

**Dr Oliver Mathematics**  
**GCSE Mathematics**  
**2015 Paper 2H: Calculator**  
**1 hour 45 minutes**

The total number of marks available is 100.

You must write down all the stages in your working.

1. Udit has a bag of chocolate sweets.

There are 30 sweets in the bag.

This table shows the types of sweets in the bag.

	Strawberry	Caramel	Nut
Dark chocolate	3	1	6
Milk chocolate	4	5	2
White chocolate	1	4	4

Udit takes at random a sweet from the bag.

- (a) Write down the probability that the sweet is a dark chocolate caramel. (1)

**Solution**

$$P(\text{dark chocolate caramel}) = \frac{1}{\underline{\underline{30}}}.$$

- (b) Work out the probability that the sweet is a white chocolate. (2)

**Solution**

$$\begin{aligned} P(\text{white chocolate}) &= \frac{1+4+4}{30} \\ &= \frac{9}{30} \\ &= \frac{3}{\underline{\underline{10}}}. \end{aligned}$$

There are some dark chocolates, some milk chocolates, and some white chocolates in a box.

The table below shows the probabilities that a chocolate taken at random from the box is a dark chocolate or is a milk chocolate.

	Dark chocolate	Milk chocolate	White chocolate
Probability	0.35	0.17	

A chocolate is taken at random from the box.

- (c) Work out the probability that the chocolate is a white chocolate. (2)

**Solution**

Let the probability of taking a white chocolate be  $x$ . Then

$$\begin{aligned} 0.35 + 0.17 + x &= 1 \Rightarrow 0.52 + x = 1 \\ &\Rightarrow \underline{x = 0.48}. \end{aligned}$$

2. The equation

$$x^3 - x^2 = 30$$

(4)

has a solution between 3 and 4.

Use a trial and improvement method to find this solution.

Give your answer correct to one decimal place.

You must show all your working.

**Solution**

You must be in TABLE mode; on my calculator (Casio fx-991) it is Mode 3.

$x$	$f(x)$	Comment
3.45	29.161	too low
3.5	30.625	too high

Clearly,

$$3.45 < x < 3.5$$

and the answer is

$$\underline{x = 3.5 \text{ (1 dp)}}.$$

3. Use your calculator to work out

$$\frac{\sqrt{70.25}}{4.2 - 2.37}$$

- (a) Write down all the figures on your calculator display. (2)  
You must give your answer as a decimal.

**Solution**

$$\begin{aligned} \frac{\sqrt{70.25}}{4.2 - 2.37} &= \frac{\sqrt{70.25}}{1.83} \\ &= \underline{\underline{4.580\ 069\ 567}} \text{ (FCD)}. \end{aligned}$$

