Dr Oliver Mathematics
Mathematics: National Qualifications N5
2018 Paper 2: Calculator
1 hour 50 minutes

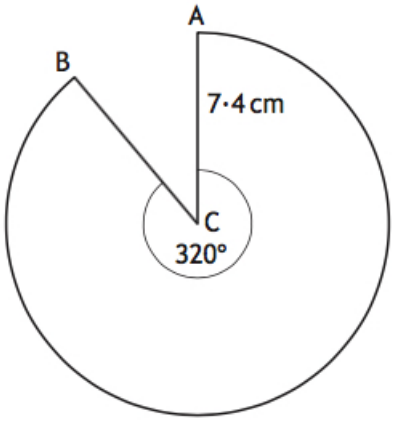
The total number of marks available is 60.
You must write down all the stages in your working.

1. Households in a city produced a total of 125 000 tonnes of waste in 2017. (3)
   The total amount of waste is expected to fall by 2% each year.
   Calculate the total amount of waste these households are expected to produce in 2020.

Solution

$$\begin{aligned}\text{Total amount of waste} &= 125\,000 \times (1 - 0.02)^3 \\ &= 125\,000 \times (0.98)^3 \\ &= \underline{\underline{117\,649 \text{ tonnes}}}.\end{aligned}$$

2. The diagram below shows a sector of a circle, centre C. (3)



   The radius of the circle is 7.4 centimetres.
   Calculate the length of the major arc AB.

Solution

$$\frac{320}{360} = \frac{\text{major arc}}{2 \times \pi \times 7.4} \Rightarrow \text{major arc} = \frac{320 \times 2 \times \pi \times 7.4}{360}$$
$$\Rightarrow \text{major arc} = 41.329\,396\,69 \text{ (FCD)}$$
$$\Rightarrow \text{major arc} = \underline{\underline{41.3 \text{ cm (3 sf)}}}.$$

3. Find $|\mathbf{r}|$, the magnitude of vector (2)

$$\mathbf{r} = \begin{pmatrix} 24 \\ -12 \\ 8 \end{pmatrix}.$$

Solution

$$|\mathbf{r}| = \sqrt{24^2 + (-12)^2 + 8^2}$$
$$= \sqrt{576 + 144 + 64}$$
$$= \sqrt{784}$$
$$= \underline{\underline{28}}.$$

4. Solve, algebraically, the inequation (3)

$$3x < 6(x-1) - 12.$$

Solution

$$3x < 6(x-1) - 12 \Rightarrow 3x < (6x - 6) - 12$$
$$\Rightarrow 3x > 18$$
$$\Rightarrow \underline{\underline{x > 6}}.$$

5. A farmers' market took place one weekend.
Stallholders were asked to record the number of customers who visited their stall.
The number of customers who visited six of the stalls on Saturday were as follows:

120 126 125 131 130 124

(a) Calculate the mean and standard deviation of the number of customers. (4)

Solution

x	x^2
120	14 400
126	15 876
125	15 625
131	17 161
130	16 900
124	15 376
756	95 338

$$\begin{aligned}\text{Mean} &= \frac{\sum x}{n}\\ &= \frac{756}{6}\\ &= \underline{\underline{126}}\end{aligned}$$

and

$$\begin{aligned}\text{standard deviation} &= \sqrt{\frac{\sum x^2 - (\sum x)^2/n}{n-1}}\\ &= \sqrt{\frac{95\,338 - (756)^2/6}{5}}\\ &= \sqrt{16\tfrac{2}{5}}\\ &= 4.049\,691\,346 \text{ (FCD)}\\ &= \underline{\underline{4.05 \text{ (3 sf)}}}.\end{aligned}$$

The mean number of customers who visited these six stalls on Sunday was 117 and the standard deviation was 6.2.

(b) Make two valid comments comparing the number of customers who visited these stalls on Saturday and Sunday. (2)

Solution

There were more visitors on Saturday ($126 > 117$) but the standard deviation has increased on Sunday ($4.05 < 6.2$).

6. A function is defined as (2)

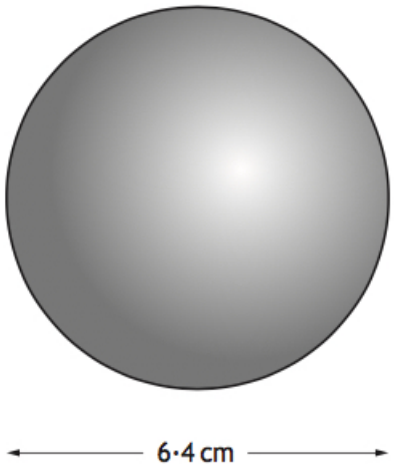
$$\mathrm{f}(x) = 5 + 4x.$$

Given that $\mathrm{f}(a) = 73$, calculate a.

