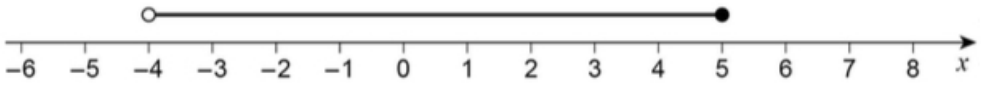
Dr Oliver Mathematics
AQA GCSE Mathematics
2017 November Paper 3: Calculator
1 hour 30 minutes

The total number of marks available is 80.
You must write down all the stages in your working.

1. Circle the inequality shown by the diagram. (1)



$-4 \leqslant x < 5 \quad -4 \leqslant x \leqslant 5 \quad -4 < x < 5 \quad -4 < x \leqslant 5.$

Solution

$-4 \leqslant x < 5 \quad -4 \leqslant x \leqslant 5 \quad -4 < x < 5 \quad \underline{-4 < x \leqslant 5}.$

2. y is 100% more than x. (1)

Circle the ratio $x : y$.

$1 : 100 \quad 100 : 1 \quad 1 : 2 \quad 2 : 1.$

Solution

$1 : 100 \quad 100 : 1 \quad \underline{1 : 2} \quad 2 : 1.$

3. The first four terms of a sequence are (1)

$-10 \quad -8 \quad -6 \quad -4.$

Circle the expression for the nth term of the sequence.

$$-12-2n \quad -8-2n \quad n+2 \quad 2n-12.$$

Solution

Let the

$$n\text{th term} = an + b.$$

$$\begin{array}{cccc} \text{-10} & -8 & -6 & -4 \\ \quad 2 & \quad 2 & \quad 2 & \end{array}$$

$$\begin{array}{cccc} a+b & 2a+b & 3a+b & 4a+b \\ \quad a & \quad a & \quad a & \end{array}$$

We compare terms:

$$a = 2$$

and

$$\begin{aligned} a + b = -10 &\Rightarrow 2 + b = -10 \\ &\Rightarrow b = -12. \end{aligned}$$

Hence,

$$n\text{th term} = 2n - 12$$

so

$$-12-2n \quad -8-2n \quad n+2 \quad \underline{2n-12}.$$

4. Circle the equation of the line that is parallel to the x-axis. (1)

$$y = -5 \quad x - y = 0 \quad x = 3 \quad x + y = 0.$$

Solution

$$\underline{y = -5} \quad x - y = 0 \quad x = 3 \quad x + y = 0.$$

5. Multiply out and simplify (2)

$$(x-8)^2.$$

Solution

$\times$	x	-8
x	x^2	$-8x$
-8	$-8x$	$+64$

$$(x-8)^2 = \underline{\underline{x^2 - 16x + 64}}.$$

6. Show that 268 can be written as the sum of a power of 3 and a square number. (2)

Solution

Hmm:

$$\begin{aligned} 268 &= 3 + 263 - \text{ not a square number} \\ &= 3^2 + 259 - \text{ not a square number} \\ &= 3^3 + 241 - \text{ not a square number} \\ &= 3^4 + 187 - \text{ not a square number} \\ &= 3^5 + 25 - \text{ a square number!} \\ &= \underline{\underline{3^5 + 5^2}}. \end{aligned}$$

7. Here is some information about the times taken by 40 people to fill in a form. (1)

Time, t minutes	Number of people
$0 < t \leqslant 5$	3
$5 < t \leqslant 10$	9
$10 < t \leqslant 15$	11
$15 < t \leqslant 20$	17

In which class interval is the median?
Circle your answer.

$$0 < t \leqslant 5 \quad 5 < t \leqslant 10 \quad 10 < t \leqslant 15 \quad 15 < t \leqslant 20.$$