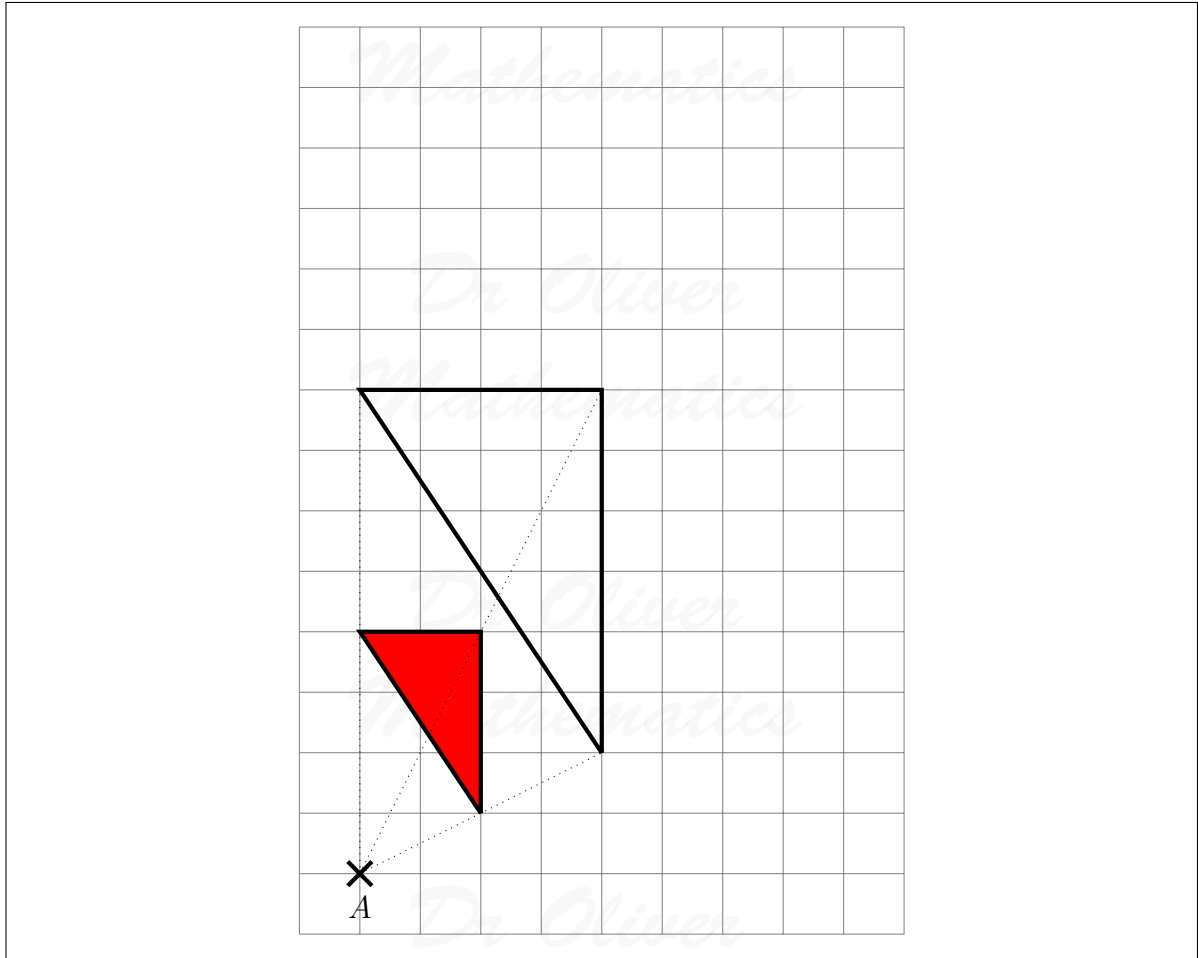
- (b) Write your answer to part (a) correct to 4 decimal places. (1)

**Solution**

The answer is 4.5801 (4 dp).

4. A shaded shape is shown on the grid. (2)





5. Redlands School sent  $x$  students to a revision day. (5)  
St Samuel's School sent twice as many students as Redlands School.  
Francis Long School sent 7 fewer students than Redlands School.  
Each student paid £15 for the revision day.  
The students paid a total of £1155.  
Work out how many students were sent by each school to the revision day.  
You must show all your working.

**Solution**

$$\text{Redlands} + \text{St Samuel} + \text{Francis Long} = \frac{\text{total cost}}{\text{price per student}}$$

$$\Rightarrow x + 2x + (x - 7) = \frac{1155}{15}$$

$$\Rightarrow 4x - 7 = 77$$

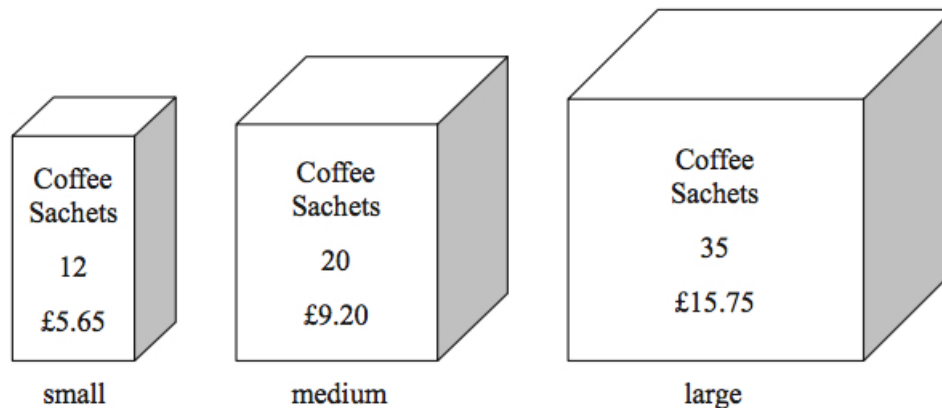
$$\Rightarrow 4x = 84$$

$$\Rightarrow x = 21;$$

Redlands School sent 21 students, St Samuel's School sent 42 students, and Francis Long School sent 14 students.

