Dr Oliver Mathematics
AQA GCSE Mathematics
2018 November Paper 2: Calculator
1 hour 30 minutes

The total number of marks available is 80.
You must write down all the stages in your working.

1. What does $(A \cap B)$ represent in $\mathrm{P}(A \cap B)$? (1)
Circle your answer.

A or B or both A but not B not A and not B A and B

Solution

A or B or both A but not B not A and not B $\underline{\underline{A \text{ and } B}}$

2. P is $(4, 9)$ and Q is $(-2, 1)$. (1)
Circle the midpoint of PQ.

$(1, 5)$ $(3, 4)$ $(3, 5)$ $(6, 8)$

Solution

Well,

$$\left(\frac{4 + (-2)}{2}, \frac{9 + 1}{2}\right) = (1, 5)$$

so

$\underline{\underline{(1, 5)}}$ $(3, 4)$ $(3, 5)$ $(6, 8)$

3. Which of these is a geometric progression? (1)
Circle your answer.

1, 3, 5, 7, 9 1, 3, 6, 10, 15 1, 4, 9, 16, 25 1, 3, 9, 27, 81

Solution

1, 3, 5, 7, 9 1, 3, 6, 10, 15 1, 4, 9, 16, 25 <u>1, 3, 9, 27, 81</u>

4. The bearing of A from B is $310°$. (1)
Circle the bearing of B from A.

$050°$ $110°$ $130°$ $220°$

Solution

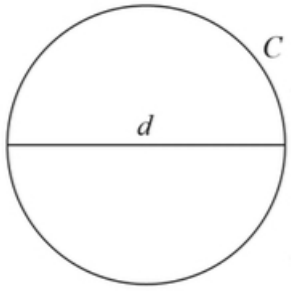
The bearing of B from A is

$$90 + 40 = 130°$$

so

$050°$ $110°$ <u>$130°$</u> $220°$

5. A circle has circumference C and diameter d. (1)



$C = kd$.

What value does the constant k represent?

Solution

<u>$k = \pi$</u>.

6. Here is some information about 20 trains leaving a station.

Number of minutes late, t	Number of trains	Midpoint
$0 \leqslant t < 5$	12	
$5 \leqslant t < 10$	7	
$10 \leqslant t < 15$	1	
$t \geqslant 15$	0	

(a) Work out an estimate of the mean number of minutes late. (3)

Solution

t	Number of trains	Midpoint	Trains × Midpoint
$0 \leqslant t < 5$	12	2.5	$12 \times 2.5 = 30$
$5 \leqslant t < 10$	7	7.5	$7 \times 7.5 = 52.5$
$10 \leqslant t < 15$	1	12.5	$1 \times 12.5 = 12.5$
Total	20		95

Now,

$$\begin{aligned} \text{mean} &= \frac{\sum ft}{\sum t} \\ &\approx \frac{95}{20} \\ &= \underline{\underline{4.75 \text{ minutes}}}. \end{aligned}$$

The station manager looks at the information in more detail.

Number of minutes late, t	Number of trains
$0 \leqslant t < 2$	12
$2 \leqslant t < 4$	0
$4 \leqslant t < 6$	7
$6 \leqslant t < 8$	0
$8 \leqslant t < 10$	0
$10 \leqslant t < 12$	1

He works out an estimate of the mean using this information.

(b) How does his estimate compare with the answer to part (a)? (1)
Tick **one** box.

☐ Higher than part (a)

☐ Same as part (a)

☐ Lower than part (a)

☐ Not possible to tell

Solution

t	Number of trains	Midpoint	Trains × Midpoint
$0 \leqslant t < 2$	12	1	$12 \times 1 = 12$
$4 \leqslant t < 6$	7	5	$7 \times 5 = 35$
$10 \leqslant t < 12$	1	11	$1 \times 11 = 11$
Total	20		58

Now,

$$\begin{aligned} \text{mean} &= \frac{\sum ft}{\sum t} \\ &\approx \frac{58}{20} \\ &= 2.9 \text{ minutes} \end{aligned}$$

so Lower than part (a).

