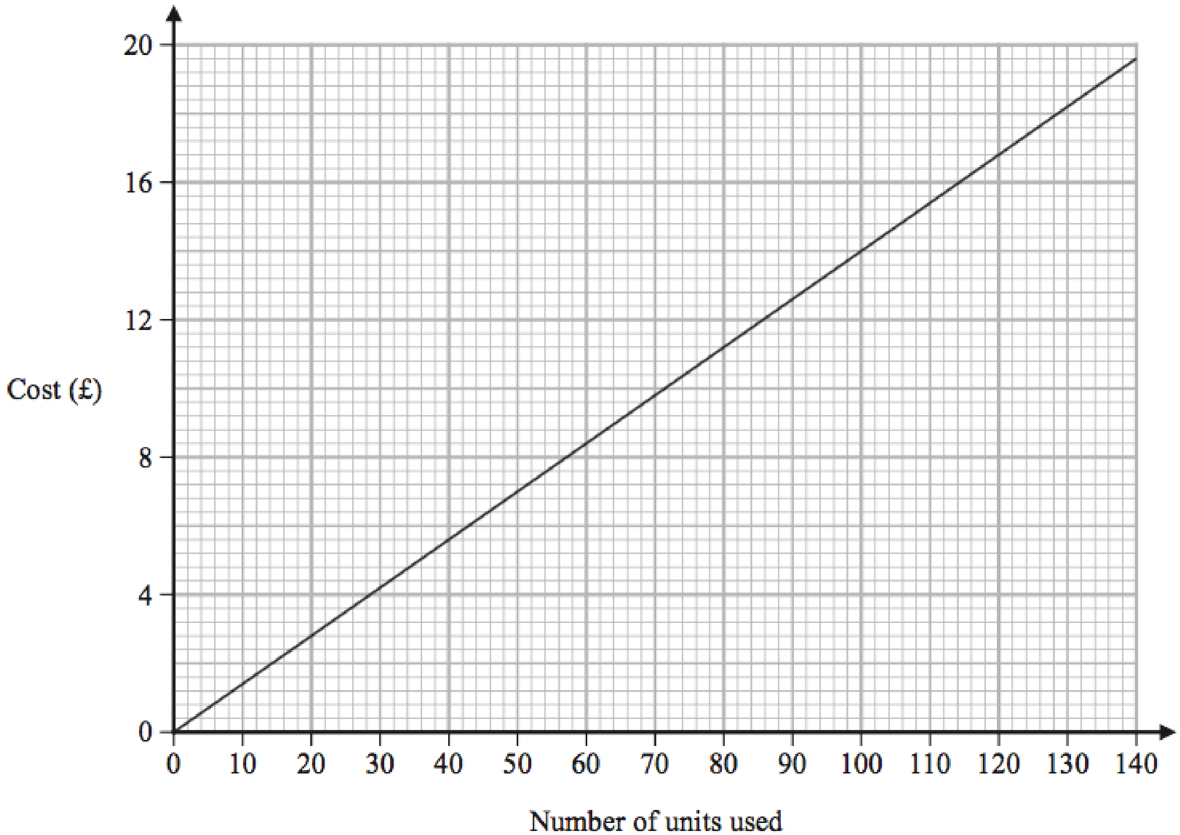
Finally,

$$\begin{aligned}\text{area of trapezium } QUVR &= \text{area of rectangle } STUV - \text{area of trapezium } RSTQ\\ &= (12x^2 + 20x) - 10x^2\\ &= \underline{\underline{2x^2 + 20x}},\end{aligned}$$

as required.

8. An electricity company charges the same fixed amount for each unit of electricity used.

   David uses this graph to work out the total cost of the electricity he has used.



(a) Work out the gradient of the straight line. (2)

Solution

E.g., take the points $(0, 0)$ and $(80, 11.2)$:

$$\begin{aligned} \text{gradient} &= \frac{11.2 - 0}{80 - 0} \\ &= \underline{\underline{0.14}}. \end{aligned}$$

(b) What does the gradient of this line represent? (1)

Solution

E.g., <u>cost per unit of electricity</u>.

9. (a) Express (3)

$$\sqrt{\frac{10^{360}}{10^{150} \times 10^{90}}}$$

as a power of 10.