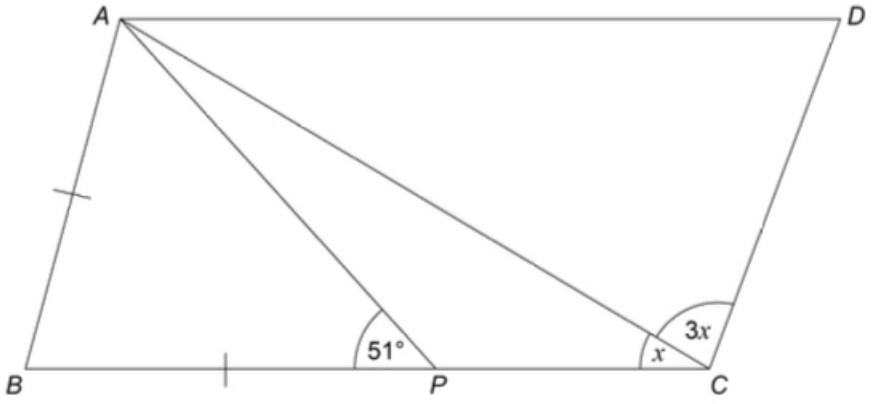
Solution

Time, t minutes	Number of people	Cumulative Total
$0 < t \leqslant 5$	3	3
$5 < t \leqslant 10$	9	$3 + 9 = 12$
$10 < t \leqslant 15$	11	$12 + 11 = 23$
$15 < t \leqslant 20$	17	$23 + 17 = 40$

so

$$0 < t \leqslant 5 \quad 5 < t \leqslant 10 \quad \underline{\underline{10 < t \leqslant 15}} \quad 15 < t \leqslant 20.$$

8. $ABCD$ is a parallelogram. (4)
$AB = BP$.



Work out the size of angle x.

Solution

Well, $\angle PAB = 51^\circ$ (base angles)
$\angle ABP = 180 - (2 \times 51) = 78^\circ$ (completing the triangle)
$\angle BCD = 180 - 78 = 102^\circ$ (interior angles)
$\angle PCA = \frac{1}{4} \times 102 = \underline{\underline{25\frac{1}{2}^\circ}}$ (dividing by 4)

9. (a) Rearrange (2)

$$v = u + at$$

to make t the subject of the formula.

Solution

$$v = u + at \Rightarrow v - u = at$$
$$\Rightarrow \underline{\underline{t = \frac{v-u}{a}}}.$$

(b) Complete this table with consistent metric units. (2)

Distance	Time	Speed	Acceleration
m	s		

Solution

Distance	Time	Speed	Acceleration
m	s	$\underline{\underline{\text{m/s}}}$	$\underline{\underline{\text{m/s}^2}}$

10. Construct a locus of points that are the same distance from points A and B. (2)

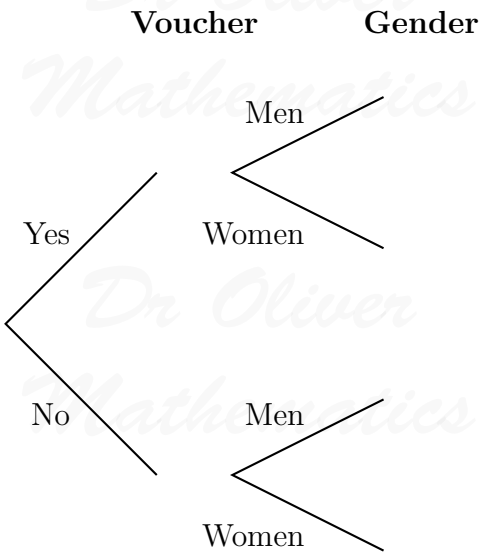
$A\bullet$ $\bullet B$

Solution

- Open your pair of compasses about $\frac{3}{4}$ between the distance between A and B.
- Place your compasses at A.
- Draw a light (but visible!) arc from about 1 o'clock down to about 5 o'clock.
- Do not change the separation of the points!
- Place your compasses at B.
- Draw a light (but visible!) arc from about 7 o'clock down to about 11 o'clock.
- Next, take your ruler and line up the two points made, one above the line and the one other below.
- Draw a straight line. And that's it!

