Dr Oliver Mathematics
GCSE Mathematics
2024 November Paper 1H: Non-Calculator
1 hour 30 minutes

The total number of marks available is 80.
You must write down all the stages in your working.

1. Work out (3)

$$818.4 \div 1.2.$$

Solution

Well,

$$\begin{array}{rr|rrr} & & 6 & 8 & .2 \\ \cline{3-5} 1.2 & 8 & 1 & 8 & .4 \\ & 7 & 2 & & \\ \cline{2-5} & & 9 & 8 & .4 \\ & & 9 & 6 & \\ \cline{2-5} & & & 2 & .4 \\ & & & 2 & .4 \\ \cline{4-5} & & & & 0 \end{array}$$

Hence,

$$818.4 \div 1.2 = \underline{\underline{68.2}}.$$

2. The table shows the probabilities that a biased dice will land on 3, on 4, on 5, and on 6.

Number on dice	1	2	3	4	5	6
Probability			0.10	0.30	0.05	0.25

Karim assumes that the probabilities that the dice will land on 1 and on 2 are the same.

Karim rolls the biased dice 500 times.

(a) Assuming Karim is right, work out an estimate for the number of times the dice will land on 2. (3)

Solution

Well,

$$0.10 + 0.30 + 0.05 + 0.25 = 0.7$$

so the probability that lands on 2 is

$$\begin{aligned} \tfrac{1}{2}(1 - 0.7) &= \tfrac{1}{2}(0.3) \\ &= 0.15. \end{aligned}$$

So,

$$\begin{aligned} \text{estimate} &= 500 \times 0.15 \\ &= 5 \times 15 \\ &= \underline{\underline{75}}. \end{aligned}$$

Karim is wrong.

The probability that the dice will land on 2 is greater than the probability that the dice will land on 1.

(b) How does this information affect your answer to part (a)? (1)

Solution

E.g., the probability that the dice will land on 2 will be <u>greater</u>.

3. (a) Work out (2)

$$3\tfrac{1}{2} - 1\tfrac{1}{6}.$$

Give your answer as a mixed number.

Solution

Well,

$$\begin{aligned} 3\tfrac{1}{2} - 1\tfrac{1}{6} &= (3 + \tfrac{3}{6}) - (1 + \tfrac{1}{6}) \\ &= (3 - 1) + (\tfrac{3}{6} - \tfrac{1}{6}) \\ &= 2 + \tfrac{2}{6} \\ &= \underline{\underline{2\tfrac{1}{3}}}. \end{aligned}$$

(b) Show that (3)

$$5\tfrac{1}{4} \div 2\tfrac{1}{3} = 2\tfrac{1}{4}.$$

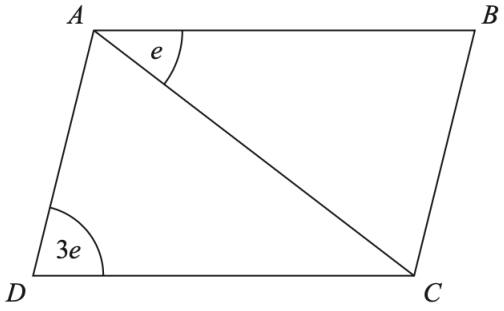
Solution

Convert each fraction into a top-heavy fraction:

$$\begin{aligned} 5\tfrac{1}{4} \div 2\tfrac{1}{3} &= \tfrac{21}{4} \div \tfrac{7}{3} \\ &= \tfrac{21}{4} \times \tfrac{3}{7} \\ &= \tfrac{3}{4} \times \tfrac{3}{1} \\ &= \tfrac{9}{4} \\ &= \underline{\underline{2\tfrac{1}{4}}}, \end{aligned}$$

as required.

4. $ABCD$ is a parallelogram. (3)



All angles are measured in degrees.

Find an expression, in terms of e, for the size of angle CAD.
Give a reason for each stage of your working.

Solution

Interior angles with $\angle CDAB$:

$$\begin{aligned} \angle CDA + \angle CAD + \angle CAB = 180^\circ &\Rightarrow 3e + \angle CAD + e = 180^\circ \\ &\Rightarrow \underline{\underline{\angle CAD = (180 - 4e)^\circ}}. \end{aligned}$$

5. A car travelled 4.96 miles at an average speed of 30.4 miles per hour.

(a) Work out an estimate for the time taken by the car. (3)
Give your answer in minutes.