Solution

$$\begin{aligned}\sqrt{\frac{10^{360}}{10^{150}\times 10^{90}}} &= \sqrt{\frac{10^{360}}{10^{240}}}\\ &= \sqrt{10^{120}}\\ &= \sqrt{(10^{60})^2}\\ &= \underline{\underline{10^{60}}}.\end{aligned}$$

Liam was asked to express $(12^{50})^2$ as a power of 12.

Liam wrote

$$(12^{50})^2 = 12^{50^2} = 12^{2\,500}.$$

Liam's method is wrong.

(b) Explain why. (1)

Solution

He is $\underline{\underline{\text{wrong}}}$ because

$$\begin{aligned}(12^{50})^2 &= 12^{50\times 2}\\ &= 12^{100}.\end{aligned}$$

10. Jane bought a new car three years ago. (3)

At the end of the first year the value of the car had decreased by 12.5%.
The value of the car then decreased by 10% each year for the next two years.

At the end of the three years, the value of the car was £17 010.

Work out the value of the car when Jane bought it three years ago.

Solution

Well,

$$\begin{aligned} (1 - 0.125) \times (1 - 0.1)^2 \times \text{original price} &= 17\,010 \\ \Rightarrow \quad 0.875 \times 0.81 \times \text{original price} &= 17\,010 \\ \Rightarrow \quad \text{original price} &= \frac{17\,010}{0.875 \times 0.81} \\ \Rightarrow \quad \text{original price} &= \underline{\underline{\pounds 24\,000}}. \end{aligned}$$

11. Rayheem has (2)

- 16 shirts,
- 5 pairs of jeans, and
- 3 jackets.

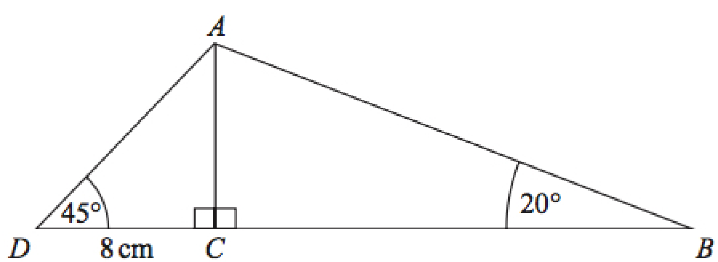
Rayheem chooses an outfit to wear.
An outfit is 1 shirt, 1 pair of jeans, and 1 jacket.

Work out how many different outfits Rayheem can choose.

Solution

$$16 \times 5 \times 3 = \underline{\underline{240}}.$$

12. ABC and ACD are right-angled triangles. (3)



- $DC = 8$ cm.
- Angle $ADC = 45°$.
- Angle $ABC = 20°$.

Work out the length of AB.
Give your answer correct to 3 significant figures.

Solution

$$\begin{aligned} \tan = \frac{\text{opp}}{\text{adj}} &\Rightarrow \tan 45^\circ = \frac{AC}{8} \\ &\Rightarrow AC = 8 \tan 45^\circ \\ &\Rightarrow AC = 8 \end{aligned}$$

and

$$\begin{aligned} \sin = \frac{\text{opp}}{\text{hyp}} &\Rightarrow \sin 20^\circ = \frac{8}{AB} \\ &\Rightarrow AB = \frac{8}{\sin 20^\circ} \\ &\Rightarrow AB = 23.390\,435\,2 \text{ (FCD)} \\ &\Rightarrow \underline{\underline{AB = 23.4 \text{ cm (3 sf)}}}. \end{aligned}$$

13. **a** and **b** are vectors such that (3)

$$\mathbf{a} = \begin{pmatrix} 2 \\ -3 \end{pmatrix} \text{ and } 3\mathbf{a} - 2\mathbf{b} = \begin{pmatrix} 8 \\ -17 \end{pmatrix}.$$

Find **b** as a column vector.