7. Work out the values of a and b in the identity (4)

$$5(7x + 8) + 3(2x + b) \equiv ax + 13.$$

Solution

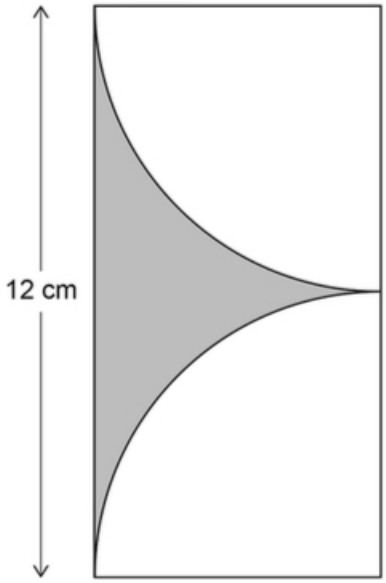
Well,

$$5(7x+8)+3(2x+b) \equiv ax+13 \Rightarrow 35x+40+6x+3b \equiv ax+13$$
$$\Rightarrow 41x+(3b+40) \equiv ax+13$$

so $\underline{\underline{a=41}}$ and

$$3b+40=13 \Rightarrow 3b=-27$$
$$\Rightarrow \underline{\underline{b=-9}}.$$

8. Two identical quarter circles are cut from a rectangle as shown. (4)



Not drawn accurately

Work out the shaded area.

Solution

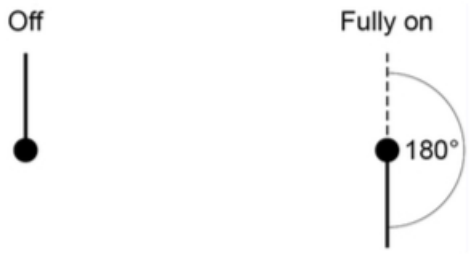
Obviously, the two squares are

$$\frac{12}{2}=6 \text{ cm}$$

each and we have a quarter of a circle in each. Now,

$$\begin{aligned}\text{shaded area} &= (12 \times 6) - (2 \times \tfrac{1}{4} \times \pi \times 6^2) \\ &= \underline{\underline{(72 - 18\pi)\ \text{cm}^2}}.\end{aligned}$$

9. he diagrams show the position of a tap when off and fully on. (4)
The tap is fully on when the angle of turn is 180°.



When fully on, water flows out of the tap at 14 litres per minute.
The rate at which water flows out is in direct proportion to the angle of turn.
The tap is turned 135°.

The water flows into a tank with a capacity of 79.8 litres.
Will it take **less than** $7\frac{1}{2}$ minutes to fill the tank?
You **must** show your working.

Solution

Now,

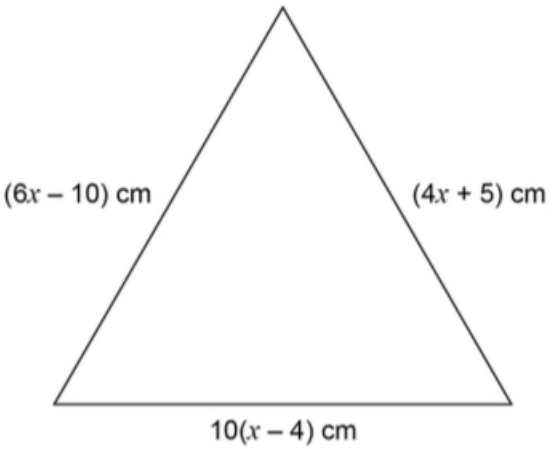
$$\begin{aligned}\text{water flows} &= \frac{135}{180} \times 14 \\ &= 10\tfrac{1}{2} \text{ litres a minute}\end{aligned}$$

and

$$\begin{aligned}\text{fill the tank} &= \frac{79.8}{1012} \\ &= 7.6 \text{ minutes;}\end{aligned}$$

so, it will take longer.

10. This triangle is equilateral. (5)



Not drawn accurately

Is the perimeter of the triangle greater than one metre?
You **must** show your working.

Solution

Well, all sides are the same length so

$$\begin{aligned}6x - 10 = 4x + 5 &\Rightarrow 2x = 15 \\ &\Rightarrow x = 7.5\end{aligned}$$

and that gives each side of length

$$6(7.5) - 10 = 35 \text{ cm.}$$

Finally,

$$\begin{aligned}\text{perimeter} &= 3 \times 35 \\ &= 105 \text{ cm} \\ &= 1.05 \text{ m.}\end{aligned}$$

Hence, the perimeter of the triangle greater than one metre.