Solution

Round to 1 significant figure:

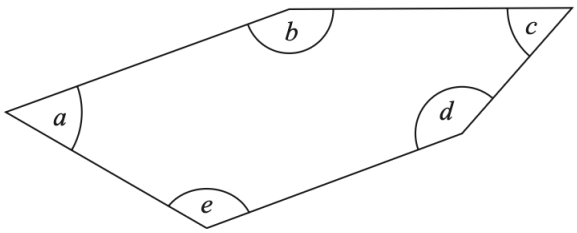
$$\begin{aligned} \text{time} &= \frac{\text{distance}}{\text{speed}} \\ &= \frac{4.96}{30.4} \\ &\approx \frac{5}{30} \\ &= \tfrac{1}{6} \text{ hours} \\ &= \underline{\underline{10 \text{ minutes}}}. \end{aligned}$$

(b) Is your answer to part (a) an underestimate or an overestimate? (1)
Give a reason for your answer.

Solution

It is an <u>overestimate</u>: it is an upper estimate for the distance and it is a lower estimate for the speed.

6. Here is a pentagon. (4)



- Angle a = angle c.
- Angle $b = 155°$.
- Angle d is three times the size of angle c.
- Angle e is two times the size of angle c.

Work out the size of angle a.

Solution

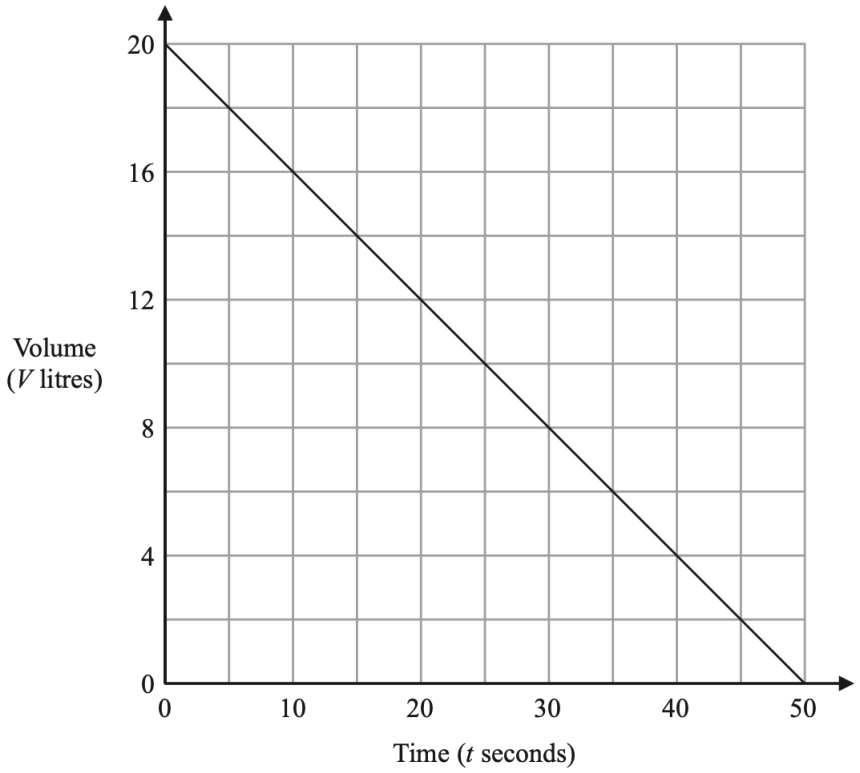
Well, the pentagram has

$$(5-2)\times 180 = 540^\circ.$$

Now,

$$\begin{aligned} a+b+c+d+e = 540 &\Rightarrow c+155+c+3c+2c = 540 \\ &\Rightarrow 7c = 385 \\ &\Rightarrow \underline{\underline{c = 55^\circ}}. \end{aligned}$$

7. The graph shows the volume of water, V litres, in a tank at time t seconds. (1)

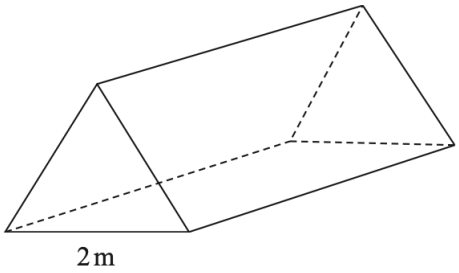


What does the gradient of this graph represent?