Solution

$$\begin{aligned} 3\begin{pmatrix} 2 \\ -3 \end{pmatrix} - 2\mathbf{b} = \begin{pmatrix} 8 \\ -17 \end{pmatrix} &\Rightarrow \begin{pmatrix} 6 \\ -9 \end{pmatrix} - 2\mathbf{b} = \begin{pmatrix} 8 \\ -17 \end{pmatrix} \\ &\Rightarrow \begin{pmatrix} 6 \\ -9 \end{pmatrix} - \begin{pmatrix} 8 \\ -17 \end{pmatrix} = 2\mathbf{b} \\ &\Rightarrow 2\mathbf{b} = \begin{pmatrix} -2 \\ 8 \end{pmatrix} \\ &\Rightarrow \underline{\underline{\mathbf{b} = \begin{pmatrix} -1 \\ 4 \end{pmatrix}}}. \end{aligned}$$

14. (a) Factorise fully (2)

$$4p^2 - 36.$$

Solution

Difference of two squares:

$$\begin{aligned} 4p^2 - 36 &= 4(p^2 - 9) \\ &= \underline{\underline{4(p-3)(p+3)}}. \end{aligned}$$

(b) Show that (3)

$$(m+4)(2m-5)(3m+1)$$

can be written in the form

$$am^3 + bm^2 + cm + d,$$

where a, b, c, and d are integers.

Solution

Well,

$\times$	m	$+4$
$2m$	$2m^2$	$+8m$
-5	$-5m$	-20

so

$$(m+4)(2m-5) = 2m^2 + 3m - 20$$

and

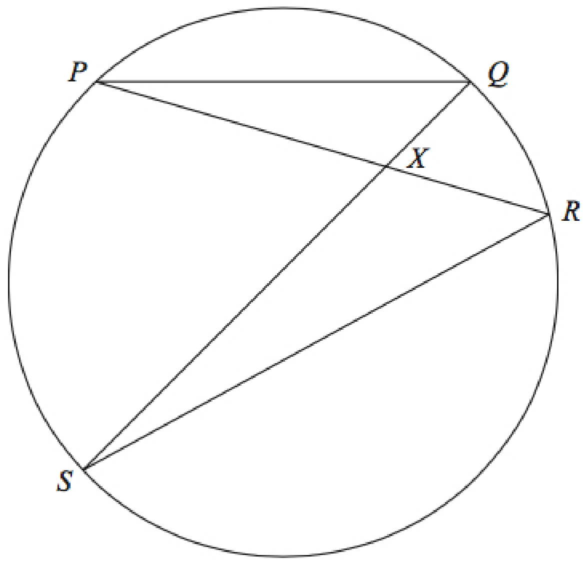
$\times$	$2m^2$	$+3m$	-20
$3m$	$6m^3$	$+9m^2$	$-60m$
$+1$	$+2m^2$	$+3m$	-20

so

$$(2m^2 + 3m - 20)(3m+1) = \underline{\underline{6m^3 + 11m^2 - 57m - 20}};$$

hence, $\underline{\underline{a = 6}}$, $\underline{\underline{b = 11}}$, $\underline{\underline{c = -57}}$, and $\underline{\underline{d = -20}}$.

15. P, Q, R, and S are four points on a circle. (3)



PXR and SXQ are straight lines.

Prove that triangle PQX and triangle SRX are similar.

Solution

$\angle PXQ = \angle SXR$ (corresponding angles)
$\angle XPQ = \angle XSR$ (angles in the same segment)
$\angle XQP = \angle XRS$ (angles in the same segment)
So, triangle PQX and triangle SRX are similar because all three pairs of corresponding angles are equal.

16. (3)

$$p = \sqrt{\frac{2e}{f}}.$$

$e = 6.8$, correct to 1 decimal place.
$f = 0.05$, correct to 1 significant figure.

Work out the upper bound for the value of p.
Give your answer correct to 3 significant figures.
You must show all your working.