Solution

$$5 + 4a = 73 \Rightarrow 4a = 68$$
$$\Rightarrow a = 17$$

7. A toy company makes juggling balls in the shape of a sphere with a diameter of 6.4 centimetres. (3)



Calculate the volume of one juggling ball.
Give your answer correct to 2 significant figures.

Solution

$$\begin{aligned}\text{Volume} &= \tfrac{4}{3} \times \pi \times \left(\frac{6.4}{2}\right)^3 \\ &= \tfrac{4}{3} \times \pi \times (3.2)^3 \\ &= 137.258\,277\,4 \text{ (FCD)} \\ &= \underline{\underline{140 \text{ cm}^3 \text{ (2 sf)}}}.\end{aligned}$$

8. Solve the equation (3)

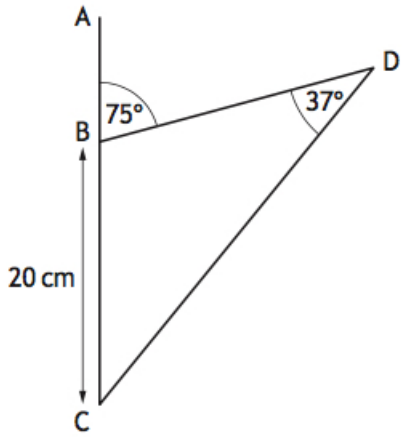
$$7 \sin x^\circ + 2 = 3, \text{ for } 0 \leqslant x < 360.$$

Solution

$$\begin{aligned}7 \sin x^\circ + 2 = 3 &\Rightarrow 7 \sin x^\circ = 1 \\ &\Rightarrow \sin x^\circ = \tfrac{1}{7} \\ &\Rightarrow x = 8.213\,210\,070\,2,\ 171.786\,789\,3 \text{ (FCD)} \\ &\Rightarrow \underline{\underline{x = 8.21,\ 172 \text{ (3 sf)}}}.\end{aligned}$$

9. In this diagram: (3)

- angle $ABD = 75^\circ$,
- angle $BDC = 37^\circ$,and
- $BC = 20$ centimetres.



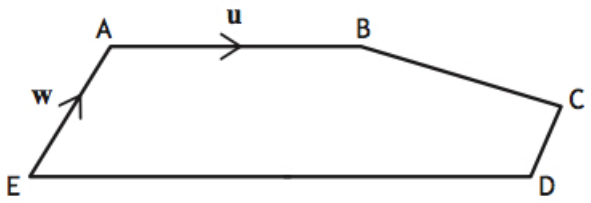
Calculate the length of DC.

Solution

$\angle CAD = 180 - 75 = 105°$ (supplementary angles) and

$$\frac{DC}{\sin CAD} = \frac{BC}{\sin BDC} \Rightarrow \frac{DC}{\sin 105°} = \frac{20}{\sin 37°}$$
$$\Rightarrow DC = \frac{20 \sin 105°}{\sin 37°}$$
$$\Rightarrow DC = 32.100\,422\,53 \text{ (FCD)}$$
$$\Rightarrow \underline{\underline{DC = 32.1 \text{ cm (3 sf)}}}.$$

10. In the diagram below, $\overrightarrow{AB}$ and $\overrightarrow{EA}$ represent the vectors **u** and **w** respectively. (2)



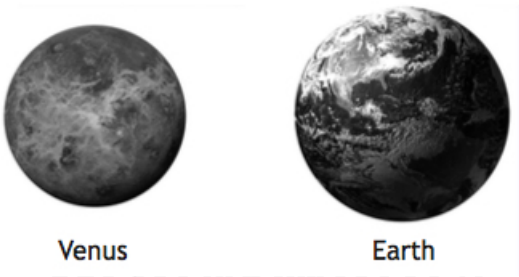
- $\overrightarrow{ED} = 2\overrightarrow{AB}$ and
- $\overrightarrow{EA} = 2\overrightarrow{DC}$.

Express $\overrightarrow{BC}$ in terms of **u** and **w**.
Give your answer in its simplest form.