11.
- 42 men and 38 women visit a restaurant.
- 44 of these people have a voucher.
- Three times as many men as women do **not** have a voucher.

(a) Complete the frequency tree. (4)

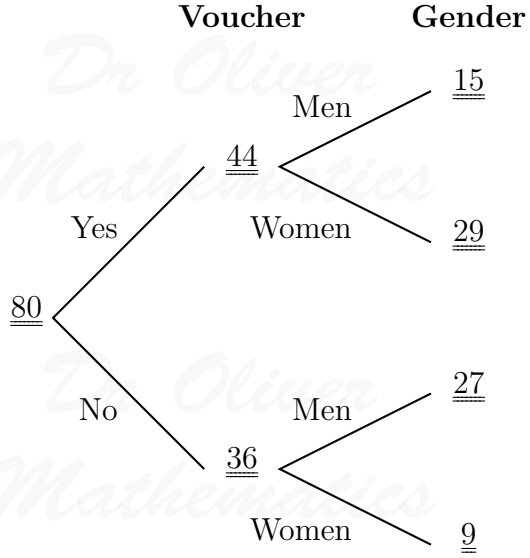


Solution

Well,

$$\tfrac{3}{4} \times 36 = 27,$$

and we complete the frequency table:



A voucher takes **15%** off the bill.

After using the voucher, the bill for a meal is £27.20.

(b) How much was the bill before using the voucher? (3)

Solution

$$\text{new price} = \text{original price} \times \text{discount} \Rightarrow \text{original price} = \frac{\text{new price}}{\text{discount}}$$
$$\Rightarrow \text{original price} = \frac{27.20}{1 - 0.15}$$
$$\Rightarrow \text{original price} = \frac{27.20}{0.85}$$
$$\Rightarrow \text{original price} = \underline{\underline{£32}}.$$

12. The distance by road from Newport to London is 140 miles.

- Tom travels by coach from Newport to London.
- The coach leaves Newport at 1.30 pm

He assumes the coach will travel at an average speed of 50 mph.

(a) Use his assumption to work out the arrival time in London. (3)

Solution

Well,

$$\text{time} = \frac{140}{50}$$
$$= 2\tfrac{4}{5} \text{ hours}$$
$$= 2 \text{ hours } 48 \text{ minutes}$$

and he will arrive at

$$1.30 + 2.48 = \underline{\underline{4.18 \text{ pm}}}.$$

In fact, the coach has a lower average speed.

(b) How does this affect the arrival time? (1)

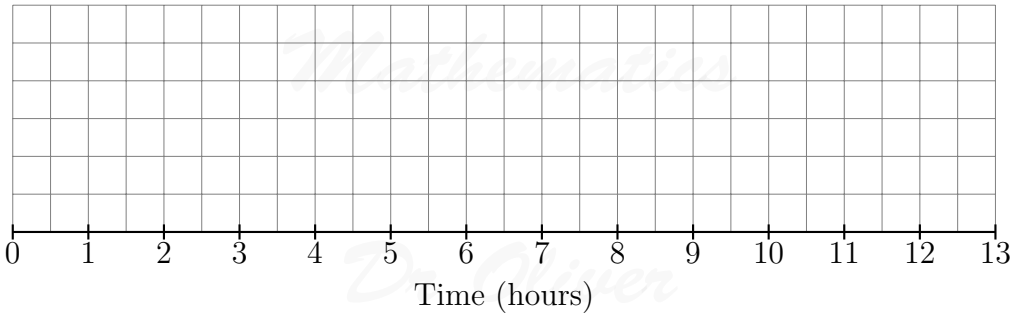
Solution

He will arrive <u>later</u> than his expected time.

