# Dr Oliver Mathematics
# GCSE Mathematics
# 2018 November Paper 1H: Non-Calculator
# 1 hour 30 minutes

The total number of marks available is 80.
You must write down all the stages in your working.

1. Work out the value of (2)

$$\frac{3^7 \times 3^{-2}}{3^3}.$$

**Solution**

$$\begin{aligned}\frac{3^7 \times 3^{-2}}{3^3} &= \frac{3^{7-2}}{3^3} \\ &= \frac{3^5}{3^3} \\ &= 3^{5-3} \\ &= 3^2 \\ &= \underline{\underline{9}}.\end{aligned}$$

2.

$$v^2 = u^2 + 2as.$$

$u = 12$, $a = -3$, and $s = 18$.

(a) Work out a value of $v$. (2)

**Solution**

$$\begin{aligned}v^2 = 12^2 + 2(-3)(18) &\Rightarrow v^2 = 144 - 108 \\ &\Rightarrow v^2 = 36 \\ &\Rightarrow \underline{\underline{v = \pm 6}}.\end{aligned}$$

(b) Make $s$ the subject of (2)

$$v^2 = u^2 + 2as.$$

**Solution**

$$v^2 = u^2 + 2as \Rightarrow 2as = u^2 - v^2$$
$$\Rightarrow s = \underline{\underline{\frac{u^2 - v^2}{2a}}}.$$

3. A bonus of £2 100 is shared by 10 people who work for a company. (5)
40% of the bonus is shared equally between 3 managers.
The rest of the bonus is shared equally between 7 salesmen.

One of the salesmen says, "If the bonus is shared equally between all 10 people I will get 25% more money."

Is the salesman correct?
You must show how you get your answer.

**Solution**

Original plan:
The managers will get

$$2\,100 \times \frac{40}{100} = 21 \times 40$$
$$= £840$$

between them and this works out at

$$\frac{840}{3} = £280$$

each. Now,

$$2\,100 - 840 = £1\,260$$

is left and this works out at

$$\frac{1\,260}{7} = £180$$

for each salesman.

Salesman:

$$\frac{2\,100}{10} = £210$$

so the percentage increase is

$$\begin{aligned}\frac{210-180}{180}\times 100\% &= \frac{30}{180}\times 100\% \\ &= \tfrac{1}{6}\times 100\% \\ &= 16\tfrac{2}{3}\%.\end{aligned}$$

So, no, the salesman is not correct.

4. It would take 120 minutes to fill a swimming pool using water from 5 taps.

(a) How many minutes will it take to fill the pool if only 3 of the taps are used? (2)

**Solution**

$$5 \text{ taps} \times 120 \text{ min} = 600$$

and

$$\frac{600}{3} = \underline{\underline{200 \text{ minutes}}}.$$

(b) State one assumption you made in working out your answer to part (a). (1)

**Solution**

E.g., the water from the taps flow at the same rate.

5. A plane travels at a speed of 213 miles per hour.

(a) Work out an estimate for the number of seconds the plane takes to travel 1 mile. (3)

**Solution**

$$\begin{aligned}\text{time} = \frac{\text{distance}}{\text{speed}} &\Rightarrow \text{time} = \frac{1}{213} \text{ miles per hour} \\ &\Rightarrow \text{time} = \frac{60}{213} \text{ miles per minute} \\ &\Rightarrow \text{time} = \frac{60\times 60}{213} \text{ miles per second} \\ &\approx \text{time} = \frac{3\,600}{200} \text{ miles per second} \\ &\Rightarrow \text{time} = \underline{\underline{18 \text{ miles per second}}}.\end{aligned}$$

(b) Is your answer to part (a) an underestimate or an overestimate? (1)
Give a reason for your answer.

**Solution**

E.g., it is an <u>overestimate</u> as the speed is rounded down.

6. Solve the simultaneous equations (3)

$$5x + y = 21$$
$$x - 3y = 9.$$

**Solution**

$$5x + y = 21 \quad (1)$$
$$x - 3y = 9 \quad (2)$$

Do $3 \times (1)$:

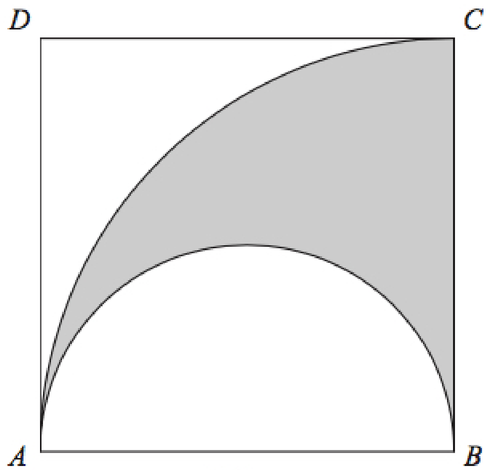
$$15x + 3y = 63 \quad (3)$$

and $(3) + (2)$:

$$16x = 72 \Rightarrow \underline{\underline{x = 4\tfrac{1}{2}}}$$

$$\Rightarrow 5(4\tfrac{1}{2}) + y = 21$$
$$\Rightarrow 22\tfrac{1}{2} + y = 21$$
$$\Rightarrow \underline{\underline{y = -1\tfrac{1}{2}}}.$$

7. The diagram shows a square $ABCD$ with sides of length 20 cm. (4)
It also shows a semicircle and an arc of a circle.




$AB$ is the diameter of the semicircle.
$AC$ is an arc of a circle with centre $B$.