11. An approximation for the value of π is given by (2)

$$4\left(1-\tfrac{22}{57}+\tfrac{22}{85}-\tfrac{22}{105}+\tfrac{22}{117}-\tfrac{22}{242}\right).$$

Use your calculator to show that this approximation is within 0.1 of 3.14.

Solution

Using a calculator, this value is

$$3.041\,836\,619 \text{ (FCD)}$$

and the value of π is

$$3.141\,592\,654 \text{ (FCD)}.$$

The difference between those values is

$$0.099\,753\,034\,66 \text{ (FCD)};$$

hence, this approximation is within 0.1 of 3.14.

12. Work out (2)

$$\frac{9.12\times10^{10}}{3.2\times10^{4}}.$$

Give your answer in standard form.

Solution

$$\begin{aligned}\frac{9.12\times10^{10}}{3.2\times10^{4}} &= 2\,850\,000\\ &= 2.85\times10^{6}.\end{aligned}$$

13. Ashraf is going to put boxes into a crate. (2)

- The crate is a cuboid measuring 2.5 m by 2 m by 1.2 m.
- Each box is a cube of length 50 cm.

He does these calculations.

volume of crate	=	$2.5 \times 2 \times 1.2$
	=	$6\ m^3$
volume of one box	=	$0.5 \times 0.5 \times 0.5$
	=	$0.125\ m^3$
number of boxes	=	$6 \div 0.125$
	=	48

He claims, "I can put 48 boxes in the crate."

Evaluate Ashraf's method **and** claim.

Solution

No, he is not correct: e.g., 50 cm will not fit in 0.2 m.

14. The cross section of a prism has n sides. (1)

Circle the expression for the number of edges of the prism.

$$2n \quad 3n \quad n+2 \quad 2n+3$$

Solution

$$2n \quad \underline{3n} \quad n+2 \quad 2n+3$$

15. The volume of a medal is 45 cm^3. (4)

The medal is made from copper and tin.

volume of copper : volume of tin = 22 : 3

- The density of copper is 8.96 g/cm^3.
- The density of tin is 7.31 g/cm^3

Work out the mass of the medal.

Solution

Well,

$$\begin{aligned}\text{volume of copper} &= 45 \times \left(\frac{22}{22+3}\right)\\ &= 39.6 \text{ cm}^3\end{aligned}$$

and

$$\begin{aligned}\text{volume of tin} &= 45 \times \left(\frac{3}{22+3}\right)\\ &= 5.4 \text{ cm}^3.\end{aligned}$$

Now,

$$\begin{aligned}\text{mass of copper} &= \text{density} \times \text{volume}\\ &= 8.96 \times 39.6\\ &= 354.816 \text{ g}\end{aligned}$$