13. Here is some information about the length of time cars stayed in a car park. (3)

Shortest time	30 minutes
Longest time	12 hours
Lower quartile	2 hours
Interquartile range	3 hours
Median time	4 hours

Draw a box plot to show this information.

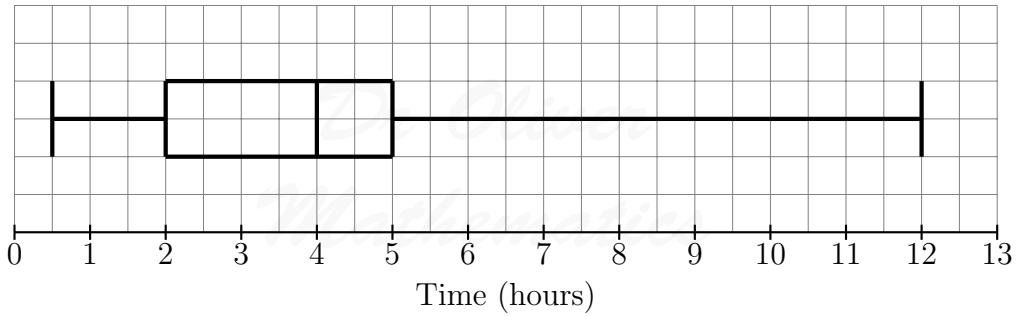


Solution

Well,

$$\begin{aligned} \text{UQ} &= \text{LQ} + \text{IQR} \\ &= 2 + 3 \\ &5 \end{aligned}$$

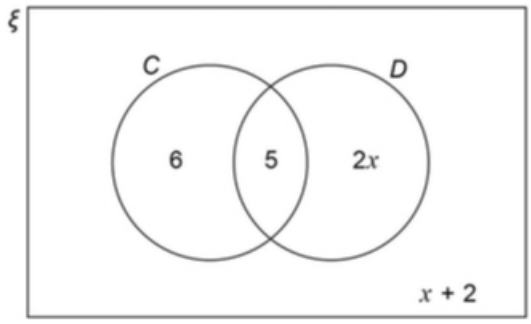
and so we draw a box plot:



14. In the Venn diagram,

    - $\mathscr{E}$ represents 31 students in a class,

- C is students who have a cat, and
- D is students who have a dog,



One student from the class is picked at random.

(a) Work out the probability that the student has a dog. (3)

Solution

Now,

$$6 + 5 + 2x + (x + 2) = 31 \Rightarrow 3x = 18$$
$$\Rightarrow x = 6$$

and so

$$\text{P(dog)} = \frac{5 + (2 \times 6)}{31}$$
$$= \underline{\underline{\tfrac{17}{31}}}.$$

One of the students who has a cat is picked at random.

(b) Work out the probability that this student has a dog. (1)

Solution

$$\frac{5}{6 + 5} = \underline{\underline{\tfrac{5}{11}}}.$$

15. Circle the highest common factor (HCF) of $6xy^2$ and $4x^3y$. (1)

$$2xy^2 \quad 2xy \quad 12x^3y^2 \quad 24x^4y^3.$$

Solution

$$2xy^2 \quad \underline{\underline{2xy}} \quad 12x^3y^2 \quad 24x^4y^3.$$

16. (1)

$$\mathrm{f}(x) = x^2 - x^3.$$

Circle the value of $\mathrm{f}(-3)$.

$$18 \quad -18 \quad 36 \quad -36.$$

Solution

$$\begin{aligned} \mathrm{f}(x) &= (-3)^2 - (-3)^3 \\ &= 9 - (-27) \\ &= 36 \end{aligned}$$

so

$$18 \quad -18 \quad \underline{\underline{36}} \quad -36.$$

17. At a football game, (3)

$$\text{number of men} : \text{number of women} : \text{number of children} = 13 : 5 : 7.$$

There are 4 152 **more** men than women.

