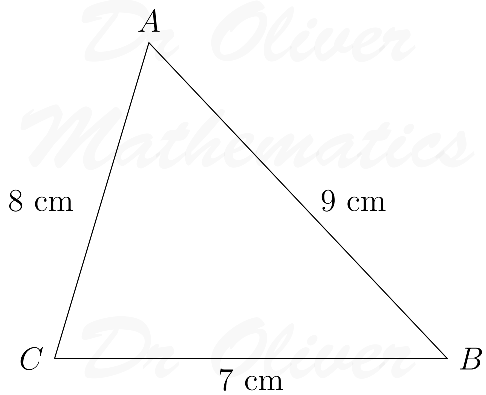
Dr Oliver Mathematics
Hero's (or Heron's) Method

In this note, we will investigate Hero's (or Heron's) Method.

Suppose we have the following triangle.



What is its area? Well, we know

$$\begin{aligned}\cos A^\circ = \frac{b^2 + c^2 - a^2}{2bc} &\Rightarrow \cos A^\circ = \frac{8^2 + 9^2 - 7^2}{2 \times 8 \times 9} \\ &\Rightarrow \cos A^\circ = \tfrac{2}{3} \\ &\Rightarrow \sin A^\circ = \tfrac{\sqrt{5}}{3}\end{aligned}$$

involving the trick

$$\sin^2 A^\circ + \cos^2 A^\circ = 1.$$

Now,

$$\begin{aligned}\text{area} &= \tfrac{1}{2} \times 8 \times 9 \times \tfrac{\sqrt{5}}{3} \\ &= \underline{\underline{12\sqrt{5} \text{ cm}^2}}.\end{aligned}$$

We will introduce the *semi-perimeter*, s, i.e.,

$$s = \tfrac{1}{2}(a + b + c).$$

Then the area will equal

$$\boxed{\text{area} = \sqrt{s(s-a)(s-b)(s-c)};}$$

the details are omitted.

In the former example, $a = 7$, $b = 8$, $c = 9$, and

$$s = \tfrac{1}{2}(7 + 8 + 9) = 12.$$

Finally,

$$\begin{aligned}\text{area} &= \sqrt{12(12-7)(12-8)(12-9)} \\ &= \sqrt{12 \times 5 \times 4 \times 3} \\ &= \sqrt{720} \\ &= \underline{\underline{12\sqrt{5}\text{ cm}^2}}.\end{aligned}$$

Second, and final, example. In triangle DEF, we have the sides 13.2 cm, 15.4 cm, and 5.1 cm. What is its area? Give your answer to 3 significant figures.

$$s = \tfrac{1}{2}(13.2 + 15.4 + 5.1) = 16.85$$

and

$$\begin{aligned}\text{area} &= \sqrt{16.85(16.85-13.2)(16.85-15.4)(16.85-5.1)} \\ &= \sqrt{16.85 \times 3.65 \times 1.45 \times 11.75} \\ &= 32.370\,493\,41 \text{ (FCD)} \\ &= \underline{\underline{32.4\text{ cm}^2\text{ (3 sf)}}}.\end{aligned}$$