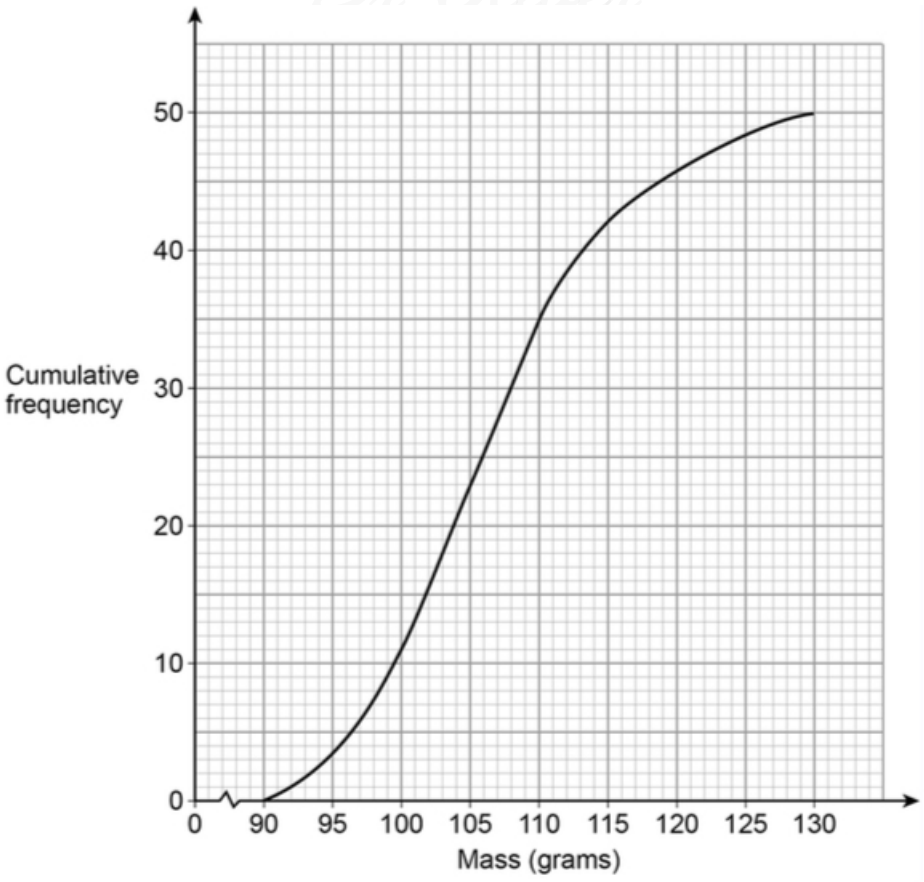
and

$$\begin{aligned}\text{mass of tin} &= \text{density} \times \text{volume}\\ &= 7.31 \times 5.4\\ &= 39.474 \text{ g}.\end{aligned}$$

Finally, add:

$$354.816 + 39.474 = \underline{\underline{394.29 \text{ g}}}.$$

16. The cumulative frequency graph shows information about the masses of 50 apples.



(a) Use the graph to estimate the median mass of the apples. (1)

Solution

106 g.

(b) Estimate the proportion of the apples that have a mass greater than 115 grams. (2)

Solution

$$\frac{50-42}{50} = \frac{4}{25}.$$

17. a is a prime number. (1)
b is an even number.

$$N = a^2 + ab.$$

Circle the correct statement about N.

could be even or odd always even always prime always odd

Solution

For example, if

$$a = 2, b = 4 \Rightarrow N = 2^2 + (2 \times 4) = 12 \text{ (not odd, not prime)}$$

but

$$a = 3, b = 4 \Rightarrow N = 3^2 + (3 \times 4) = 21 \text{ (not even, not prime)}$$

so

$$\underline{\underline{\text{could be even or odd}}} \quad \text{always even} \quad \text{always prime} \quad \text{always odd}$$

18. A bag contains 20 discs.

10 are red, 7 are blue, and 3 are green.

Marnie takes a disc at random before putting it back in the bag.

Nick then takes a disc at random before putting it back in the bag.

Olly then takes a disc at random.

(a) Work out the probability that they all take a red disc. (2)

Solution

$$\begin{aligned} \text{P}(RRR) &= \tfrac{10}{20} \times \tfrac{10}{20} \times \tfrac{10}{20} \\ &= \underline{\underline{\tfrac{1}{8}}}. \end{aligned}$$

All 20 discs are in the bag.

- Reggie takes three discs at random, one after the other.
- After he takes a disc he does **not** put it back in the bag.

Reggie's first disc is blue.

(b) Work out the probability that all three discs are different colours. (3)

Solution

Once he made the original selection, there are 10 are red and 3 are green. Now,

$$\begin{aligned}\text{P(different colours)} &= \text{P}(RW) + \text{P}(WR)\\ &= (\tfrac{10}{19} \times \tfrac{3}{18}) + (\tfrac{3}{19} \times \tfrac{10}{18})\\ &= \tfrac{30}{342} + \tfrac{30}{342}\\ &= \underline{\underline{\tfrac{10}{57}}}.\end{aligned}$$

19. There are four starters and ten main courses to choose from. (3)

Two of the starters and three of the main courses are suitable for vegans.

Lunch

Choose one starter and one main course

What percentage of the possible lunches have **both** courses suitable for vegans?

Solution

$$\begin{aligned}\text{Percentage} &= \tfrac{2}{4} \times \tfrac{3}{10} \times 100\%\\ &= \underline{\underline{15\%}}.\end{aligned}$$

20. n is a positive integer. (3)

Prove algebraically that

$$2n^2\left(\frac{3}{n} + n\right) + 6n(n^2 - 1)$$

is a cube number.