Work out the number of children at the game.

Solution

Well,

$$13 - 5 = 8$$

and

$$\frac{4\,152}{8} = 519.$$

So, there are

$$13 \times 519 = 6\,747 \text{ men}$$

and

$$5 \times 519 = 2\,595 \text{ women.}$$

$(6\,747 - 2\,595 = 4\,152\,\checkmark)$ Finally,

$$7 \times 519 = \underline{\underline{3\,633 \text{ children}}}.$$

18. Expand and simplify (4)

$$(3x^2 + 2)(2x + 5) - 6x(x^2 - 3).$$

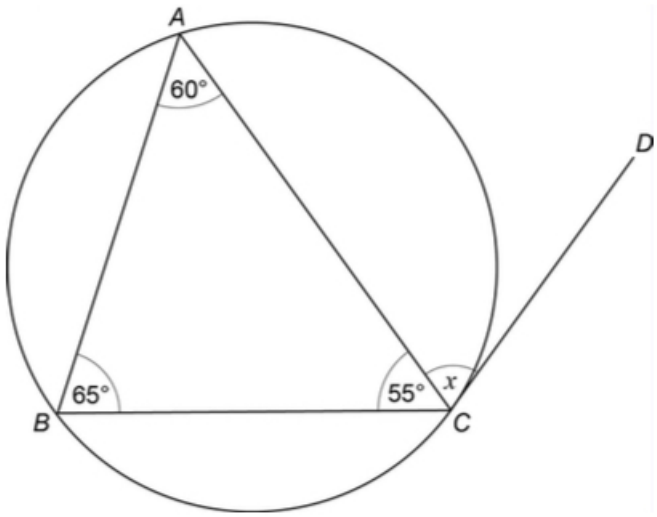
Solution

$\times$	$3x^2$	$+2$
$2x$	$6x^3$	$+4x$
$+5$	$+15x^2$	$+10$

Now,

$$\begin{aligned}(3x^2 + 2)(2x + 5) - 6x(x^2 - 3) &= 6x^3 + 15x^2 + 4x + 10 - 6x^3 + 18x \\ &= \underline{\underline{15x^2 + 22x + 10}}.\end{aligned}$$

19. A, B, and C are points on a circle. (2)
CD is a tangent to the circle.



Write down the size of angle x.
Give a reason for your answer.

Solution

$\underline{\underline{x = 65^\circ}}$ (alternate segment theorem)

20. (1)

- w is a positive number.
- x is 10% more than w.
- y is 10% less than x.

Which statement is true?
Tick **one** box.

$w < x$ and $w < y$ ☐

$w < x$ and $w = y$ ☐

$x > y$ and $w > y$ ☐

$x > y$ and $w = y$ ☐

Solution

Well,

$$x = 1.1w \text{ and } y = 0.9x.$$

Now,

$$\begin{aligned} y &= 0.9x \\ &= 0.9(1.1w) \\ &= 0.99w \end{aligned}$$

and so it is the third box ($x > y$ and $w > y$).

21. N is a number. (3)

As a product of prime factors in index form

$$N = 2 \times 3^4 \times y^3.$$

Work out

$$3N^2$$

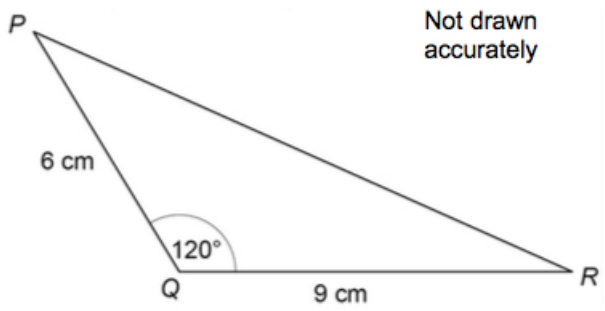
as a product of prime factors in index form.