Show that

$$\frac{\text{area of shaded region}}{\text{area of square}} = \tfrac{1}{8}\pi.$$

**Solution**

$$\begin{aligned}\text{Area of square} &= 20 \times 20\\ &= 400 \text{ cm}^2.\end{aligned}$$

Now,

$$\begin{aligned}\text{the area of sector} ABC &= \tfrac{1}{4} \times \pi \times 20^2\\ &= \tfrac{1}{4} \times \pi \times 400\\ &= 100\pi\end{aligned}$$

and

$$\begin{aligned}\text{the area of semicircle} &= \tfrac{1}{2} \times \pi \times 10^2\\ &= \tfrac{1}{2} \times \pi \times 100\\ &= 50\pi;\end{aligned}$$

$$\begin{aligned}\text{area of shaded region} &= \text{the area of sector } ABC - \text{the area of semicircle}\\ &= 100\pi - 50\pi\\ &= 50\pi.\end{aligned}$$

Finally,

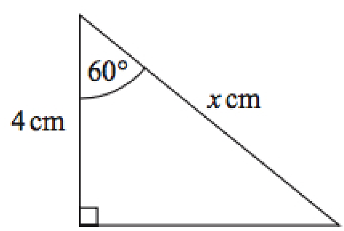
$$\frac{\text{area of shaded region}}{\text{area of square}} = \frac{50\pi}{400} = \underline{\underline{\tfrac{1}{8}\pi}},$$

as required.

8. (a) Write down the exact value of $\tan 45^\circ$. (1)

**Solution**
$\tan 45^\circ = \underline{\underline{1}}$.

Here is a right-angled triangle.




$\cos 60^\circ = 0.5$.

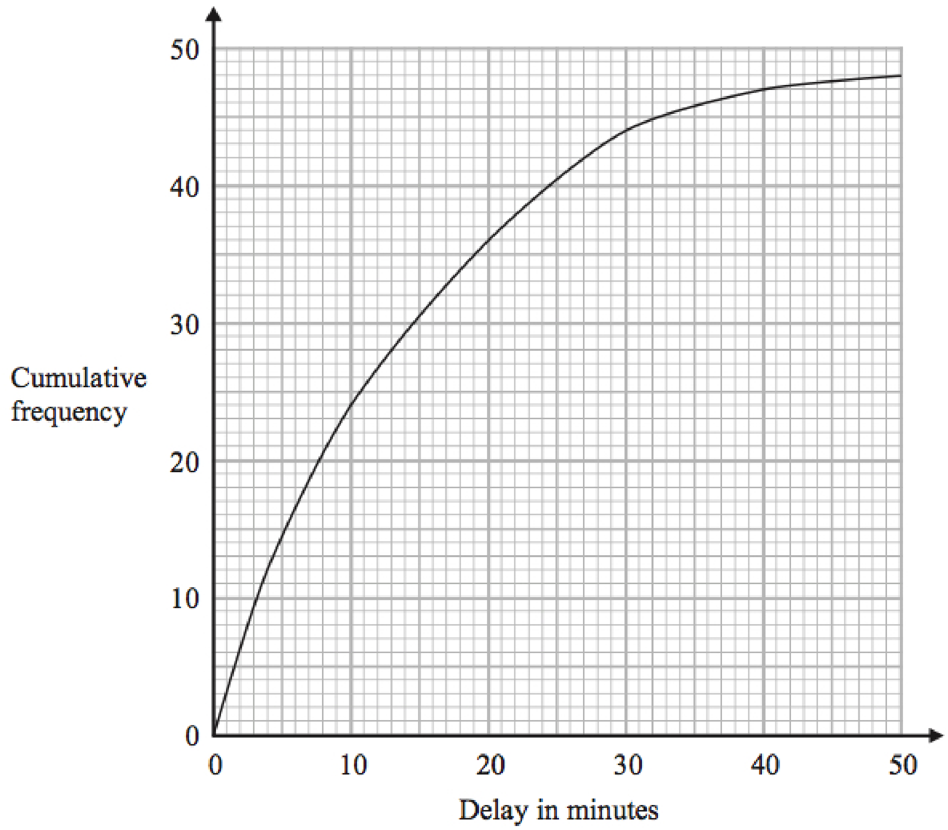
(b) Work out the value of $x$. (2)

**Solution**

$$\cos = \frac{\text{adj}}{\text{hyp}} \Rightarrow \cos 60^\circ = \frac{4}{x}$$
$$\Rightarrow x = \frac{4}{\cos 60^\circ}$$
$$\Rightarrow x = \frac{4}{0.5}$$
$$\Rightarrow \underline{\underline{x = 8}}.$$

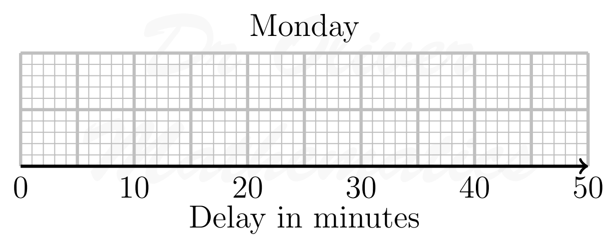
9. The times that 48 trains left a station on Monday were recorded.

The cumulative frequency graph gives information about the numbers of minutes the trains were delayed, correct to the nearest minute.



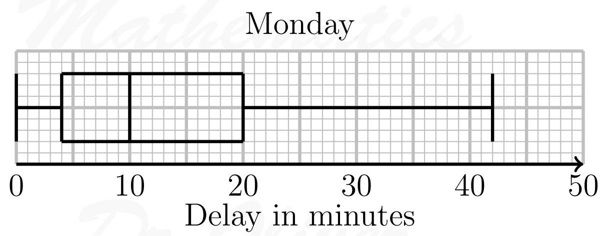

The shortest delay was 0 minutes.
The longest delay was 42 minutes.