Solution

$$2n^2\left(\frac{3}{n}+n\right)+6n(n^2-1) = 6n+2n^3+6n^3-6n$$
$$= 8n^3$$
$$= (2n)^3$$
$$= (\text{a positive number})^3;$$

hence, the expression is a cube number.

21. y is inversely proportional to $\sqrt{x}$.

$y = 4$ when $x = 9$.

(a) Work out an equation connecting y and x. (3)

Solution

Now,

$$y \propto \frac{1}{\sqrt{x}} \Rightarrow y = \frac{k}{\sqrt{x}},$$

for some constant k. Next,

$$x = 9, y = 4 \Rightarrow 4 = \frac{k}{\sqrt{9}}$$
$$\Rightarrow k = 4 \times 3$$
$$\Rightarrow k = 12$$

and

$$y = \frac{12}{\sqrt{x}}.$$

(b) Work out the value of y when $x = 25$. (2)

Solution

$$y = \frac{12}{\sqrt{25}}$$
$$= \frac{12}{5}$$
$$= 2.4.$$

22. Simplify fully (3)

$$\frac{x^5 - 4x^3}{3x - 6}.$$

Solution

Difference of two squares:

$$\begin{aligned}\frac{x^5 - 4x^3}{3x - 6} &= \frac{x^3(x^2 - 4)}{3(x - 2)} \\ &= \frac{x^3(x + 2)(x - 2)}{3(x - 2)} \\ &= \underline{\underline{\frac{x^3(x + 2)}{3}}}.\end{aligned}$$

23. PQR is a straight line. (1)
$PQ : QR = 3 : 1$.
$\overrightarrow{PQ} = \mathbf{a}$.

Not drawn accurately

Circle the vector $\overrightarrow{RQ}$.

$\frac{1}{3}\mathbf{a}$ $\frac{1}{4}\mathbf{a}$ $-\frac{1}{3}\mathbf{a}$ $-\frac{1}{4}\mathbf{a}$