Give your answer in terms of y.

Solution

$$\begin{aligned} 3N^2 &= 3(2 \times 3^4 \times y^3)^2 \\ &= 3(2^2 \times 3^8 \times y^6) \\ &= 2^2 \times 3^9 \times y^6. \end{aligned}$$

22. Here is a triangle. (3)



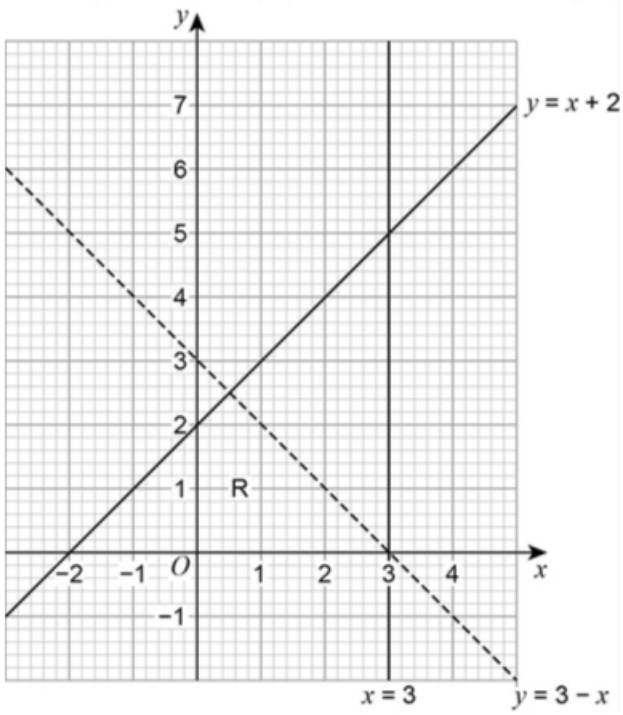
Work out the length PR.

Solution

$$
\begin{aligned}
& PR^2 = PQ^2 + QR^2 - 2 \times PQ \times QR \times \cos PQR \\
\Rightarrow \quad & PR^2 = 6^2 + 9^2 - 2 \times 6 \times 9 \times \cos 120^\circ \\
\Rightarrow \quad & PR^2 = 171 \\
\Rightarrow \quad & PR = \underline{\underline{3\sqrt{19} \text{ or } 13.1 \text{ cm (3 sf)}}}.
\end{aligned}
$$

23. Joe draws this graph to identify the region R represented by (2)

$$y \leqslant x + 2,\ y > 3 - x,\ \text{and}\ x < 3.$$



Make **two** criticisms of his graph.

Solution

E.g., Joe has made $x = 3$ solid, rather than dotted; R is in the wrong place.

24. (3)

$$a : b = 9 : 4 \text{ and } 10b = 7c.$$

Work out

$$a : c$$

in its simplest form.