Solution

Well,

$$6.75 \leqslant e < 6.85$$

and

$$0.045 \leqslant f < 0.055.$$

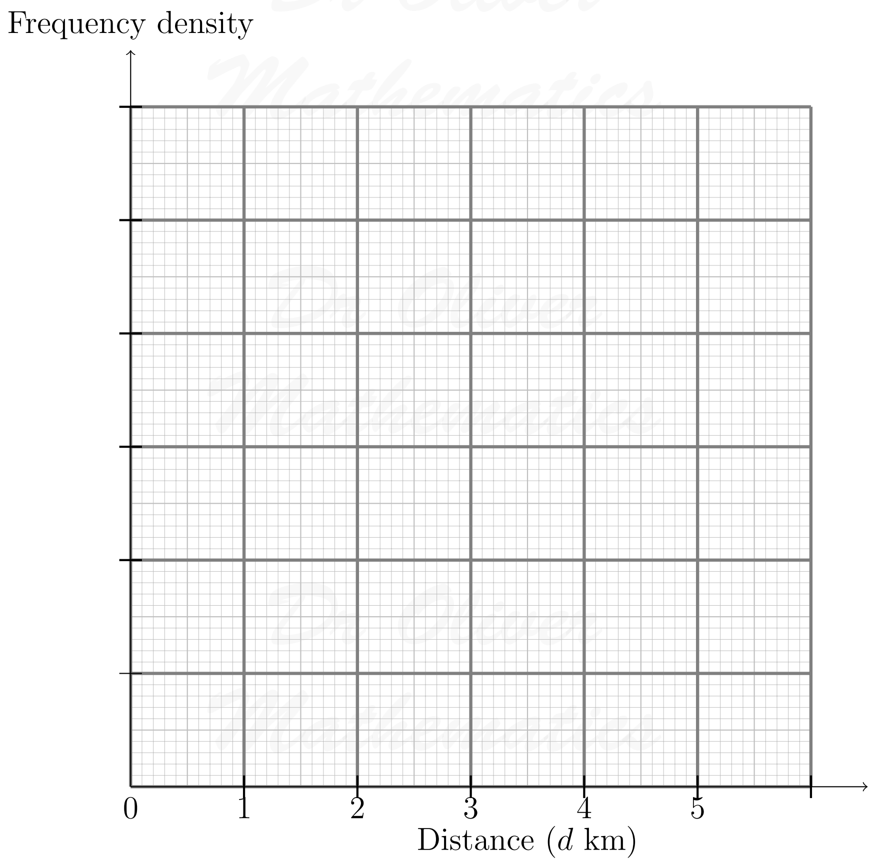
So choose the **upper** bound for e and choose the **lower** bound for f:

$$\begin{aligned} p &= \sqrt{\frac{2 \times 6.85}{0.045}} \\ &= 17.448\,336\,44 \text{ (FCD)} \\ &= \underline{\underline{17.4 \text{ (3 sf)}}}. \end{aligned}$$

17. The table gives information about the distances, in miles, that some Year 10 students live from school.

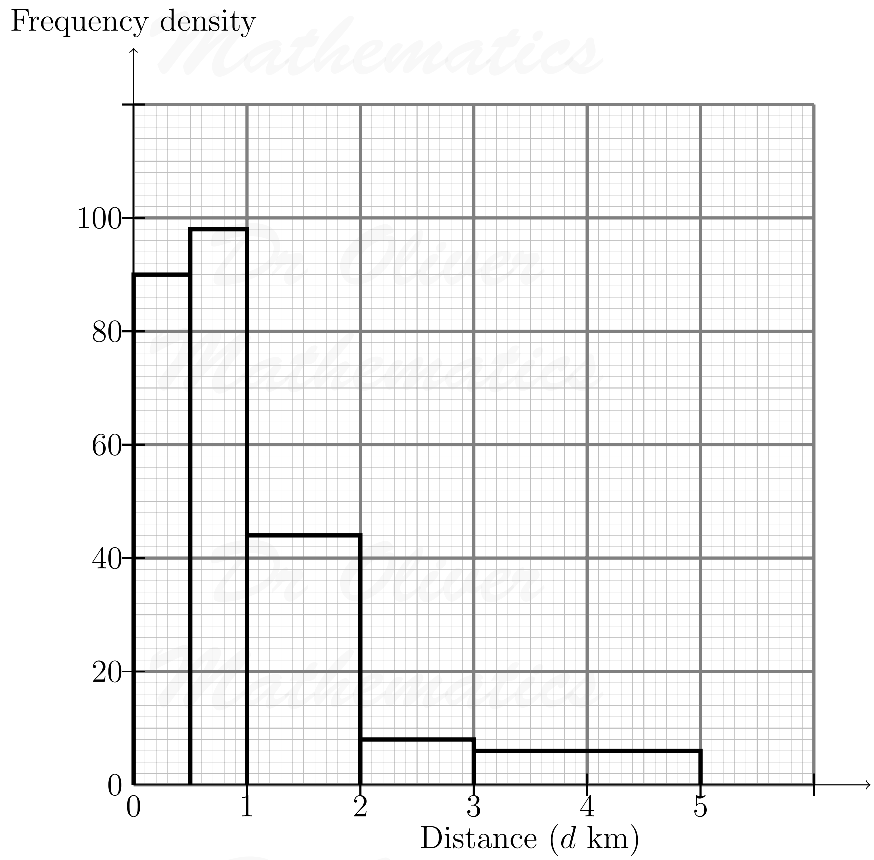
Distance (d miles)	Frequency
$0 < d \leqslant 1.0$	90
$1.0 < d \leqslant 1.5$	48
$1.5 < d \leqslant 2.0$	22
$2.0 < d \leqslant 3.0$	8
$3.0 < d \leqslant 5.0$	12

