Dr Oliver Mathematics
Worked Examples
Mass, Density, and Volume 3

From: Edexcel GCSE Mathematics (9-1) Practice Tests Set 20: Paper 2H/3H

1. The diagram shows a solid made from a cylinder and a hemisphere. (6)
The cylinder and the hemisphere are both made from the same metal.

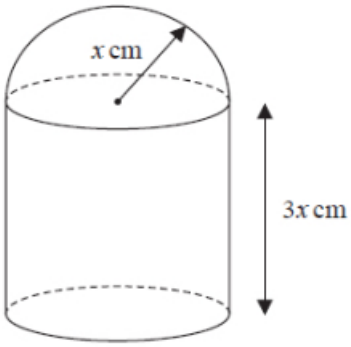

Diagram **NOT** accurately drawn

The plane face of the hemisphere coincides with the upper plane face of the cylinder.

The radius of the cylinder and the radius of the hemisphere are both x cm.
The height of the cylinder is $3x$ cm.

The total surface area of the solid is 81π cm^2.
The mass of the solid is 840 grams.

The following table gives the density of each of four metals.

Metal	Density (g/cm^3)
Aluminium	2.7
Nickel	8.9
Silver	10.5
Gold	19.3

The metal used to make the solid is one of the metals in the table.

Determine the metal used to make the solid.
Show your working clearly.

Solution

Let d g/cm^3 be the density of the material.

We need to work out

- surface area of the cylinder,
- surface area of the hemisphere,
- volume of the cylinder, and
- volume of the hemisphere.

Having done that, we can calculate

$$\text{total surface area} = \text{surface area of the cylinder} + \text{surface area of the hemisphere}$$

and

$$\text{volume of the solid} = \text{volume of the cylinder} + \text{volume of the hemisphere.}$$

Now,

$$\begin{aligned}\text{surface area of the cylinder} &= \text{base (not } \textit{two} \text{ — why?)} + \text{curved surface area}\\ &= (\pi \times x^2) + (2 \times \pi \times x \times 3x)\\ &= \pi x^2 + 6\pi x^2\\ &= 7\pi x^2\end{aligned}$$

and

$$\begin{aligned}\text{surface area of the hemisphere} &= \tfrac{1}{2} \times \text{surface area of the sphere}\\ &= \tfrac{1}{2} \times (4 \times \pi \times x^2)\\ &= 2\pi x^2.\end{aligned}$$

So,

$$\begin{aligned}\text{total surface area} &= \text{SA of the cylinder} + \text{SA of the hemisphere}\\ &= 7\pi x^2 + 2\pi x^2\\ &= 9\pi x^2.\end{aligned}$$

But we know from the question that the total surface area of the solid is 81π cm^2:

$$\begin{aligned}81\pi = 9\pi x^2 &\Rightarrow 9 = x^2\\ &\Rightarrow x = 3;\end{aligned}$$

so, the height of the cylinder is 9 cm and the radius of the cylinder and the radius of the hemisphere are both 3 cm.

Next,

$$\begin{aligned}\text{volume of the cylinder} &= \pi \times 3^2 \times 9 \\ &= 81\pi\end{aligned}$$

and

$$\begin{aligned}\text{volume of the hemisphere} &= \tfrac{1}{2} \times \text{volume of the sphere} \\ &= \tfrac{1}{2} \times (\tfrac{4}{3}\pi \times 3^3) \\ &= 18\pi.\end{aligned}$$

So,

$$\begin{aligned}\text{volume of the solid} &= \text{volume of the cylinder} + \text{volume of the hemisphere} \\ &= 81\pi + 18\pi \\ &= 99\pi.\end{aligned}$$

Finally,

$$\begin{aligned}\text{density} = \frac{\text{mass}}{\text{volume}} &\Rightarrow d = \frac{840}{99\pi} \\ &\Rightarrow d = 2.700\,811\,155 \text{ (FCD)}.\end{aligned}$$

Is there some metal in the table has a density the same?
Yes: the density of aluminium is 2.7.
Hence, we can state the metal used is aluminium.