Dr Oliver Mathematics
Mathematics: National Qualifications N5
2016 Paper 1: Non-Calculator
1 hour

The total number of marks available is 40.
You must write down all the stages in your working.

1. Give that (2)

$$\mathbf{p} = \begin{pmatrix} 4 \\ -6 \end{pmatrix} \text{ and } \mathbf{q} = \begin{pmatrix} -5 \\ -1 \end{pmatrix},$$

find the resultant vector $\frac{1}{2}\mathbf{p} + \mathbf{q}$.

Express your answer in component form.

Solution

$$\begin{aligned} \tfrac{1}{2}\mathbf{p} + \mathbf{q} &= \tfrac{1}{2}\begin{pmatrix} 4 \\ -6 \end{pmatrix} + \begin{pmatrix} -5 \\ -1 \end{pmatrix} \\ &= \begin{pmatrix} 2 \\ -3 \end{pmatrix} + \begin{pmatrix} -5 \\ -1 \end{pmatrix} \\ &= \underline{\underline{\begin{pmatrix} -3 \\ -4 \end{pmatrix}}}. \end{aligned}$$

2. Evaluate (2)

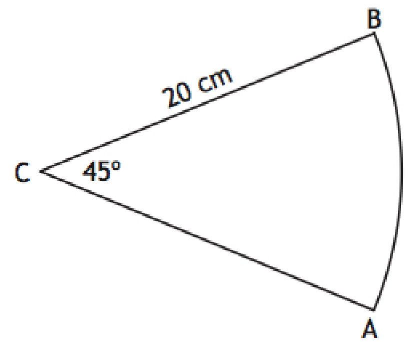
$$\tfrac{3}{4}(\tfrac{1}{3} + \tfrac{2}{7}).$$

Give your answer in its simplest form.

Solution

$$\begin{aligned} \tfrac{3}{4}(\tfrac{1}{3} + \tfrac{2}{7}) &= \tfrac{3}{4}(\tfrac{7}{21} + \tfrac{6}{21}) \\ &= \tfrac{3}{4} \times \tfrac{13}{21} \\ &= \tfrac{1}{4} \times \tfrac{13}{7} \\ &= \underline{\underline{\tfrac{13}{28}}}. \end{aligned}$$

3. The diagram shows a sector of a circle, centre C. (3)



The radius of the circle is 20 centimetres and angle ACB is $45°$.
Calculate the area of the sector.
Take $\pi = 3.14$.

Solution

$$\begin{aligned}\text{Area} &= \frac{45}{360} \times \pi \times 20^2 \\ &= \tfrac{1}{8} \times 3.14 \times 400 \\ &= \tfrac{1}{8} \times 1\,256 \\ &= \underline{\underline{157 \text{ cm}^2}}.\end{aligned}$$

4. Charlie is making costumes for a school show.
One day he made 2 cloaks and 3 dresses.
The total amount of material he used was 9.6 square metres.

(a) Write down an equation to illustrate this information. (1)

Solution
Let c and d be the material he needs to make a cloak and a dress respectively. Then

$$\underline{\underline{2c + 3d = 9.6}} \quad (1)$$

The following day Charlie made 3 cloaks and 4 dresses.
The total amount of material he used was 13.3 square metres.

(b) Write down an equation to illustrate this information. (1)

Solution

$$\underline{\underline{3c + 4d = 13.3}} \quad (2)$$

(c) Calculate the amount of material required to make one cloak and the amount of material required to make one dress. (4)

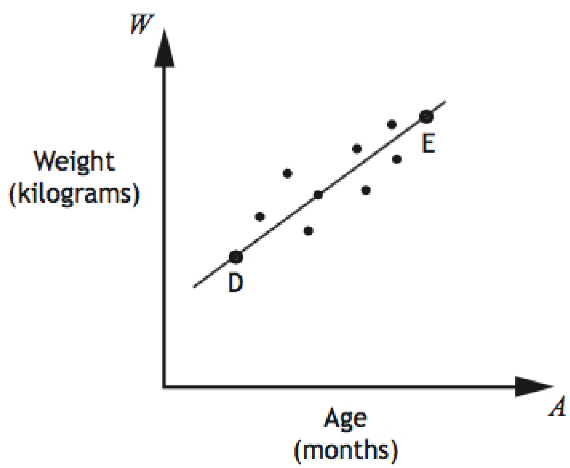
Solution

$$\begin{aligned} 3 \times (1): \quad & 6c + 9d = 28.8 \quad (3) \\ 2 \times (2): \quad & 6c + 8d = 26.6 \quad (4) \end{aligned}$$

Now, (3) − (4):

$$\begin{aligned} \underline{\underline{d = 2.2}} &\Rightarrow 2c + 6.6 = 9.6 \\ &\Rightarrow 2c = 3 \\ &\Rightarrow \underline{\underline{c = 1.5}}. \end{aligned}$$

5. A cattle farmer records the weight of some of his calves.
The scattergraph shows the relationship between the age, A months, and the weight, W kilograms, of the calves.