(a) On the grid below, draw a box plot for the information about the delays on Monday. (3)

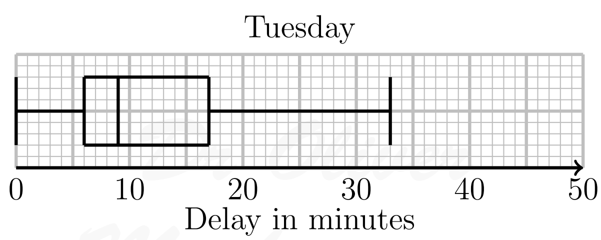



| **Solution** |
| --- |

| | Minutes |
|---|---|
| Shortest delay | 0 |
| LQ | 4 |
| Median | 10 |
| UQ | 20 |
| Longest delay | 42 |




48 trains left the station on Tuesday.
The box plot below gives information about the delays on Tuesday.




(b) Compare the distribution of the delays on Monday with the distribution of the delays on Tuesday. (2)

**Solution**

**Average**

Since the median for Mondays (10) is higher than the median for Tuesdays (9), the trains on Tuesday left more punctually on average.

**Spread**

Since the range for Mondays ($42 - 0 = 42$) is larger than the range for Tuesdays ($33 - 0 = 33$), the trains were more consistent ion Tuesdays.

**OR**

Since the IQR for the Mondays ($20 - 4 = 16$) is larger than the IQR for for Tuesdays ($17 - 6 = 9$), the trains were more consistent on Tuesdays.

Mary says, "The longest delay on Tuesday was 33 minutes. This means that there must be some delays of between 25 minutes and 30 minutes."

(c) Is Mary right? (1)
You must give a reason for your answer.

**Solution**

No, e.g., there might not be any delays between 25 minutes and 30 minutes as in the upper 25% (12 trains) the delays may all be between 17 minutes and 25 minutes.

10. (a) Simplify (1)

$$\frac{x-1}{5(x-1)^2}.$$

**Solution**

$$\begin{aligned}\frac{x-1}{5(x-1)^2} &= \frac{\cancel{(x-1)}}{5(x-1)^{\cancel{2}}}\\ &= \frac{1}{5(x-1)}.\end{aligned}$$

(b) Factorise fully (2)

$$50-2y^2.$$

**Solution**

Difference of two squares:

$$50-2y^2 = 2(25-y^2)$$

$$\left.\begin{array}{ll}\text{add to:} & 0\\ \text{multiply to:} & -1\end{array}\right\} +1,\ -1$$

$$= 2(5+y)(5-y).$$

11. Jack and Sadia work for a company that sells boxes of breakfast cereal. (3)

The company wants to have a special offer.

Here is Jack's idea for the special offer.

| Put 25% more cereal into each box and do **not** change the price. |
|---|

Here is Sadia's idea.

| Reduce the price and do **not** change the amount of cereal in each box. |
|---|

Sadia wants her idea to give the same value for money as Jack's idea.

By what percentage does she need to reduce the price?

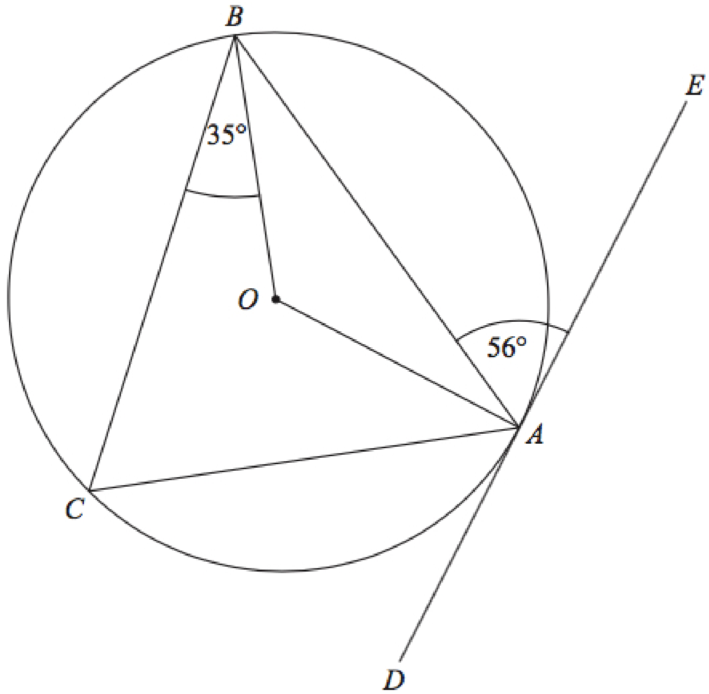
**Solution**

Let's do it with some values, e.g., 100 g of cereal costing £1.
Under Jack's idea, 125 g of cereal would cost £1.
Under Sadia's idea, 100 g of cereal would cost £$x$.

$$\frac{125}{1} = \frac{100}{x} \Rightarrow \frac{x}{1} = \frac{100}{125}$$
$$\Rightarrow x = \tfrac{4}{5}$$
$$\Rightarrow x = 0.8.$$

Hence, she needs to reduce the price by 20%.

12. $A$, $B$, and $C$ are points on the circumference of a circle, centre $O$. (3)
$DAE$ is the tangent to the circle at $A$.