Solution

E.g., the rate at which the water is poured.

8. The diagram shows a solid triangular prism on a horizontal floor. (3)



$$\text{pressure} = \frac{\text{force}}{\text{area}}$$

- The face in contact with the floor is a rectangle of width 2 m.
- The pressure on the floor due to the prism is 80 newtons/m^2.
- The force exerted by the prism on the floor is 720 newtons.

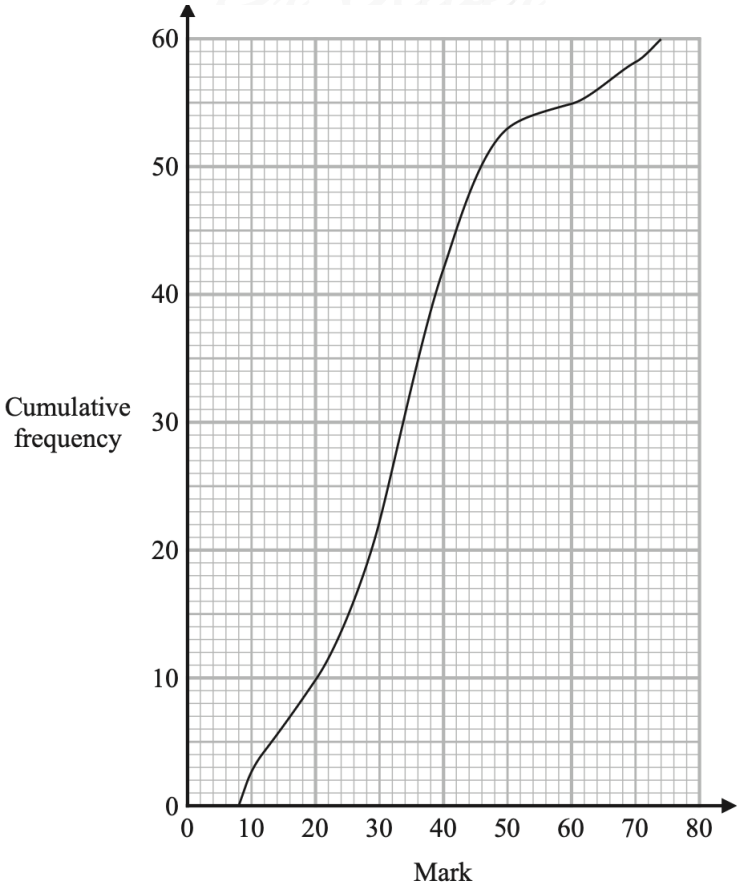
Work out the length of the prism.

Solution

Let the length of the prism be x m. Then

$$\begin{aligned} \text{pressure} = \frac{\text{force}}{\text{area}} &\Rightarrow 80 = \frac{720}{2x} \\ &\Rightarrow 2x = \frac{720}{80} \\ &\Rightarrow 2x = 9 \\ &\Rightarrow \underline{\underline{x = 4.5}}. \end{aligned}$$

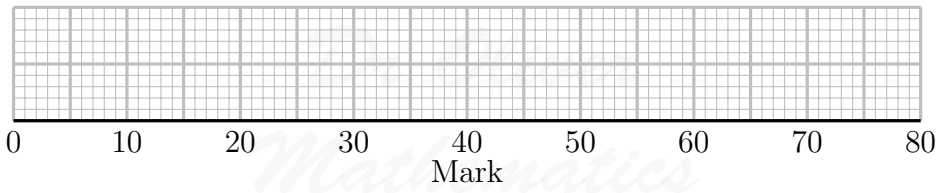
9. The cumulative frequency graph gives information about the marks that 60 students got in a test.



For these 60 students

- the highest mark was 74 and
- the lowest mark was 8.

(a) On the grid below, draw a box plot. (3)



Solution

The LQ is at

$$\frac{1}{4} \times 60 = 15\text{th position}$$

and is 25.