A line of best fit is drawn.
Point D represents a 3 month old calf which weighs 100 kilograms.
Point E represents a 15 month old calf which weighs 340 kilograms.

(a) Find the equation of the line of best fit in terms of A and W. (3)
Give the equation in its simplest form.

Solution

$$\begin{aligned}\text{Gradient} &= \frac{340-100}{15-3} \\ &= \frac{240}{12} \\ &= 20\end{aligned}$$

and the equation is

$$\begin{aligned}W - 100 = 20(A-3) &\Rightarrow W - 100 = 20A - 60 \\ &\Rightarrow \underline{\underline{W = 20A + 40}}.\end{aligned}$$

(b) Use your equation from part (a) to estimate the weight of a one year old calf. (1)
Show your working.

Solution
One-year old equals 12 months:

$$\begin{aligned}W &= (20 \times 12) + 40 \\ &= 240 + 40 \\ &= \underline{\underline{280 \text{ kilograms}}}.\end{aligned}$$

6. Determine the nature of the roots of the function (2)

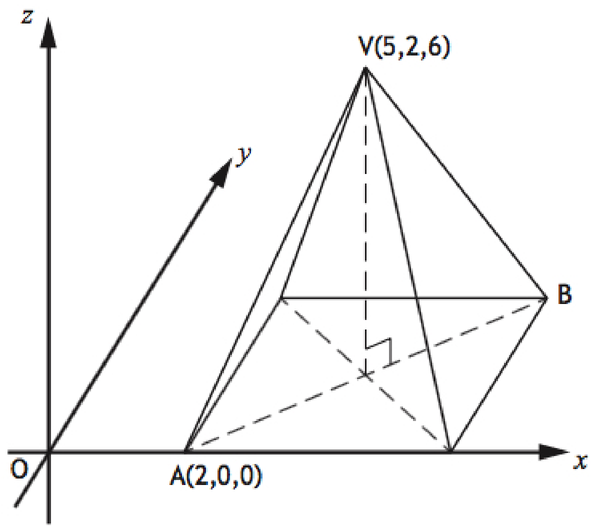
$$\mathrm{f}(x) = 7x^2 + 5x - 1.$$

Solution

$$\begin{aligned}5^2 - 4 \times 7 \times (-1) &= 25 - (-28) \\ &= 53;\end{aligned}$$

hence, they are $\underline{\underline{\text{real}}}$ and $\underline{\underline{\text{distinct}}}$.

7. The diagram shows a rectangular based pyramid, relative to the coordinate axes.



A is the point $(2, 0, 0)$.
V is the point $(5, 2, 6)$.

(a) Write down the coordinates of B. (1)

Solution
$\underline{\underline{B(8, 4, 0)}}$.

(b) Calculate the length of edge AV of the pyramid. (3)

Solution

$$\begin{aligned} AV &= \sqrt{(5-2)^2 + (2-0)^2 + (6-0)^2} \\ &= \sqrt{9+4+36} \\ &= \sqrt{49} \\ &= \underline{\underline{7}}. \end{aligned}$$

8. Solve the equation (3)

$$\tfrac{2}{3}x - \tfrac{5}{6} = 2x.$$

Give your answer in its simplest form.

Solution

$$\begin{aligned}\tfrac{2}{3}x - \tfrac{5}{6} = 2x &\Rightarrow -\tfrac{5}{6} = \tfrac{4}{3}x \\ &\Rightarrow -\tfrac{5}{2} = 4x \\ &\Rightarrow \underline{\underline{x = -\tfrac{5}{8}}}.\end{aligned}$$

9. The function $\mathrm{f}(x)$ is defined by (2)

$$\mathrm{f}(x) = \frac{2}{\sqrt{x}},\ x > 0.$$

Express $\mathrm{f}(5)$ as a fraction with a rational denominator.