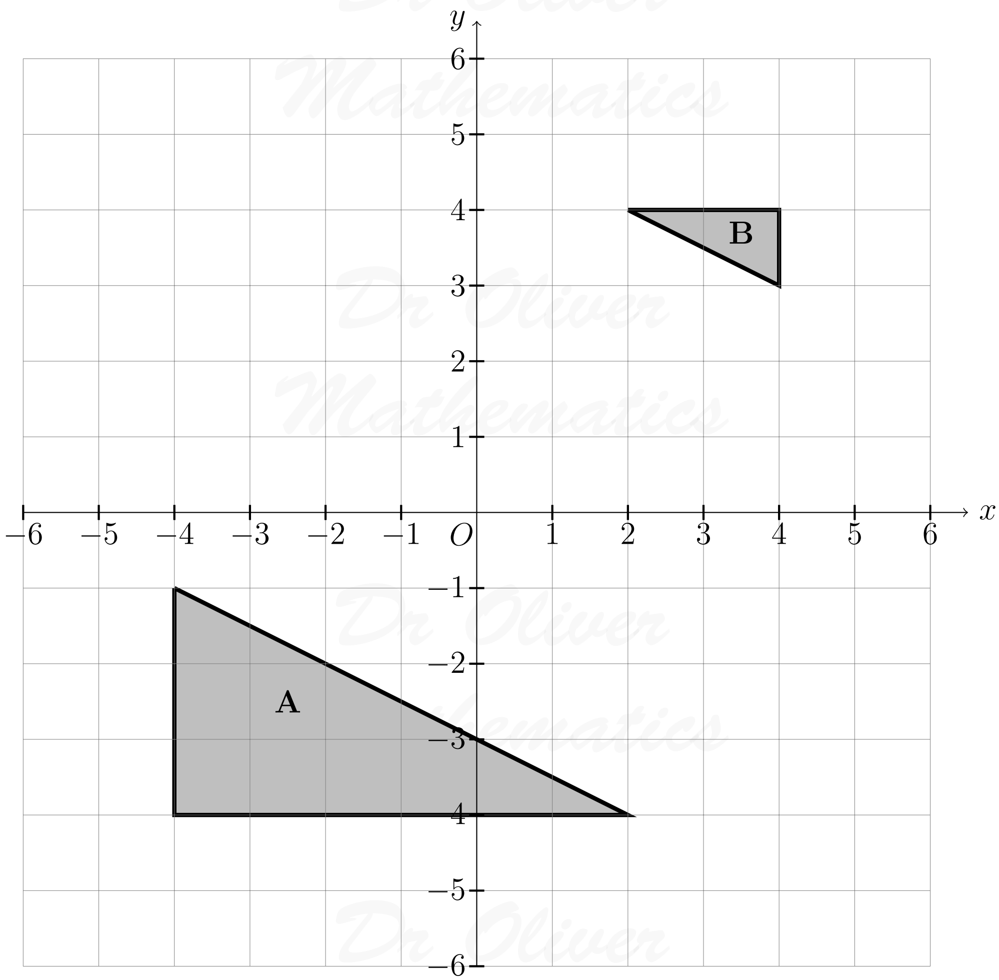

Angle $BAE = 56^\circ$.
Angle $CBO = 35^\circ$.

Work out the size of angle $CAO$.
You must show all your working.

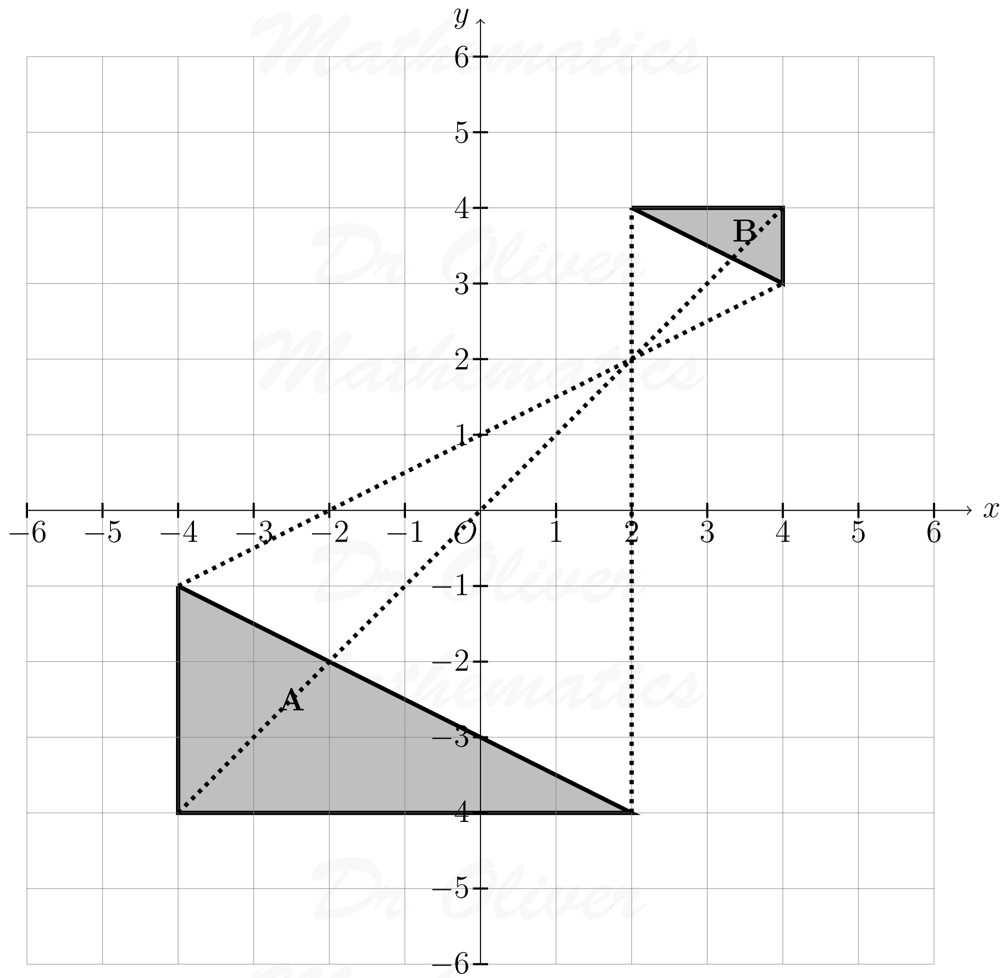
**Solution**

Well, $\angle OAB = 90 - 56 = 34^\circ$ ($\angle OAE$ is a tangent)
$\angle OBA = 34^\circ$ (base angles)
$\angle BCA = 56^\circ$ (alternating segment theorem)
$\angle CAO = 180 - (56 + 35 + 34 + 34) = 180 - 159 = \underline{\underline{21^\circ}}$ (completing the triangle)

13. Describe fully the single transformation that maps triangle **A** onto triangle **B**. (2)




**Solution**




It is a enlargement, centre (2, 2), scale factor $-\frac{1}{3}$.