The median is at

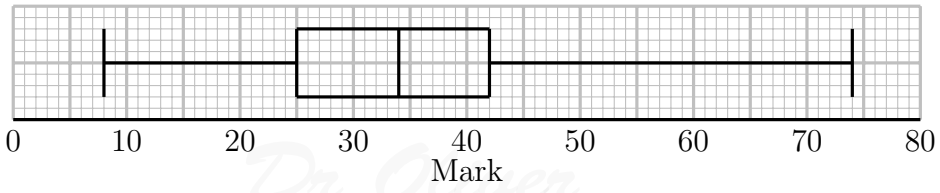
$$\frac{1}{2} \times 60 = 30\text{th position}$$

and is 34.

The UQ is at

$$\frac{3}{4} \times 60 = 45\text{th position}$$

and is 42.



The pass mark for the test was 40.

Sian says, "30% of the 60 students passed the test."

(b) Is Sian correct? (3)
You must show how you get your answer.

Solution

Draw a line up from 40 up to the curve and read-off: 42.
The means $60 - 42 = 18$ passed.

Now,

$$60 \times 0.3 = 18;$$

so, Sian is correct.

10. (a) Work out (2)

$$25^{\frac{1}{2}} \times 8^{\frac{1}{3}}.$$

Solution

Well,

$$25^{\frac{1}{2}} \times 8^{\frac{1}{3}} = 5 \times 2$$
$$= \underline{\underline{10}}.$$

(b) Find the value of (2)

$$\left(\tfrac{1}{32}\right)^{\frac{3}{5}}.$$

Solution

Now,

$$\left(\tfrac{1}{32}\right)^{\frac{3}{5}} = \frac{1}{32^{\frac{3}{5}}}$$
$$= \frac{1}{(32^{\frac{1}{5}})^3}$$
$$= \frac{1}{2^3}$$
$$= \underline{\underline{\tfrac{1}{8}}}.$$

11. Kate was asked to factorise

$$x^2 + 5x + 6$$

in the form

$$(x + a)(x + b).$$

Kate says, "The sum of a and b must be 6 and the product of a and b must be 5."

(a) Explain what is wrong with Kate's statement. (1)

Solution

She has got her terms the <u>wrong way around</u>: "The sum of a and b must be 5 and the product of a and b must be 6."

(b) Factorise fully (2)

$$2m^2 - 2.$$

Solution
Difference of two squares:

$$\begin{aligned} 2m^2 - 2 &= 2(m^2 - 1) \\ &= \underline{\underline{2(m-1)(m+1)}}. \end{aligned}$$

(c) Factorise fully (2)

$$ax + bx - ay - by.$$

Solution
Well,

$$\begin{aligned} ax + bx - ay - by &= x(a+b) - y(a+b) \\ &= \underline{\underline{(x-y)(a+b)}}. \end{aligned}$$

12. **A**, **B**, and **C** are three solid spheres. (4)

- Sphere **A** has a volume of 64 cm^3.
- Sphere **B** has a volume of 125 cm^3.
- The radius of sphere **C** is 50% of the radius of sphere **B**.

Work out the ratio of the surface area of sphere **A** to the surface area of sphere **C**.
Give your answer in the form $a : b$ where a and b are integers.

Solution
The volume scale ratio (VSR) between **A** and **B** is

$$64 : 125 = 4^3 : 5^3$$

and the length scale ratio (LSR) between **A** and **B** is

$$4 : 5.$$

The length scale ratio (LSR) between **B** and **C** is

$$1 : 0.5 = 5 : 2.5.$$

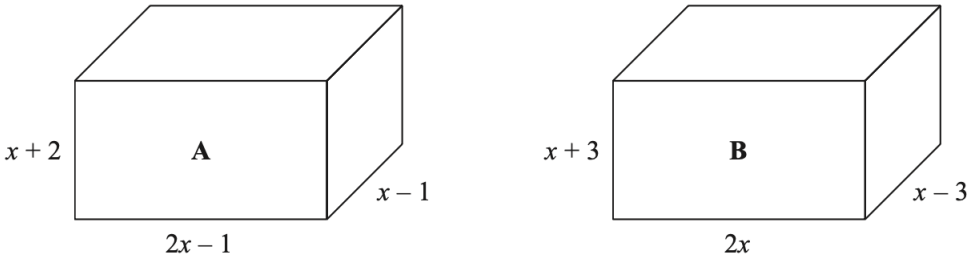
So, the length scale ratio (LSR) between **A** and **C** is

$$4 : 2.5.$$

Finally, the area scale ratio (ASR) between **A** and **C** is

$$\begin{aligned} 4^2 : 2.5^2 &= 16 : 6.25 \\ &= 16 \times 4 : 6.25 \times 4 \\ &= \underline{\underline{64 : 25}}. \end{aligned}$$

13. Here are two cuboids. (5)



All lengths are measured in centimetres.