Solution

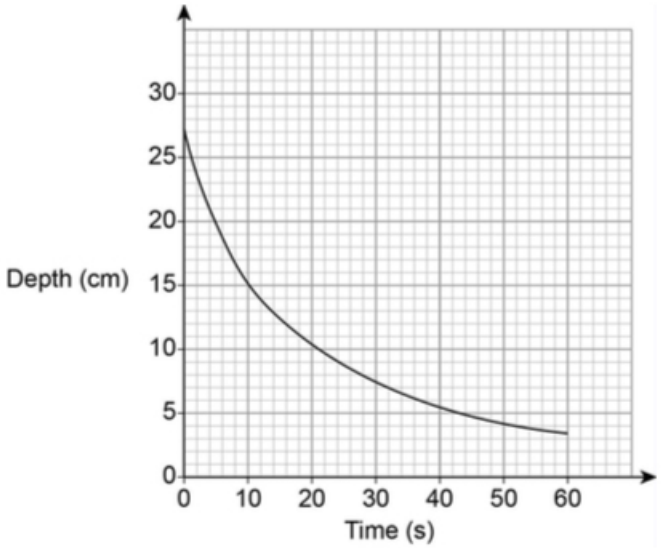
Now,

$$10b = 7c \Rightarrow b = \tfrac{7}{10}c$$

and

$$\begin{aligned} a : b = 9 : 4 &\Rightarrow 4a = 9b \\ &\Rightarrow 4a = 9(\tfrac{7}{10}c) \\ &\Rightarrow 4a = \tfrac{63}{10}c \\ &\Rightarrow 40a = 63c \\ &\Rightarrow \underline{\underline{a : c = 63 : 40}}. \end{aligned}$$

25. Liquid is leaking out of a container. (3)
The graph shows the depth of the liquid for 60 seconds.



Use the graph to work out an estimate of the rate of decrease of depth at 10 seconds.
You **must** show your working.

Solution

The gradient of the tangent goes through $(0, 20)$ and $(30, 0)$ and

$$\begin{aligned} \text{gradient} &= \frac{20 - 0}{0 - 30} \\ &= -\tfrac{2}{3}; \end{aligned}$$

hence, an estimate of the rate of decrease is $\underline{\underline{\tfrac{2}{3}\ \text{cm}\,\text{s}^{-1}}}$.

26. (2)

$$a^2 - b^2 \equiv (a + b)(a - b).$$

- a and b are positive whole numbers with $a > b$.
- $a^2 - b^2$ is a prime number.

Why are a and b consecutive numbers?

Solution

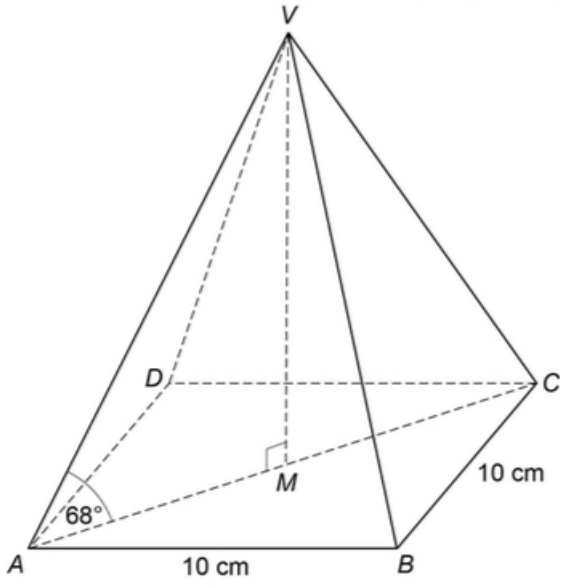
Well,

$$a^2 - b^2 = (a + b)(a - b)$$

is a prime number.
So either $a + b = 1$ (no! why?) and so $a - b = 1 \Rightarrow a = b + 1$.
Hence, a and b consecutive numbers

27. $VABCD$ is a square-based pyramid. (6)

- The horizontal base $ABCD$ has side length 10 cm and centre M.
- Angle $VMA = 90°$.
- Angle $VAM = 68°$.



Volume of pyramid = $\frac{1}{3}$ × area of base × perpendicular height

Work out the volume of the pyramid.

Solution

Well,

$$\begin{aligned} AC^2 = AB^2 + BC^2 &\Rightarrow AC^2 = 10^2 + 10^2 \\ &\Rightarrow AC^2 = 200 \\ &\Rightarrow AC = 10\sqrt{2} \\ &\Rightarrow AM = 5\sqrt{2} \text{ cm} \end{aligned}$$

and

$$\begin{aligned} \tan = \frac{\text{opp}}{\text{adj}} &\Rightarrow \tan 68^\circ = \frac{VM}{5\sqrt{2}} \\ &\Rightarrow VM = 5\sqrt{2}\tan 68^\circ. \end{aligned}$$

Finally,

$$\begin{aligned} \text{volume} &= \tfrac{1}{3} \times 10 \times 10 \times 5\sqrt{2}\tan 68^\circ \\ &= 583.383\,566 \text{ (FCD)} \\ &= \underline{\underline{583 \text{ cm}^3 \text{ (3 sf)}}}. \end{aligned}$$

28. (5)

$$y = p \times q^{x-1},$$

where p and q are numbers.