14. (a) Work out the value of (2)

$$\left(\frac{16}{81}\right)^{\frac{3}{4}}.$$

**Solution**

$$\left(\frac{16}{81}\right)^{\frac{3}{4}} = \left(\frac{2^4}{3^4}\right)^{\frac{3}{4}}$$
$$= \left(\left[\frac{2}{3}\right]^4\right)^{\frac{3}{4}}$$
$$= \left(\frac{2}{3}\right)^3$$
$$= \underline{\underline{\tfrac{8}{27}}}.$$

$$3^a = \tfrac{1}{9} \quad 3^b = 9\sqrt{3} \quad 3^c = \tfrac{1}{\sqrt{3}}$$

(b) Work out the value of (2)

$$a + b + c.$$

**Solution**

$$\begin{aligned} 3^a = \tfrac{1}{9} &\Rightarrow 3^a = 3^{-2} \\ &\Rightarrow a = -2, \\ 3^b = 9\sqrt{3} &\Rightarrow 3^b = 3^2 \times 3^{\frac{1}{2}} \\ &\Rightarrow 3^b = 3^{\frac{5}{2}} \\ &\Rightarrow b = \tfrac{5}{2}, \text{ and} \\ 3^c = \tfrac{1}{\sqrt{3}} &\Rightarrow 3^c = 3^{-\frac{1}{2}} \\ &\Rightarrow c = -\tfrac{1}{2}. \end{aligned}$$

Hence,

$$a + b + c = -2 + \tfrac{5}{2} - \tfrac{1}{2} = \underline{\underline{0}}.$$

15. Three solid shapes **A**, **B**, and **C** are similar. (4)

The surface area of shape **A** is 4 $\text{cm}^2$.
The surface area of shape **B** is 25 $\text{cm}^2$.

The ratio of the volume of shape **B** to the volume of shape **C** is 27 : 64.

Work out the ratio of the height of shape **A** to the height of shape **C**.
Give your answer in its simplest form.

**Solution**

Going from **A** to **B**, the area scale factor (ASF) is

$$4 : 25 = 2^2 : 5^2$$

which means the length scale factor (LSF) is

$$2 : 5$$

and the volume scale factor (VSF) is

$$2^3 : 5^3 = 8 : 125.$$

Now, the volume of shape **C** is

$$\begin{aligned} 125 \times \frac{64}{27} &= 5^3 \times \frac{4^3}{3^3} \\ &= \left(5 \times \frac{4}{3}\right)^3 \\ &= \left(\tfrac{20}{3}\right)^3 \end{aligned}$$

and the length of **C** is $\frac{20}{3}$.

Hence, the ratio of the height of shape **A** to the height of shape **C** is

$$2 : \tfrac{20}{3} = 6 : 20 = \underline{\underline{3 : 10}}.$$

16. Prove algebraically that (3)

$$0.2\dot{5}\dot{6}$$

can be written as $\frac{127}{495}$.

**Solution**

Let $x = 0.2\dot{5}\dot{6}$. Then

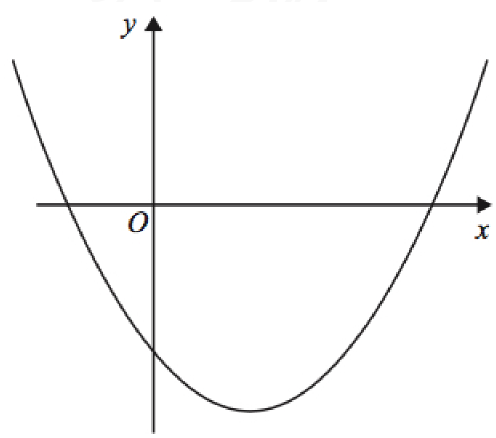
$$10x = 2.\dot{5}\dot{6} \quad (1)$$
$$1\,000x = 256.\dot{5}\dot{6} \quad (2)$$

Do (2) − (1):

$$990x = 254 \Rightarrow 495x = 127$$
$$\Rightarrow \underline{\underline{x = \tfrac{127}{495}}},$$

as required.

17. Here is a sketch of a curve. (4)




The equation of the curve is

$$y = x^2 + ax + b,$$

where $a$ and $b$ are integers.

The points $(0, -5)$ and $(5, 0)$ lie on the curve.

Find the coordinates of the turning point of the curve.