(a) On the grid, draw a histogram for this information. (3)

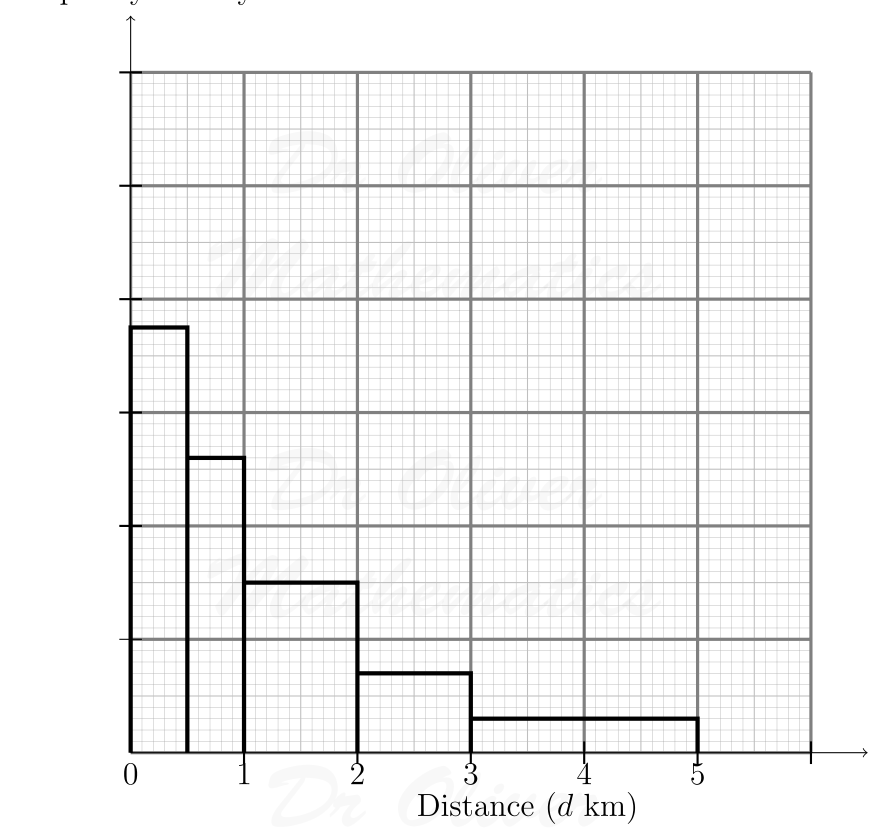


Solution

Distance (d miles)	Frequency	Width	Frequency Density
$0 < d \leqslant 1.0$	90	1	90
$1.0 < d \leqslant 1.5$	48	0.5	98
$1.5 < d \leqslant 2.0$	22	0.5	44
$2.0 < d \leqslant 3.0$	8	1	8
$3.0 < d \leqslant 5.0$	12	2	6


Frequency density
100
80
60
40
20
0
0
1
2
3
4
5
Distance (d km)

The histogram below shows information about the distances, in miles, that some Year 11 students live from school.



The number of Year 11 students who live between 1 and 2 miles from school is n.