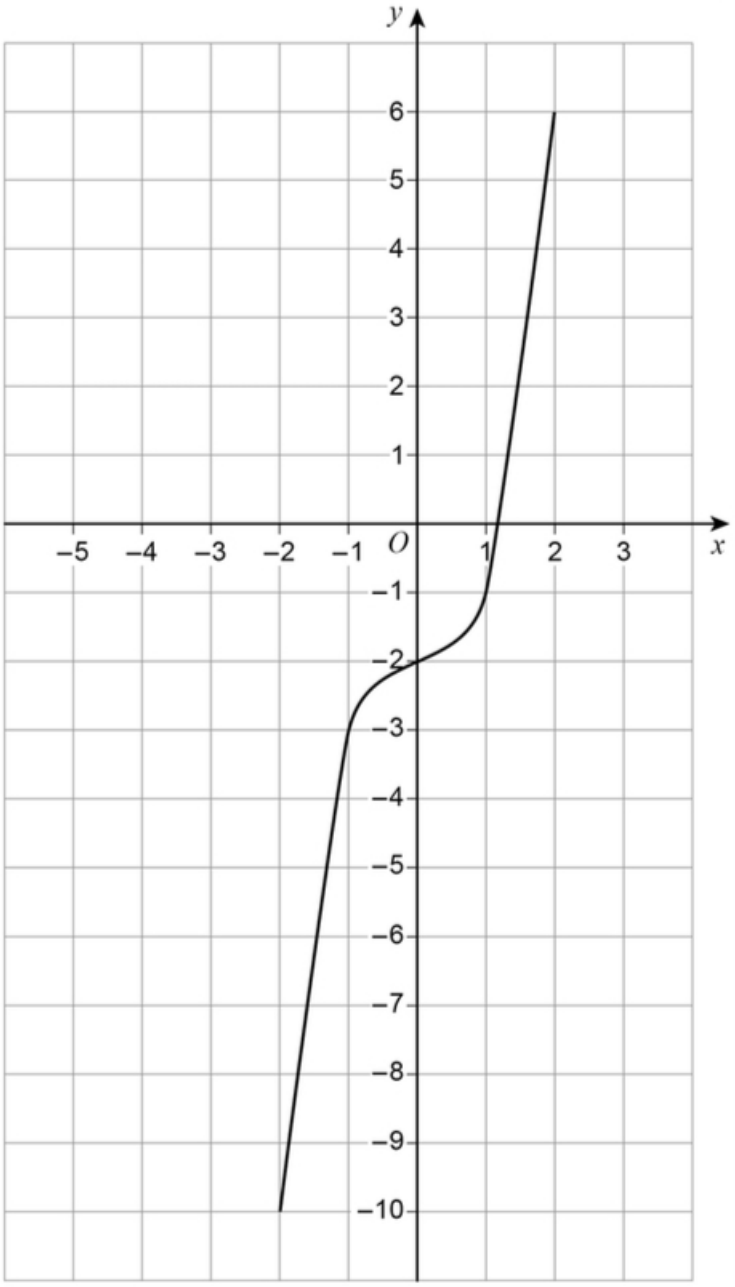
Solution

$$\begin{aligned}\overrightarrow{RQ} &= \overrightarrow{RP} + \overrightarrow{PQ} \\ &= -\overrightarrow{PR} + \overrightarrow{PQ} \\ &= -\tfrac{4}{3}\mathbf{a} + \mathbf{a} \\ &= -\tfrac{1}{3}\mathbf{a}\end{aligned}$$

so

$$\tfrac{1}{3}\mathbf{a} \quad \tfrac{1}{4}\mathbf{a} \quad \underline{\underline{-\tfrac{1}{3}\mathbf{a}}} \quad -\tfrac{1}{4}\mathbf{a}$$

24. Here is a sketch of $y = \mathrm{f}(x)$. (2)
The curve passes through the points $(-2, -10)$, $(-1, -3)$, $(0, -2)$, $(1, -1)$, and $(2, 6)$.



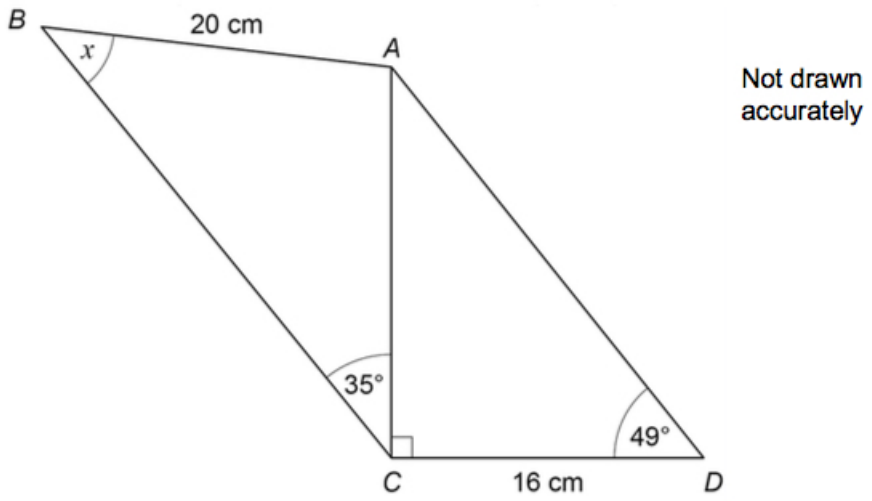
On the grid, sketch the curve $y = \mathrm{f}(x + 2)$.

Solution

The curve passes through the points

$$\underline{\underline{(-4, -10),\ (-3, -3),\ (-2, -2),\ (-1, -1) \text{ and } (0, 6)}}.$$

25. ABC and ACD are triangles. (5)



Work out the size of angle x.

Solution

$$\tan = \frac{\text{opp}}{\text{adj}} \Rightarrow \tan 49^\circ = \frac{AC}{16}$$
$$\Rightarrow AC = 16 \tan 49^\circ$$

and we apply the sine rule:

$$\frac{\sin ABC}{AC} = \frac{\sin ACB}{AB} \Rightarrow \frac{\sin x}{16 \tan 49^\circ} = \frac{\sin 35^\circ}{20}$$
$$\Rightarrow \sin x = \frac{16 \tan 49^\circ \sin 35^\circ}{20}$$
$$\Rightarrow x = 31.860\,934\,8 \text{ (FCD)}$$
$$\Rightarrow \underline{\underline{x = 31.9^\circ \text{ (3 sf)}}}.$$

26. (3)

$$\mathrm{f}(x) = \frac{x}{x+2} \text{ and } \mathrm{g}(x) = x^2 - 2.$$

Work out $\mathrm{f\,g}(x)$.
Give your answer in the form

$$a + bx^n,$$

where a, b, and n are integers.