**Solution**

$$x = 0, y = -5 \Rightarrow -5 = 0 + 0 + b$$
$$\Rightarrow b = -5$$

and

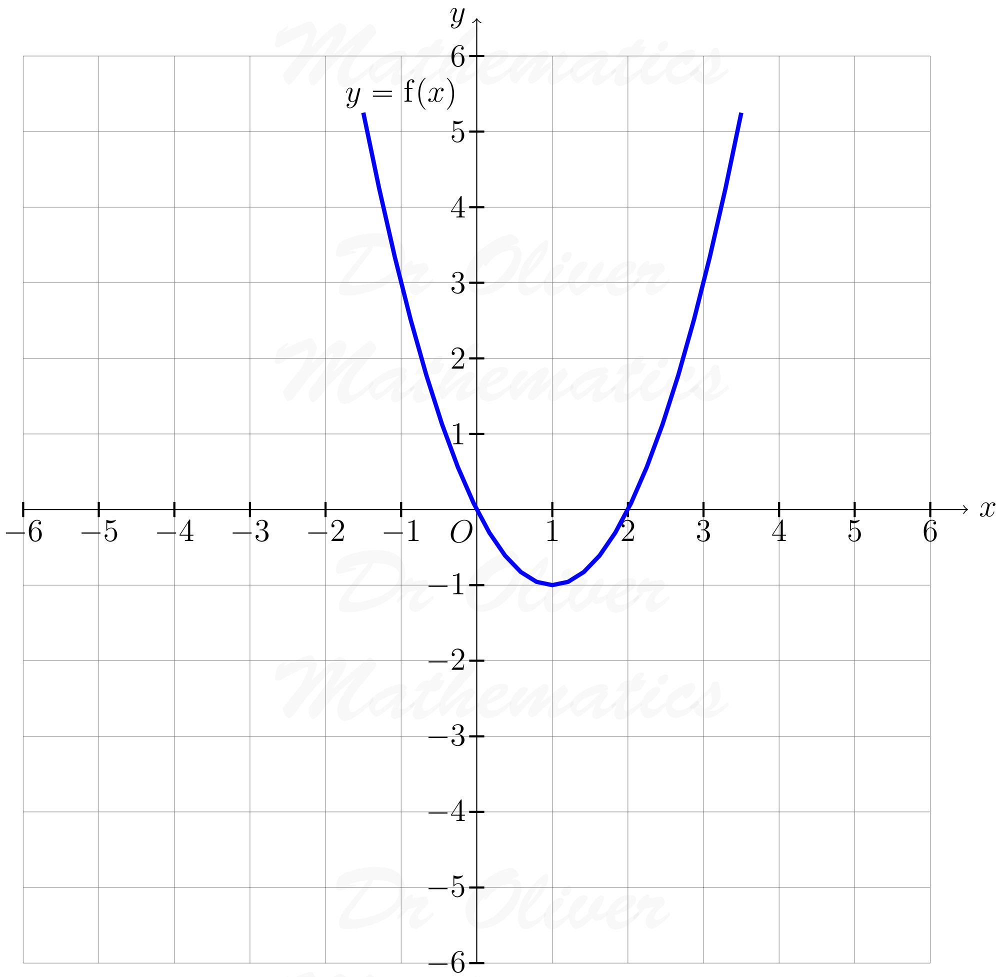
$$\begin{aligned} x = 5, y = 0 &\Rightarrow 0 = 5^2 + 5a - 5 \\ &\Rightarrow 5a = -20 \\ &\Rightarrow a = -4. \end{aligned}$$

So, the equation of the curve is

$$\begin{aligned} y = x^2 - 4x - 5 &\Rightarrow y = (x^2 - 4x + 4) - 5 - 4 \\ &\Rightarrow y = (x-2)^2 - 9; \end{aligned}$$

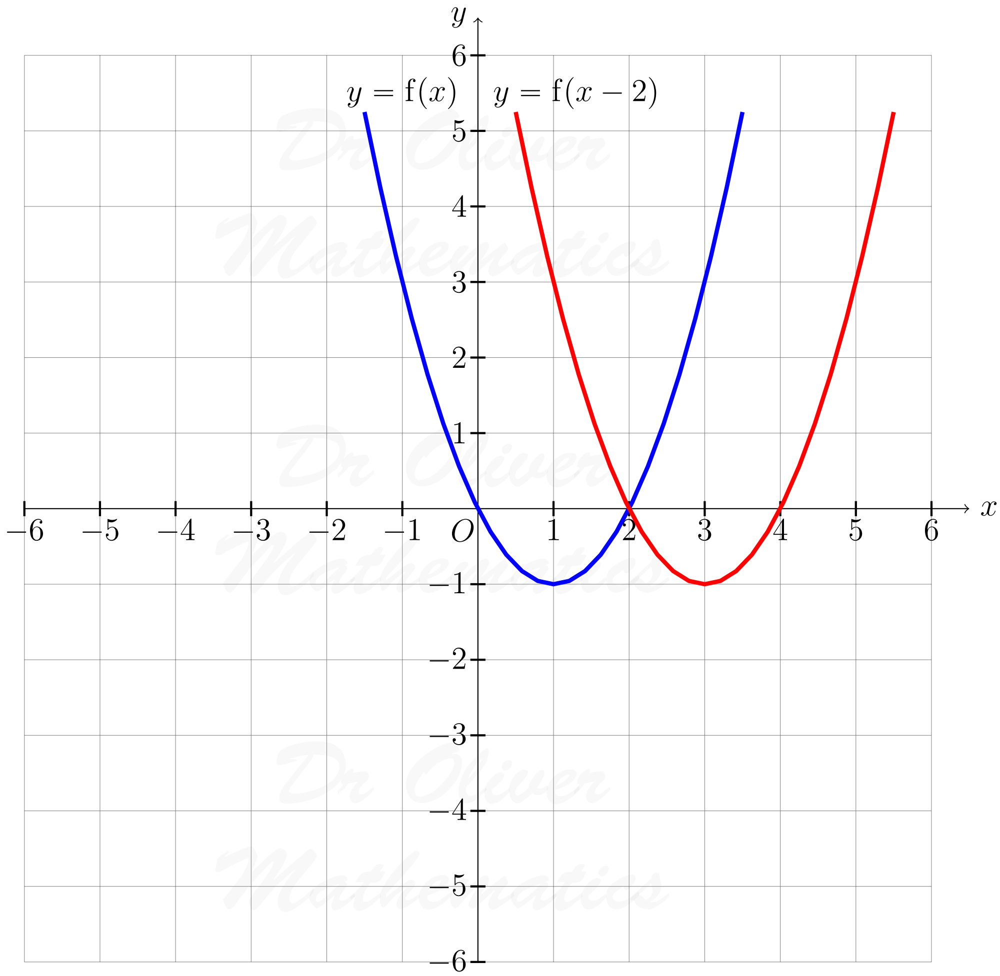
hence, the coordinates of the turning point of the curve is $\underline{\underline{(2, -9)}}$.