The volume of cuboid **A** is 142 cm^3 greater than the volume of cuboid **B**.

Work out the value of x.

Solution

Well,

×	x	$+2$
x	x^2	$+2x$
-1	$-x$	-2

and

$$(x + 2)(x - 1) = x^2 + x - 2.$$

Now,

$\times$	x^2	$+x$	-2
$2x$	$2x^3$	$+2x^2$	$-4x$
-1	$-x^2$	$-x$	$+2$

and the volume of cuboid **A** is

$$2x^3 + x^2 - 5x + 2.$$

Next, difference of two squares:

$$\begin{aligned} \text{volume of cuboid } \mathbf{B} &= 2x(x+3)(x-3) \\ &= 2x(x^2-9) \\ &= 2x^3 - 18x. \end{aligned}$$

Then

$$2x^3 + x^2 - 5x + 2 = (2x^3 - 18x) + 142 \Rightarrow x^2 + 13x - 140 = 0$$

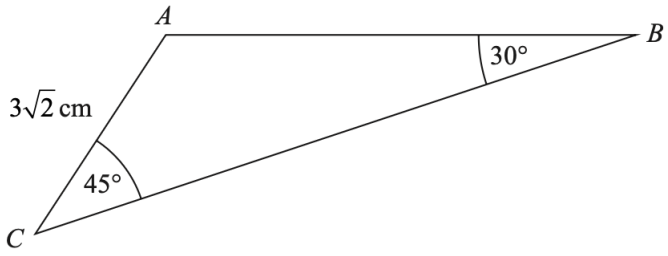
$$\left.\begin{array}{ll} \text{add to:} & +13 \\ \text{multiply to:} & -140 \end{array}\right\} +20, -7$$

$$\begin{aligned} &\Rightarrow (x+20)(x-7) = 0 \\ &\Rightarrow x + 20 = 0 \text{ or } x - 7 = 0 \\ &\Rightarrow x = -20 \text{ or } x = 7; \end{aligned}$$

but $x > 0$ (why?).

Hence, $\underline{\underline{x = 7}}$.

14. ABC is a triangle. (3)



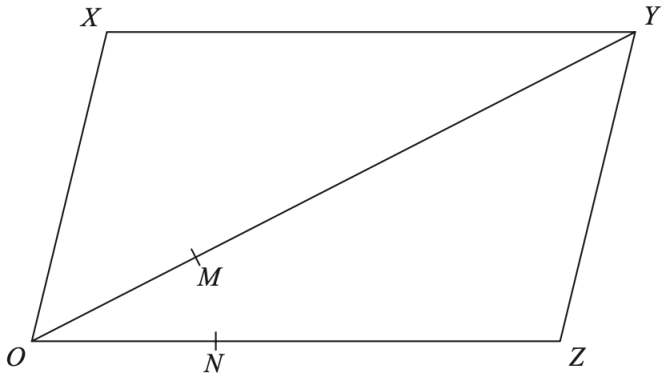
Work out the length of AB.

Solution

Sine rule:

$$\frac{AB}{\sin ACB} = \frac{AC}{\sin ABC} \Rightarrow \frac{AB}{\sin 45^\circ} = \frac{3\sqrt{2}}{\sin 30^\circ}$$
$$\Rightarrow \frac{AB}{\frac{1}{\sqrt{2}}} = \frac{3\sqrt{2}}{\frac{1}{2}}$$
$$\Rightarrow \sqrt{2}AB = 6\sqrt{2}$$
$$\Rightarrow \underline{\underline{AB = 6 \text{ cm}}}.$$

15. $OXYZ$ is a parallelogram. (4)



- $\overrightarrow{OY} = \mathbf{a}$.
- $\overrightarrow{OZ} = \mathbf{b}$.
- M is the point on OY such that

$$OM : MY = 1 : 3.$$

- N is the point on OZ such that

$$ON : NZ = 1 : 2.$$

Work out the ratio

$$XN : MN.$$

You must show all your working.