Solution

$$\begin{aligned}
\mathrm{f\,g}(x) &= \mathrm{f}(\mathrm{g}(x)) \\
&= \mathrm{f}(x^2 - 2) \\
&= \frac{x^2 - 2}{(x^2 - 2) + 2} \\
&= \frac{x^2 - 2}{x^2} \\
&= \underline{\underline{1 - 2x^{-2}}};
\end{aligned}$$

hence, $\underline{\underline{a = 1}}$, $\underline{\underline{b = -2}}$, and $\underline{\underline{n = -2}}$.

27. The point $(3, \frac{1}{64})$ lies on the curve $y = k^x$, where k is a constant. (3)

Show that the point $(\frac{1}{2}, \frac{1}{2})$ lies on the curve.

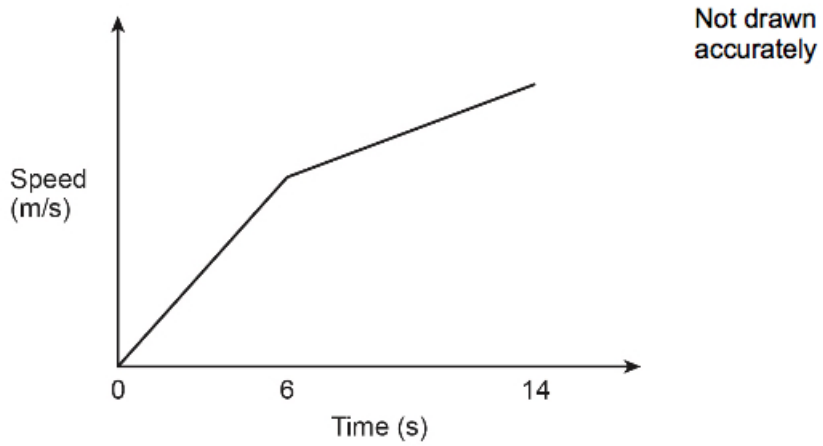
Solution

$$\begin{aligned}
x = 3, y = \tfrac{1}{64} &\Rightarrow \tfrac{1}{64} = k^3 \\
&\Rightarrow k = \sqrt[3]{\tfrac{1}{64}} \\
&\Rightarrow \underline{\underline{k = \tfrac{1}{4}}}.
\end{aligned}$$

28. Izzy runs an 80-metre race in 14 seconds.

- During the first 6 seconds her speed increases at a constant rate.
- During the last 8 seconds her speed increases at a different constant rate.
- Her speed at 14 seconds is 2 m/s more than her speed at 6 seconds.

Here is a sketch of her speed-time graph.



(a) Work out her acceleration during the last 8 seconds. (2)
State the units of your answer.

Solution

$$\text{Acceleration} = \frac{2}{8}$$
$$= \underline{\underline{\tfrac{1}{4} \text{ m/s}^2}}.$$

When Izzy finishes the 80-metre race, her speed is v m/s.

(b) Work out the value of v. (4)

Solution

Let v m/s be her speed during the first 8 seconds.
Then $(v-2)$ m/s be her speed during the first 6 seconds.
Now,

$$\begin{aligned} & 6 \text{ seconds} + 8 \text{ seconds} = \text{distance} \\ \Rightarrow \quad & [\tfrac{1}{2} \times 6 \times (v-2)] + [\tfrac{1}{2}(8)[v + (v-2)] = 80 \\ \Rightarrow \quad & 3(v-2) + 4(2v-2) = 80 \\ \Rightarrow \quad & 3v - 6 + 8v - 8 = 80 \\ \Rightarrow \quad & 11v = 94 \\ \Rightarrow \quad & \underline{\underline{v = 8\tfrac{6}{11} \text{ m/s}}}. \end{aligned}$$