18. The graph of $y = \mathrm{f}(x)$ is shown on the grid below.

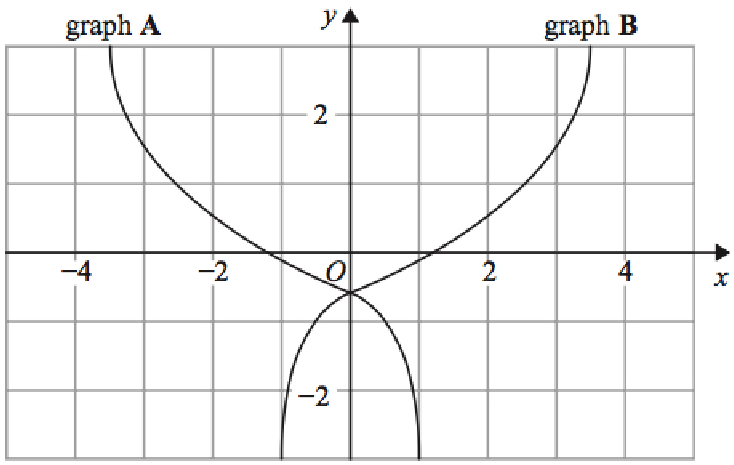


(a) Sketch the graph of $y = \mathrm{f}(x-2)$. (1)

**Solution**



On the grid, graph **A** has been reflected to give graph **B**.




The equation of graph **A** is $y = \mathrm{g}(x)$.

(b) Write down the equation of graph **B**. (1)

**Solution**

$y = \mathrm{g}(-x)$.

19. For all values of x

$$\mathrm{f}(x) = (x+1)^2 \text{ and } \mathrm{g}(x) = 2(x-1).$$

(a) Show that (2)

$$\mathrm{g}\,\mathrm{f}(x) = 2x(x+2).$$

**Solution**

$$\begin{aligned} \mathrm{g}\,\mathrm{f}(x) &= \mathrm{g}(\mathrm{f}(x)) \\ &= \mathrm{g}((x+1)^2) \\ &= 2[(x+1)^2 - 1] \\ &= 2[(x^2+2x+1) - 1] \\ &= 2(x^2+2x) \\ &= 2x(x+2), \end{aligned}$$

as required.

(b) Find (2)

$$g^{-1}(7).$$

**Solution**

$$\begin{aligned} y = 2(x-1) &\Rightarrow x - 1 = \tfrac{1}{2}y \\ &\Rightarrow x = \tfrac{1}{2}y + 1 \end{aligned}$$

and so

$$g^{-1}(x) = \tfrac{1}{2}x + 1.$$

Finally,

$$g^{-1}(7) = \tfrac{1}{2}(7) + 1 = \tfrac{7}{2} + 1 = \underline{\underline{4\tfrac{1}{2}}}.$$

20. Show that (3)

$$\frac{(\sqrt{18}+\sqrt{2})^2}{\sqrt{8}-2}$$

can be written in the form

$$a(b+\sqrt{2}),$$

where $a$ and $b$ are integers.

**Solution**

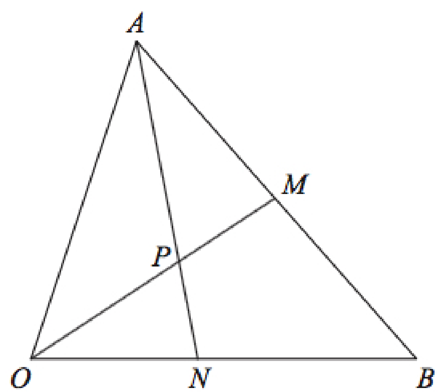
$$\begin{aligned} \frac{(\sqrt{18}+\sqrt{2})^2}{\sqrt{8}-2} &= \frac{(\sqrt{9\times 2}+\sqrt{2})^2}{\sqrt{4\times 2}-2} \\ &= \frac{(\sqrt{9}\times\sqrt{2}+\sqrt{2})^2}{\sqrt{4}\times\sqrt{2}-2} \\ &= \frac{(3\sqrt{2}+\sqrt{2})^2}{2\sqrt{2}-2} \\ &= \frac{(4\sqrt{2})^2}{2\sqrt{2}-2} \\ &= \frac{32}{2\sqrt{2}-2} \\ &= \frac{32}{2\sqrt{2}-2}\times\frac{2\sqrt{2}+2}{2\sqrt{2}+2} \end{aligned}$$

| $\times$ | $2\sqrt{2}$ | $-2$ |
|---|---|---|
| $2\sqrt{2}$ | $8$ | $-4\sqrt{2}$ |
| $+2$ | $+4\sqrt{2}$ | $-4$ |

$$= \frac{32(2\sqrt{2}+2)}{8-4}$$
$$= \frac{32(2\sqrt{2}+2)}{4}$$
$$= 8(2\sqrt{2}+2)$$
$$= \underline{\underline{16\sqrt{2}+16}};$$

hence, $\underline{\underline{a = b = 16}}$.

21. $OAB$ is a triangle. (5)
$OPM$ and $APN$ are straight lines.
$M$ is the midpoint of $AB$.




$\overrightarrow{OA} = \mathbf{a}$.
$\overrightarrow{OB} = \mathbf{b}$.
$OP : PM = 3 : 2$.

Work out the ratio $ON : NB$.