Solution

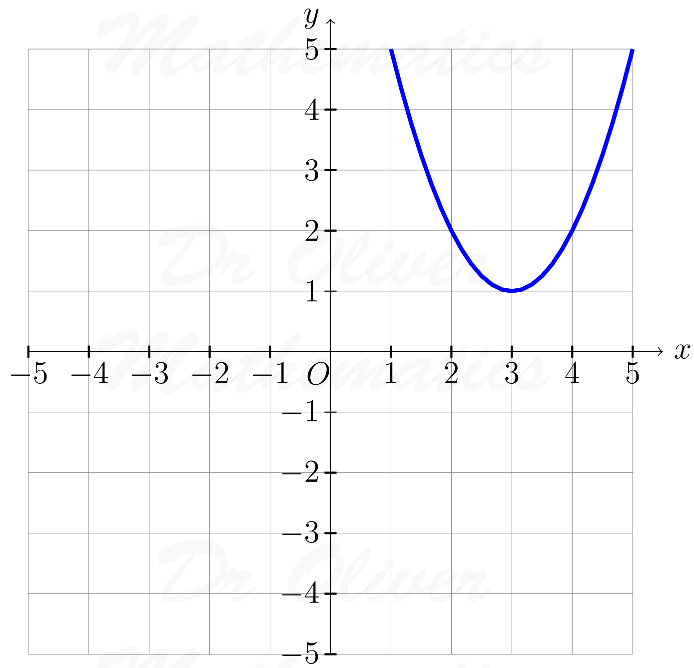
$$\begin{aligned}\mathrm{f}(5) &= \frac{2}{\sqrt{5}} \\ &= \frac{2}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} \\ &= \underline{\underline{\frac{2\sqrt{5}}{5}}}.\end{aligned}$$

10. Sketch the graph of (3)

$$y = (x-3)^2 + 1.$$

On your sketch, show clearly the coordinates of the turning point and the point of intersection with the y-axis.

Solution



The turning point is at $\underline{\underline{(3, 1)}}$ and the y-intercept is $\underline{\underline{10}}$.

11. Simplify (2)

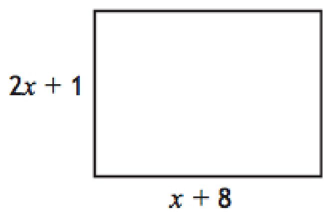
$$\tan^2 x^\circ \cos^2 x^\circ.$$

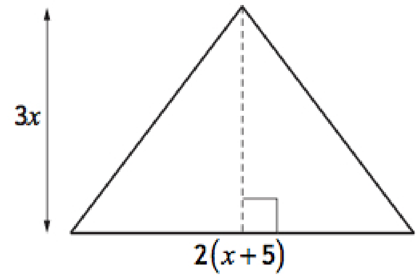
Show your working.

Solution

$$\begin{aligned}\tan^2 x^\circ \cos^2 x^\circ &= \frac{\sin^2 x^\circ}{\cos^2 x^\circ} \times \cos^2 x^\circ \\ &= \underline{\underline{\sin^2 x^\circ}}.\end{aligned}$$

12. The diagrams below show a rectangle and a triangle.
All measurements are in centimetres.





(a) Find an expression for the area of the **rectangle**. (1)

Solution

$\times$	$2x$	$+1$
x	$2x^2$	$+x$
$+8$	$+16x$	$+8$

Hence,

$$\text{rectangle} = \underline{\underline{(2x^2 + 17x + 8) \text{ cm}^2}}.$$

(b) Given that the area of the rectangle is equal to the area of the triangle, show that (3)

$$x^2 - 2x - 8 = 0.$$

Solution

$$\begin{aligned} \text{Triangle} &= \tfrac{1}{2} \times 2(x+5) \times 3x \\ &= 3x(x+5) \end{aligned}$$

and

$$\begin{aligned} 3x(x+5) = (2x+1)(x+8) &\Rightarrow 3x^2 + 15x = 2x^2 + 17x + 8 \\ &\Rightarrow \underline{\underline{x^2 - 2x - 8 = 0}}, \end{aligned}$$

as required.

(c) Hence find, **algebraically**, the length and breadth of the rectangle. (3)

Solution

$$\left.\begin{array}{ll}\text{add to:} & -2 \\ \text{multiply to:} & -8\end{array}\right\} -4,\ +2$$

$$\begin{aligned} x^2 - 2x - 8 = 0 &\Rightarrow (x-4)(x+2) = 0 \\ &\Rightarrow x - 4 = 0 \text{ or } x + 2 = 0 \\ &\Rightarrow x = 4 \text{ or } x = -2; \end{aligned}$$

hence, the only solution is $x = 4$ and that makes the length $\underline{\underline{9\text{ cm}}}$ and the breadth $\underline{\underline{12\text{ cm}}}$.