6. Coffee sachets are sold in three different sizes of box.

(4)



A small box has 12 coffee sachets and costs £5.65.

A medium box has 20 coffee sachets and costs £9.20.

A large box has 35 coffee sachets and costs £15.75.

Work out which size of box gives the best value for money.

You must show all your working.

**Solution**

Small:

$$\frac{5.65}{12} = 0.4708\dot{3}$$

Medium:

$$\frac{9.20}{20} = 0.46$$

Large:

$$\frac{15.75}{35} = 0.45$$

The best value for money is the 35 coffee sachets.

7. (a) Expand  $7(x + 5)$ .

(1)

**Solution**

$$7(x + 5) = \underline{7x + 35}.$$

- (b) Expand  $3y(4y - 3)$ .

(1)

**Solution**

$$3y(4y - 3) = \underline{12y^2 - 9y}.$$

- (c) Expand and simplify  $(t + 2)(t + 4)$ .

(2)

**Solution**

$\times$	$t$	$+2$
$t$	$t^2$	$+2t$
$+4$	$+4t$	$+8$

Hence

$$(t + 2)(t + 4) = \underline{t^2 + 6t + 8}.$$

8. Sandra has a piece of string 153 cm long.

(3)

She cuts the string into three lengths in the ratio 4 : 2 : 3.

Work out the length, in centimetres, of each piece of string.

**Solution**

$$4 + 2 + 3 = 9.$$

Now,

$$\text{First piece} = \frac{4}{9} \times 153 = \underline{68 \text{ cm}},$$

$$\text{Second piece} = \frac{2}{9} \times 153 = \underline{34 \text{ cm}},$$

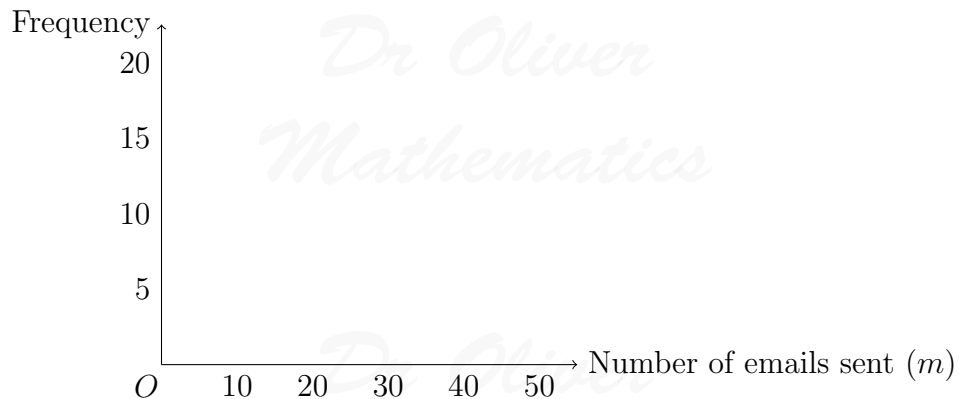
$$\text{Third piece} = \frac{3}{9} \times 153 = \underline{51 \text{ cm}}.$$

9. The frequency table gives information about the numbers of emails sent by 51 teachers on Monday.

Number of emails sent ( $m$ )	Frequency
$0 < m \leq 10$	5
$10 < m \leq 20$	17
$20 < m \leq 30$	14
$30 < m \leq 40$	9
$40 < m \leq 50$	6