**Solution**

$$\begin{aligned}\overrightarrow{AB} &= \overrightarrow{AO} + \overrightarrow{OB} \\ &= -\mathbf{a} + \mathbf{b},\end{aligned}$$

$$\begin{aligned}\overrightarrow{OM} &= \overrightarrow{OA} + \overrightarrow{AM} \\ &= \overrightarrow{OA} + \tfrac{1}{2}\overrightarrow{AB} \\ &= \mathbf{a} + \tfrac{1}{2}(-\mathbf{a} + \mathbf{b}) \\ &= \mathbf{a} - \tfrac{1}{2}\mathbf{a} + \tfrac{1}{2}\mathbf{b} \\ &= \tfrac{1}{2}\mathbf{a} + \tfrac{1}{2}\mathbf{b},\end{aligned}$$

$$\begin{aligned}\overrightarrow{OP} &= \tfrac{3}{5}\overrightarrow{OM} \\ &= \tfrac{3}{5}(\overrightarrow{OA} + \overrightarrow{AM}) \\ &= \tfrac{3}{5}(\mathbf{a} + \tfrac{1}{2}\overrightarrow{AB}) \\ &= \tfrac{3}{5}\mathbf{a} + \tfrac{3}{10}\overrightarrow{AB} \\ &= \tfrac{3}{5}\mathbf{a} + \tfrac{3}{10}(-\mathbf{a} + \mathbf{b}) \\ &= \tfrac{3}{5}\mathbf{a} - \tfrac{3}{10}\mathbf{a} + \tfrac{3}{10}\mathbf{b} \\ &= \tfrac{3}{10}\mathbf{a} + \tfrac{3}{10}\mathbf{b},\end{aligned}$$

and

$$\begin{aligned}\overrightarrow{AP} &= \overrightarrow{AO} + \overrightarrow{OP} \\ &= -\mathbf{a} + (\tfrac{3}{10}\mathbf{a} + \tfrac{3}{10}\mathbf{b}) \\ &= -\tfrac{7}{10}\mathbf{a} + \tfrac{3}{10}\mathbf{b}.\end{aligned}$$

Now,

$$\begin{aligned}\overrightarrow{AN} &= \overrightarrow{AO} + \overrightarrow{ON} \\ &= -\mathbf{a} + k\mathbf{b},\end{aligned}$$

for some constant $k$. But

$$\overrightarrow{AN} = l\overrightarrow{AP},$$

for some constant $l$:

$$\begin{aligned}& -\mathbf{a} + k\mathbf{b} = l(-\tfrac{7}{10}\mathbf{a} + \tfrac{3}{10}\mathbf{b}) \\ \Rightarrow \quad & -\mathbf{a} + k\mathbf{b} = -\tfrac{7}{10}l\mathbf{a} + \tfrac{3}{10}l\mathbf{b}\end{aligned}$$

and so

$$\tfrac{7}{10}l = 1 \text{ and } \tfrac{3}{10}l = k.$$

$$\begin{aligned}\tfrac{7}{10}l = 1 &\Rightarrow l = \tfrac{10}{7} \\ &\Rightarrow \tfrac{3}{10} \times \tfrac{10}{7} = k \\ &\Rightarrow k = \tfrac{3}{7};\end{aligned}$$

$$\overrightarrow{ON} = \tfrac{3}{7}\mathbf{b} \text{ and } \overrightarrow{NB} = \tfrac{4}{7}\mathbf{b}.$$

Hence,

$$ON : NB = \tfrac{3}{7} : \tfrac{4}{7} = \underline{\underline{3:4}}.$$

22. There are only green pens and blue pens in a box. (6)

There are three more blue pens than green pens in the box.
There are more than 12 pens in the box.

Simon is going to take at random two pens from the box.
The probability that Simon will take two pens of the same colour is $\frac{27}{55}$.

Work out the number of green pens in the box.

**Solution**

Let $n$ be the number of green pens. Then $(n+3)$ is the number of blue pens and $(2n+3)$ is the total number of pens. Now,

$$\begin{aligned}\text{P(same colour)} = \tfrac{27}{55} &\Rightarrow \text{P}(GG) + \text{P}(BB) = \tfrac{27}{55} \\ &\Rightarrow \left(\tfrac{n}{2n+3} \times \tfrac{n-1}{2n+2}\right) + \left(\tfrac{n+3}{2n+3} \times \tfrac{n+2}{2n+2}\right) = \tfrac{27}{55} \\ &\Rightarrow \tfrac{n(n-1)+(n+3)(n+2)}{2(n+1)(2n+3)} = \tfrac{27}{55} \\ &\Rightarrow 55[n(n-1) + (n+3)(n+2)] = 54(n+1)(2n+3)\end{aligned}$$

| × | $2n$ | $+3$ |
|---|---|---|
| $n$ | $2n^2$ | $+3n$ |
| $+1$ | $+2n$ | $+3$ |

$$\Rightarrow 55[(n^2 - n) + (n^2 + 5n + 6)] = 54(2n^2 + 5n + 3)$$
$$\Rightarrow 55(2n^2 + 4n + 6) = 54(2n^2 + 5n + 3)$$
$$\Rightarrow 110n^2 + 220n + 330 = 108n^2 + 270n + 162$$
$$\Rightarrow 2n^2 - 50n + 168 = 0$$
$$\Rightarrow 2(n^2 - 25n + 84) = 0$$

$$\left.\begin{array}{ll}\text{add to:} & -25 \\ \text{multiply to:} & +84\end{array}\right\} -4, -21$$

$$\Rightarrow 2(n-4)(n-21) = 0$$
$$\Rightarrow n = 4 \text{ or } n = 21;$$

because there are more than 12 pens in the box, $\underline{\underline{n = 21}}$.