(b) Find an expression, in terms of n, for the number of Year 11 students who live between 3 and 5 miles from school. (2)

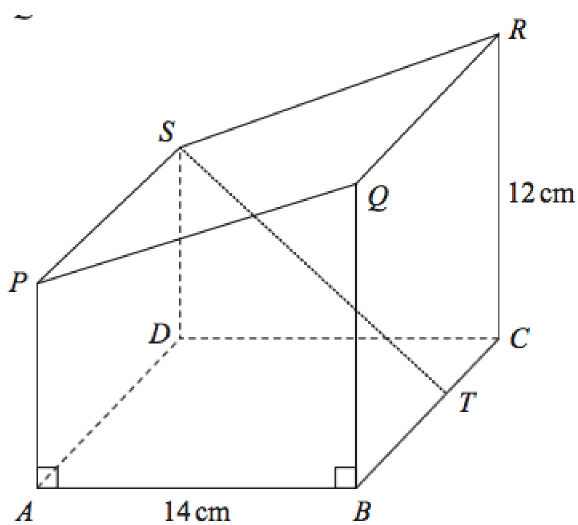
Solution

Distance (d miles)	Frequency Density	Width	Frequency
$1.0 < d \leqslant 2$	1.5	1	$1.5 \times 1 = 1.5$
$3.0 < d \leqslant 5.0$	0.3	2	$0.3 \times 2 = 0.6$

Let us call the number of Year 11 students who live between 3 and 5 miles from school m. Now,

$$\begin{aligned} m : n = 0.6 : 1.5 &\Rightarrow \frac{m}{n} = \frac{0.6}{1.5} \\ &\Rightarrow \frac{m}{n} = 0.4 \\ &\Rightarrow \underline{\underline{m = 0.4n}}. \end{aligned}$$

18. Here is a prism $ABCDSPQR$. (5)



The base $ABCD$ of the prism is a square of side 14 cm.
T is the point on BC such that $BT : TC = 4 : 3$.

The cross-section of the prism is in the shape of a trapezium of area 147 cm^2.
$CR = 12$ cm.

Find the size of the angle between the line ST and the base $ABCD$.
Give your answer correct to 1 decimal place.

Solution

We want to find the $\angle DTS$. Now,

$$BT : TC = 4 : 3 \Rightarrow BT : TC = 8 : 6$$

so that makes $TC = 6$ cm. Next,

$$\begin{aligned} DT^2 = CD^2 + TC^2 &\Rightarrow DT^2 = 14^2 + 6^2 \\ &\Rightarrow DT^2 = 196 + 36 \\ &\Rightarrow DT^2 = 232 \\ &\Rightarrow DT = 2\sqrt{58} \text{ cm}. \end{aligned}$$

Now, the cross-section of the prism is in the shape of a trapezium of area 147 cm^2:

$$\begin{aligned} \tfrac{1}{2} \times 14 \times (SD + 12) = 147 &\Rightarrow SD + 12 = 21 \\ &\Rightarrow SD = 9. \end{aligned}$$

Finally,

$$\begin{aligned} \tan = \frac{\text{opp}}{\text{adj}} &\Rightarrow \tan DTS = \frac{9}{2\sqrt{58}} \\ &\Rightarrow \angle DTS = 30.577\,946\,78 \text{ (FCD)} \\ &\Rightarrow \underline{\underline{\angle DTS = 30.6^\circ \text{ (1 dp)}}}. \end{aligned}$$

19. Show that (4)

$$\frac{3x}{x+2} - \frac{2x+1}{x-2} - 1$$

can be written in the form

$$\frac{ax+b}{x^2-4},$$

where a and b are integers.

Solution

$$\frac{3x}{x+2} - \frac{2x+1}{x-2} - 1 = \frac{3x(x-2) - (2x+1)(x+2) - (x+2)(x-2)}{(x+2)(x-2)}$$

$\times$	x	$+2$
$2x$	$2x^2$	$+4x$
$+1$	$+x$	$+2$

$\times$	x	-2
x	x^2	$-2x$
$+2$	$+2x$	-4

$$\begin{aligned} &= \frac{(3x^2 - 6x) - (2x^2 + 5x + 2) - (x^2 - 4)}{x^2 - 4} \\ &= \frac{3x^2 - 6x - 2x^2 - 5x - 2 - x^2 + 4}{x^2 - 4} \\ &= \underline{\underline{\frac{-11x + 2}{x^2 - 4}}}; \end{aligned}$$

hence, $\underline{\underline{a = -3}}$ and $\underline{\underline{b = 6}}$.