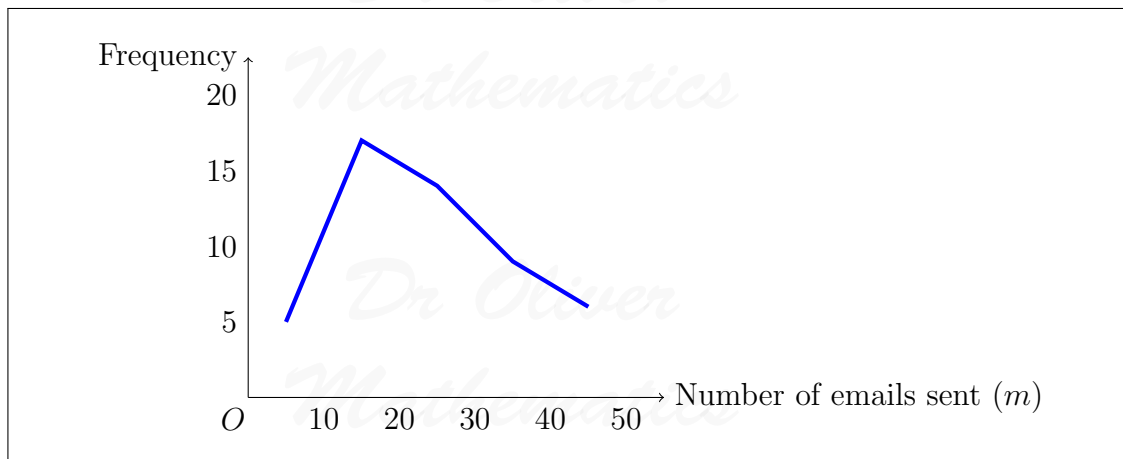
- (a) On the grid below, draw a frequency polygon for this information.

(2)



**Solution**





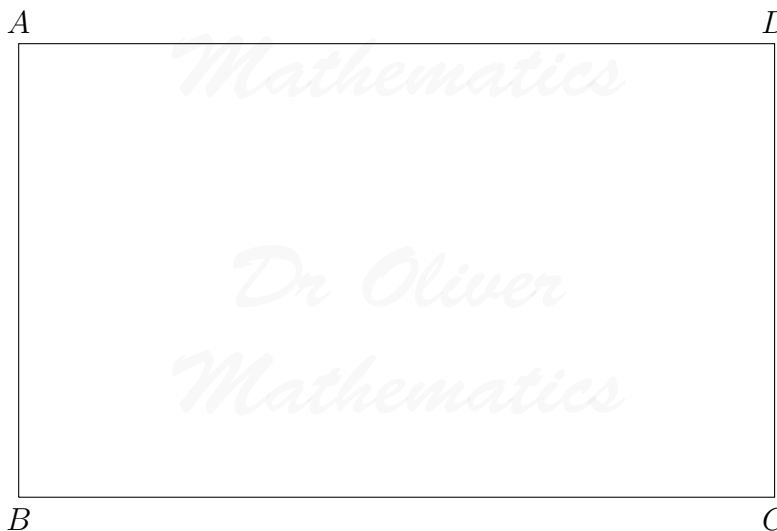
- (b) Nalini says that at least a quarter of these teachers sent more than 30 emails. (2)  
 Is Nalini correct?  
 You must explain your answer.