Solution

$$\begin{aligned}\overrightarrow{BC} &= \overrightarrow{BA} + \overrightarrow{AE} + \overrightarrow{ED} + \overrightarrow{DC} \\ &= -\mathbf{u} - \mathbf{w} + 2\mathbf{u} + \tfrac{1}{2}\mathbf{w} \\ &= \underline{\underline{\mathbf{u} - \tfrac{1}{2}\mathbf{w}}}.\end{aligned}$$

11. Venus and Earth are two planets within our solar system. (3)

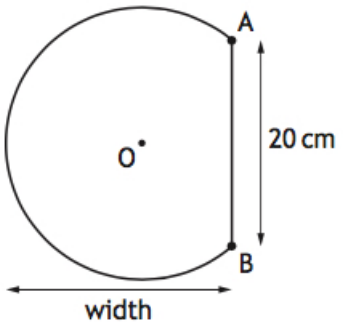


The volume of Venus is approximately 9.3×10^{11} cubic kilometres.
This is 85% of the volume of Earth.
Calculate the volume of Earth.

Solution

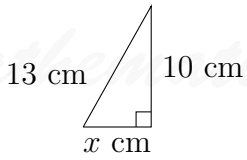
$$\begin{aligned}\frac{\text{Volume of Earth}}{9.3 \times 10^{11}} = \frac{100}{85} &\Rightarrow \text{Volume of Earth} = \frac{100 \times (9.3 \times 10^{11})}{85} \\ &\Rightarrow \text{Volume of Earth} = 1.094\,117\,647 \times 10^{12} \text{ (FCD)} \\ &\Rightarrow \text{Volume of Earth} = \underline{\underline{1.09 \times 10^{12} \text{ cubic kilometres (3 sf)}}}.\end{aligned}$$

12. The shape below is part of a circle, centre O. (4)



The circle has radius 13 centimetres.
AB is a chord of length 20 centimetres.
Calculate the width of the shape.

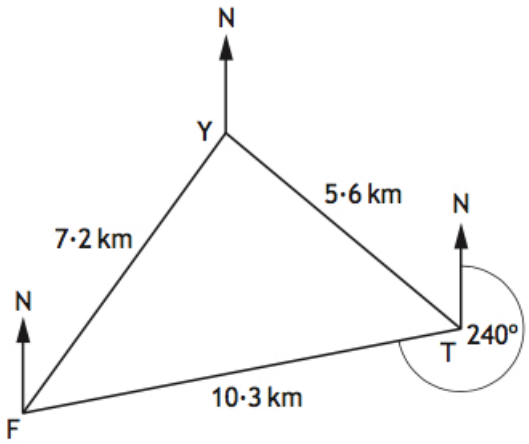
Solution



Now,

$$\begin{aligned} \text{width} &= 13 + \sqrt{13^2 - 10^2} \\ &= 13 + \sqrt{169 - 100} \\ &= \underline{\underline{13 + \sqrt{69} \text{ or } 21.3 \text{ cm } (3 \text{ sf})}}. \end{aligned}$$

13. A ferry and a trawler receive a request for help from a stranded yacht. (4)
On the diagram the points F, T, and Y show the positions of the ferry, the trawler and the yacht respectively.



- FY is 7.2 kilometres.
- TY is 5.6 kilometres.
- FT is 10.3 kilometres.
- F is on a bearing of 240° from T.

Calculate the bearing of the yacht from the trawler.

Solution

$$\cos FTY = \frac{10.3^2 + 5.6^2 - 7.2^2}{2 \times 10.3 \times 5.6} \Rightarrow \cos FTY = \tfrac{1\,223}{1\,648}$$
$$\Rightarrow \angle FTY = 42.088\,392\,19 \text{ (FCD)}$$
$$\Rightarrow \angle NTY = 240 + 42.088\ldots$$
$$\Rightarrow \angle NTY = 282.088\,392\,2 \text{ (FCD)}$$
$$\Rightarrow \underline{\underline{\angle NTY = 282^\circ \text{ (3 sf)}}}.$$

14. A straight line has equation (2)

$$2x - 5y = 20.$$

Find the coordinates of the point where this line crosses the y-axis.

Solution

$$x = 0 \Rightarrow -5y = 20$$
$$\Rightarrow y = -4;$$

hence, the coordinates are $\underline{\underline{(0, -4)}}$.

15. Express (3)

$$\frac{n}{n^2-4} \div \frac{3}{n-2},\ n \neq -2,\ n \neq 2,$$

as a single fraction in its simplest form.