- $y = 10$ when $x = 1$.
- $y = 0.312\,5$ when $x = 6$.

Work out the value of y when $x = 3$.

Solution

$$\begin{aligned} x = 1, y = 10 &\Rightarrow 10 = p \times q^0 \\ &\Rightarrow p = 10 \end{aligned}$$

and

$$x = 6, y = 0.312\,5 \Rightarrow 0.312\,5 = 10 \times q^5$$
$$\Rightarrow q^5 = 0.031\,25$$
$$\Rightarrow q = 0.5$$

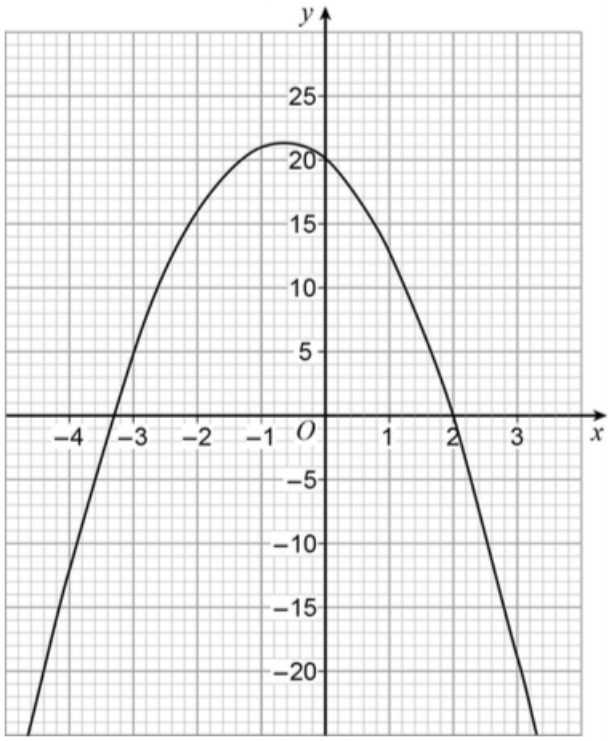
so the equation is

$$y = 10 \times 0.5^{(x-1)}.$$

Finally,

$$x = 3 \Rightarrow y = 10 \times 0.5^2$$
$$\Rightarrow y = 10 \times 0.25$$
$$\Rightarrow \underline{\underline{y = 2.5}}.$$

29. Here is the graph of $y = \mathrm{f}(x)$ where $\mathrm{f}(x)$ is a quadratic function. (2)



Write down all the integer solutions of $\mathrm{f}(x) \geqslant 0$.

Solution

They are

$$\underline{\underline{x = -3,\ -2\ -1,\ 0,\ 1,\ \text{or } 2}}.$$

30. (2)

$$\text{f}(x) = \tfrac{1}{3}x + 4 \text{ for all values of } x.$$
$$\text{g}(x) = 6x^2 + 3 \text{ for all values of } x.$$

Work out $\text{f}\,\text{g}(x)$.
Give your answer in the form

$$ax^2 + b,$$

where a and b are integers.

Solution

Now,

$$\begin{aligned}\text{f}\,\text{g}(x) &= \text{f}(6x^2 + 3)\\ &= \tfrac{1}{3}(6x^2 + 3) + 4\\ &= 2x^2 + 1 + 4\\ &= \underline{\underline{2x^2 + 5}};\end{aligned}$$

hence, $\underline{\underline{a = 2}}$ and $\underline{\underline{b = 5}}$.