**Solution**

$$\frac{9 + 6}{51} \times 100\% = 29\frac{7}{17}\%;$$

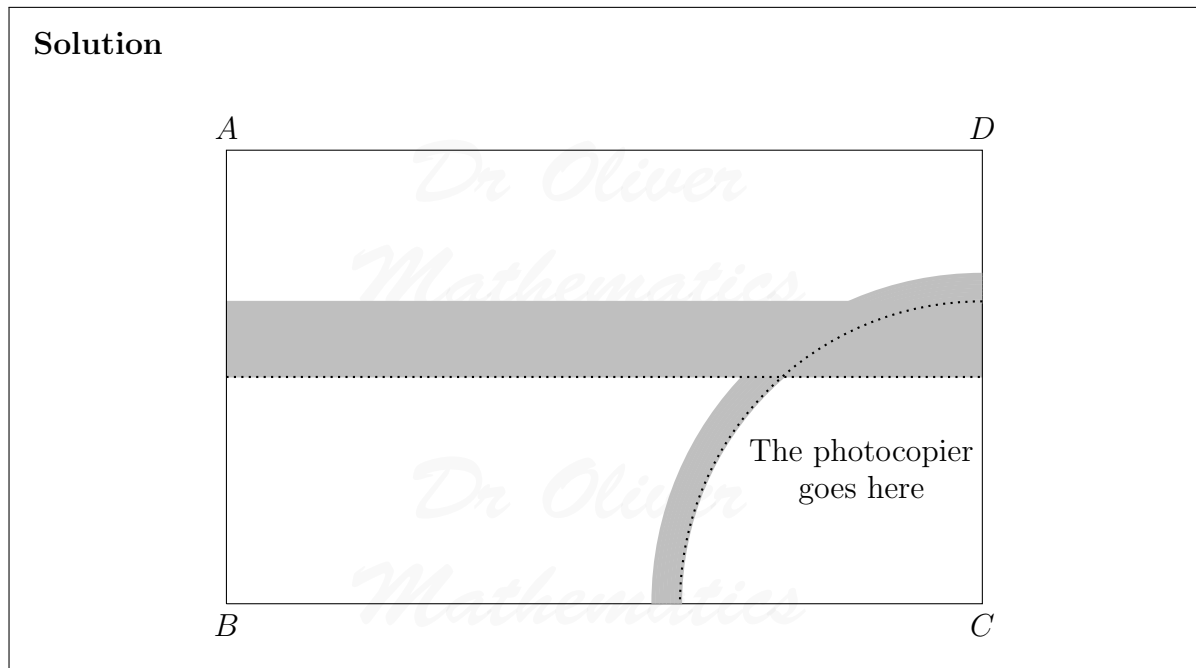
so, yes, Nalini is correct.

10. Here is a scale drawing of an office. (3)  
 The scale is 1 cm to 2 metres.



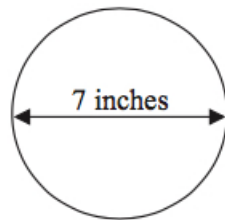
A photocopier is going to be put in the office.  
 The photocopier has to be closer to  $B$  than it is to  $A$ .

The photocopier also has to be less than 8 metres from  $C$ .  
 Show, by shading, the region where the photocopier can be put.



11. The diagram shows the top of Levi's birthday cake.

(4)



**Diagram NOT  
 accurately drawn**

The top of the cake is in the shape of a circle.

The diameter of the circle is 7 inches.

A ribbon is going to be put around the side of the cake.

Ribbons are sold in 50 cm lengths.

1 inch is 2.54 cm.

Work out if one length of ribbon is long enough to go all the way around the cake.

You must show your working.

**Solution**

$$\text{Radius} = 3.5 \text{ in} = 8.89 \text{ cm.}$$

Now,

$$2 \times \pi \times 8.8 = 55.857 \dots$$

and so we need 2 ribbons.

12. The points  $A$ ,  $B$ , and  $C$  lie in order on a straight line. (3)  
The coordinates of  $A$  are  $(2, 5)$ .  
The coordinates of  $B$  are  $(4, p)$ .  
The coordinates of  $C$  are  $(q, 17)$ .  
Given that  $AC = 4AB$ , find the values of  $p$  and  $q$ .

**Solution**

$$\begin{aligned} AC = 4AB &\Rightarrow q - 2 = 4(4 - 2) \\ &\Rightarrow q - 2 = 8 \\ &\Rightarrow \underline{\underline{q = 10}} \end{aligned}$$

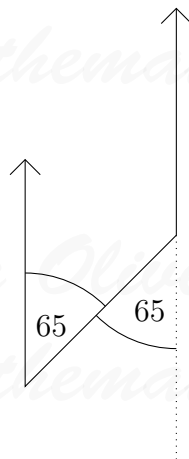
and

$$\begin{aligned} AC = 4AB &\Rightarrow 17 - 5 = 4(p - 5) \\ &\Rightarrow 12 = 4(p - 5) \\ &\Rightarrow p - 5 = 3 \\ &\Rightarrow \underline{\underline{p = 8}}. \end{aligned}$$

13. Martin and Janet are in an orienteering race. (2)  
Martin runs from checkpoint  $A$  to checkpoint  $B$ , on a bearing of  $065^\circ$ .  
Janet is going to run from checkpoint  $B$  to checkpoint  $A$ .  
Work out the bearing of  $A$  from  $B$ .