Solution

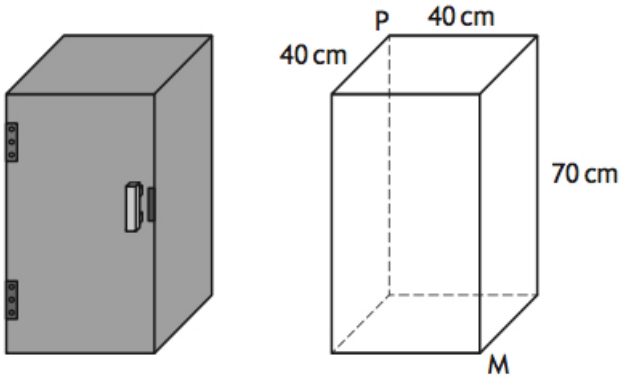
$$\left.\begin{array}{ll} \text{add to:} & 0 \\ \text{multiply to:} & -4 \end{array}\right\} -2,\ +2$$

$$n^2 - 4 = (n-2)(n+2)$$

Now,

$$\frac{n}{n^2-4} \div \frac{3}{n-2} = \frac{n}{(n-2)(n+2)} \times \frac{(n-2)}{3}$$
$$= \underline{\underline{\frac{n}{3(n+2)}}}.$$

16. Chris wants to store his umbrella in a locker. (4)
The locker is a cuboid with internal dimensions of length 40 centimetres, breadth 40 centimetres, and height 70 centimetres.



The umbrella is 85 centimetres long.
He thinks it will fit into the locker from corner P to corner M.

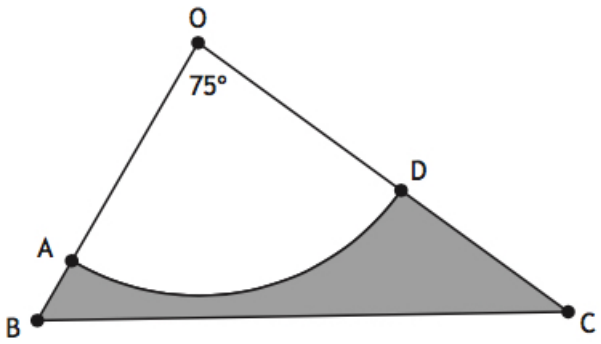
Is he correct?
Justify your answer.

Solution

$$\begin{aligned}\text{Space diagonal} &= \sqrt{40^2 + 40^2 + 70^2} \\ &= \sqrt{1\,600 + 1\,600 + 4\,900} \\ &= \sqrt{8\,100} \\ &= 90 \\ &> 85;\end{aligned}$$

so, yes, Chris's umbrella will fit in to the locker (with 5 cm to spare!).

17. In the diagram below AOD is a sector of a circle, with centre O, and BOC is a triangle. (5)



In sector AOD:

- radius = 30 centimetres and
- angle $AOD = 75°$.

In triangle OBC:

- $OB = 38$ centimetres and
- $OC = 55$ centimetres.

Calculate the area of the shaded region, $ABCD$.

Solution

$$\begin{aligned}\text{Shaded region} &= \text{triangle} - \text{circle}\\ &= \left(\tfrac{1}{2} \times 38 \times 55 \times \sin 75^\circ\right) - \left(\frac{75}{360} \times \pi \times 30^2\right)\\ &= 420.343\,865\,9 \text{ (FCD)}\\ &= \underline{\underline{420 \text{ cm}^2 \text{ (3 sf)}}}.\end{aligned}$$

18. A cinema sells popcorn in two different sized cartons.

The small carton is 16 centimetres deep and has a volume of 576 cubic centimetres.
The large carton is 24 centimetres deep and has a volume of 1 125 cubic centimetres.

(a) Show that the two cartons are **not** mathematically similar. (3)

Solution

$$\begin{aligned}\text{Large volume} &= 576 \times \left(\frac{24}{16}\right)^3\\ &= 1\,944 \text{ cubic centimetres;}\end{aligned}$$

hence, that the two cartons are <u>not</u> mathematically similar.

The large carton is redesigned so that the two cartons are now mathematically similar.
The volume of the redesigned large carton is 1500 cubic centimetres.

(b) Calculate the depth of the redesigned large carton. (2)

Solution

The volume scale ratio (VSR) is

$$\frac{1\,500}{576} = \frac{125}{48}$$

which means the length scale ratio (LSR) is

$$\left(\frac{125}{48}\right)^{\frac{1}{3}}.$$

Hence,

$$\begin{aligned}\text{redesigned large carton} &= 16 \times \left(\frac{125}{48}\right)^{\frac{1}{3}} \\ &= 22.012\,848\,33 \text{ (FCD)} \\ &= \underline{\underline{22.0 \text{ cm (3 sf)}}}.\end{aligned}$$