20. The profit made by a shop increases each year. (4)

The profit made by the shop in year n is £P_n.

Given that the profit made by the shop in the next year is£P_{n+1} then

$$P_{n+1} = aP_n + 800,$$

where a is a constant.

The table shows the profit made by the shop in 2018 and in 2019.

Year	2018	2019
Profit	£24 000	£29 600

Work out the profit predicted to be made by the shop in 2021.

Solution

Now,

$$\begin{aligned} 29\,600 = 24\,000a + 800 &\Rightarrow 24\,000a = 28\,800 \\ &\Rightarrow a = 1.2. \end{aligned}$$

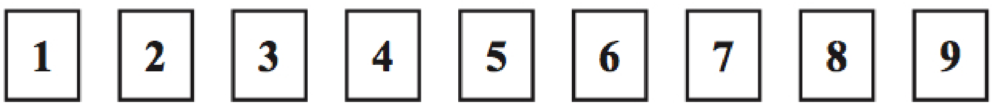
Finally,

$$\begin{aligned} P_{2000} &= 29\,600(1.2) + 800 \\ &= 35\,520 + 800 \\ &= 36\,320 \end{aligned}$$

and

$$\begin{aligned} P_{2001} &= 36\,320(1.2) + 800 \\ &= 43\,584 + 800 \\ &= \underline{\underline{44\,384}}. \end{aligned}$$

21. Ray has nine cards numbered 1 to 9. (4)



Ray takes at random three of these cards.

He works out the sum of the numbers on the three cards and records the result.

Work out the probability that the result is an even number.

Solution

He has to choose

- three evens numbers or
- one even number and two odd numbers (in whichever order: EOO, OEO, or OOE).

Now,

$$\begin{aligned} \text{P(three evens numbers)} &= \tfrac{4}{9} \times \tfrac{3}{8} \times \tfrac{2}{7} \\ &= \tfrac{1}{21} \end{aligned}$$

and

$$\begin{aligned}\text{P(one even, two odd)} &= 3 \times \tfrac{4}{9} \times \tfrac{5}{8} \times \tfrac{4}{7} \\ &= \tfrac{10}{21}.\end{aligned}$$

Finally,

$$\begin{aligned}\text{P(result is an even number)} &= \tfrac{1}{21} + \tfrac{10}{21} \\ &= \underline{\underline{\tfrac{11}{21}}}.\end{aligned}$$

22. **L** is the straight line with equation (5)

$$y = 2x - 5.$$

C is a graph with equation

$$y^2 = 6x^2 - 25x - 8.$$

Using algebra, find the coordinates of the points of intersection of **L** and **C**.
You must show all your working.

Solution

$\times$	$2x$	-5
$2x$	$4x^2$	$-10x$
-5	$-10x$	$+25$

Well,

$$\begin{aligned}y^2 = 6x^2 - 25x - 8 &\Rightarrow (2x-5)^2 = 6x^2 - 25x - 8 \\ &\Rightarrow 4x^2 - 20x + 25 = 6x^2 - 25x - 8 \\ &\Rightarrow 2x^2 - 5x - 33 = 0\end{aligned}$$

$$\left.\begin{array}{ll}\text{add to:} & -5 \\ \text{multiply to:} & (+2)\times(-33) = -66\end{array}\right\} -11,\ +6$$

$$\begin{aligned}
&\Rightarrow 2x^2 - 11x + 6x - 33 = 0 \\
&\Rightarrow x(2x-11) + 3(2x-11) = 0 \\
&\Rightarrow (x+3)(2x-11) = 0 \\
&\Rightarrow x+3 = 0 \text{ or } 2x-11 = 0 \\
&\Rightarrow x = -3 \text{ or } x = 5.5 \\
&\Rightarrow y = -11 \text{ or } y = 6;
\end{aligned}$$

hence, the intersection of **L** and **C** are $\underline{\underline{(-3,-11)}}$ and $\underline{\underline{(5.5,6)}}$.