Solution

Well,

$$\overrightarrow{OM} = \tfrac{1}{4}\mathbf{a} \text{ and } \overrightarrow{ON} = \tfrac{1}{3}\mathbf{b}.$$

Now,

$$\begin{aligned}
\overrightarrow{XN} &= \overrightarrow{XO} + \overrightarrow{ON} \\
&= (\overrightarrow{YO} + \overrightarrow{OZ}) + \overrightarrow{ON} \\
&= (-\overrightarrow{OY} + \overrightarrow{OZ}) + \overrightarrow{ON} \\
&= -\mathbf{a} + \mathbf{b} + \tfrac{1}{3}\mathbf{b} \\
&= -\mathbf{a} + \tfrac{4}{3}\mathbf{b} \\
&= -\tfrac{4}{4}\mathbf{a} + \tfrac{4}{3}\mathbf{b} \\
&= 4(-\tfrac{1}{4}\mathbf{a} + \tfrac{1}{3}\mathbf{b})
\end{aligned}$$

and

$$\begin{aligned}
\overrightarrow{MN} &= \overrightarrow{MO} + \overrightarrow{ON} \\
&= -\overrightarrow{OM} + \overrightarrow{ON} \\
&= -\tfrac{1}{4}\mathbf{a} + \tfrac{1}{3}\mathbf{b}.
\end{aligned}$$

Hence,

$$XN : MN = \underline{\underline{4 : 1}}.$$

16. (a) Rationalise the denominator of (2)

$$\frac{15}{\sqrt{5}}.$$

Give your answer in its simplest form.

Solution

Well,

$$\begin{aligned}
\frac{15}{\sqrt{5}} &= \frac{15}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} \\
&= \frac{15\sqrt{5}}{5} \\
&= \underline{\underline{3\sqrt{5}}}.
\end{aligned}$$

(b) Write (4)

$$\frac{\sqrt{75}-2}{1+2\sqrt{3}}$$

in the form

$$\frac{a-b\sqrt{3}}{c},$$

where a, b, and c are integers.

Solution

Well,

$$\begin{aligned}\sqrt{75} &= \sqrt{25\times 3}\\ &= \sqrt{25}\times\sqrt{3}\\ &= 5\sqrt{3}.\end{aligned}$$

Now,

$\times$	$5\sqrt{3}$	-2
1	$5\sqrt{3}$	-2
$-2\sqrt{3}$	-30	$+4\sqrt{3}$

and

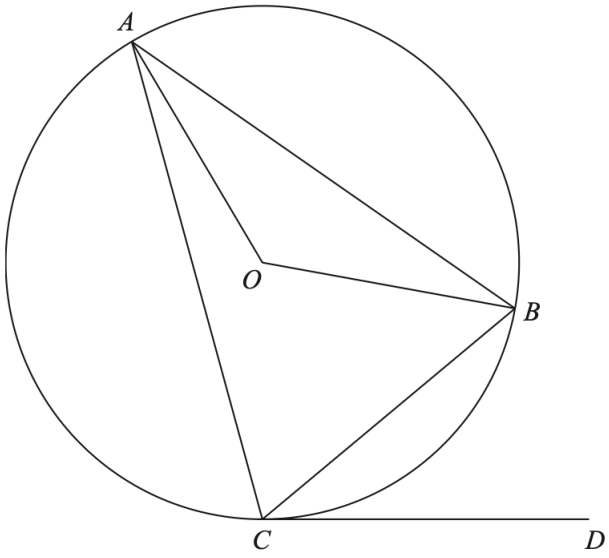
$\times$	1	$+2\sqrt{3}$
1	1	$+2\sqrt{3}$
$-2\sqrt{3}$	$+2\sqrt{3}$	-12

which means

$$\begin{aligned}\frac{\sqrt{75}-2}{1+2\sqrt{3}} &= \frac{5\sqrt{3}-2}{1+2\sqrt{3}}\\ &= \frac{5\sqrt{3}-2}{1+2\sqrt{3}}\times\frac{1-2\sqrt{3}}{1-2\sqrt{3}}\\ &= \frac{-32+9\sqrt{3}}{-11}\\ &= \underline{\underline{\frac{32-9\sqrt{3}}{11}}};\end{aligned}$$

hence, $\underline{\underline{a=32}}$, $\underline{\underline{b=-9}}$, and $\underline{\underline{c=11}}$.

17. A, B, and C are points on a circle, centre O. (4)



- CD is a tangent to the circle.
- Angle $BCD = 40°$.
- Angle $OAB = 3 \times$ angle OAC.

Work out the size of angle ACD.
Write down any circle theorems that you use.

Solution

Well, $\angle CAB = 40°$ (alternating segment theorem)
$\angle OAC = 10°$
$\angle OAB = \angle OBA = 30°$
$\angle AOB = 180 - 30 - 30 = 120°$ (completing the triangle)
$\angle ACB = 120 \div 2 = 60°$ (angle at the centre is twice the angle at the circumference)
$\angle ACD = 60 + 40 = \underline{\underline{100°}}$.

18.

$$\mathrm{f}(x) = \frac{5x - 3}{4}.$$

(a) Find $\mathrm{f}^{-1}(x)$. (2)

Solution
Well,

$$y = \frac{5x-3}{4} \Rightarrow 4y = 5x - 3$$
$$\Rightarrow 4y + 3 = 5x$$
$$\Rightarrow \frac{4y+3}{5} = x,$$

and so

$$\underline{\underline{\mathrm{f}^{-1}(x) = \frac{4x+3}{5}}}.$$

For all values of x,

$$\mathrm{g}(x) = (x-1)^2 \text{ and } \mathrm{h}(x) = 1 - 2x.$$

(b) Work out the value of $\mathrm{g}\,\mathrm{h}(5)$. (2)

Solution
Now,

$$\begin{aligned} \mathrm{g}\,\mathrm{h}(5) &= \mathrm{g}(\mathrm{h}(5)) \\ &= \mathrm{g}(-9) \\ &= \underline{\underline{100}}. \end{aligned}$$

19. In the semi-finals of a chess tournament, (4)

- player **A** will play player **B** and
- player **C** will play player **D**.

The two winners will then play each other in the final.

- The probability that player **A** will win against player **B** is 0.6.
- The probability that player **A** will win against player **C** is 0.5.
- The probability that player **A** will win against player **D** is 0.3.
- The probability that player **C** will win against player **D** is 0.2.

Work out the probability that player **A** will win the chess tournament.

Solution

Well,

$$\begin{aligned} \mathrm{P}(\mathbf{A} \text{ wins}) &= \mathrm{P}(\mathbf{A} \text{ beats } \mathbf{C}) + \mathrm{P}(\mathbf{A} \text{ beats } \mathbf{D}) \\ &= (0.6 \times 0.2 \times 0.5) + (0.6 \times 0.8 \times 0.3) \\ &= 0.06 + 0.144 \\ &= \underline{\underline{0.204}}. \end{aligned}$$

20. **C** is the circle with equation (4)

$$x^2 + y^2 = 4.$$

Find an equation of the tangent to **C** at the point $(p, 1)$ where $p > 0$.

Give your answer in the form

$$y + x\sqrt{a} = b,$$

where a and b are integers.

Solution

Well,

$$\begin{aligned} y = 1 &\Rightarrow x^2 + 1 = 4 \\ &\Rightarrow x^2 = 3 \\ &\Rightarrow x = \sqrt{3}, \end{aligned}$$

as $x > 0$.

Now,

$$\begin{aligned} m &= \frac{1 - 0}{\sqrt{3} - 0} \\ &= \frac{1}{\sqrt{3}}, \end{aligned}$$

so

$$m_{\text{tangent}} = -\sqrt{3}.$$

Next, an equation of the tangent is

$$\begin{aligned} y - 1 = -\sqrt{3}(x - \sqrt{3}) &\Rightarrow y - 1 = -\sqrt{3}x + 3 \\ &\Rightarrow \underline{\underline{y + x\sqrt{3} = 4}}; \end{aligned}$$

hence, $\underline{\underline{a = 3}}$ and $\underline{\underline{b = 4}}$.