**Solution**

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The bearing is

$$180 + 65 = \underline{\underline{245^\circ}}.$$

14. Sumeet records the times, in minutes, for 40 runners to finish a half marathon. Information about these times is shown in the table.

(4)

Time ( $t$ minutes)	Frequency
$60 < t \leq 90$	10
$90 < t \leq 120$	14
$120 < t \leq 150$	9
$150 < t \leq 180$	5
$180 < m \leq 50$	2

Calculate an estimate for the mean time.

**Solution**

Time ( $t$ minutes)	Frequency	Midpoint	Freq $\times$ Mid
$60 < t \leq 90$	10	75	750
$90 < t \leq 120$	14	105	1 470
$120 < t \leq 150$	9	135	1 215
$150 < t \leq 180$	5	165	825
$180 < m \leq 210$	2	195	390
Total			4 650

An estimate for the mean time is

$$\frac{4650}{40} = \underline{\underline{116.25 \text{ minutes}}}.$$

15. (a) Work out the value of  $25^{-3}$ . (1)

**Solution**

$$25^{-3} = \frac{1}{25^3} = \frac{1}{\underline{\underline{15625}}} \text{ or } 0.000064.$$

- (b) Work out the value of  $350^3$ . (2)  
Give your answer in standard form.

**Solution**

$$350^3 = 42\,875\,000 = \underline{\underline{4.2875 \times 10^7}}.$$

16. Liquid  $A$  has a density of  $0.7 \text{ g/cm}^3$ . (4)  
Liquid  $B$  has a density of  $1.6 \text{ g/cm}^3$ .  
140 g of liquid  $A$  and 128 g of liquid  $B$  are mixed to make liquid  $C$ .  
Work out the density of liquid  $C$ .

**Solution**

$$\text{Volume } A = \frac{140}{0.7} = 200 \text{ cm}^3,$$

$$\text{Volume } B = \frac{128}{1.6} = 80 \text{ cm}^3,$$

and so the required volume is

$$200 + 80 = 280 \text{ cm}^3.$$

The mass is

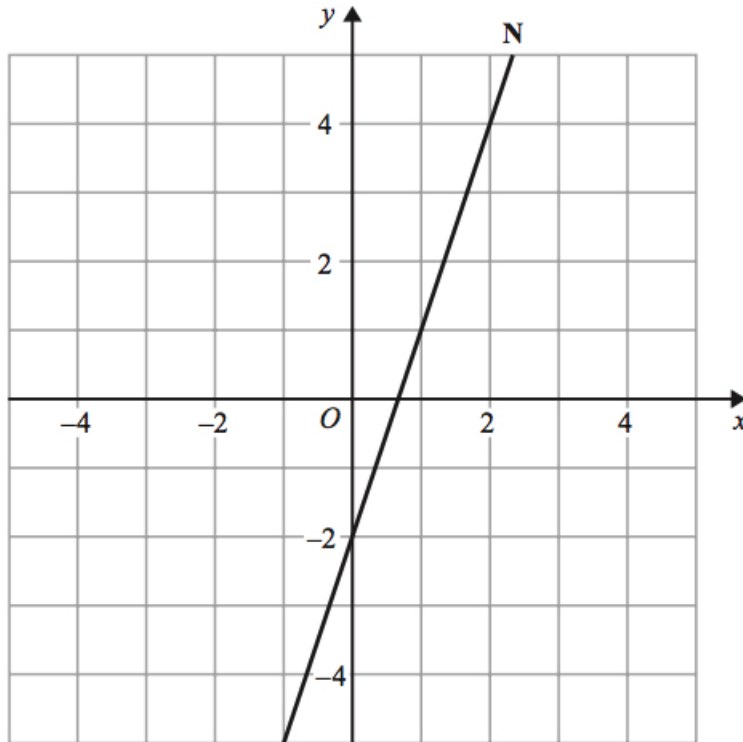
$$140 + 128 = 268 \text{ g}$$

and density of liquid  $C$  is

$$\begin{aligned} \frac{268}{280} &= \frac{67}{70} \\ &= 0.9571428 \\ &= \underline{\underline{0.957 \text{ g/cm}^3}} \text{ (3 sf)} \end{aligned}$$

17. The line **N** is drawn below.

(3)



Find an equation of the line perpendicular to line **N** that passes through the point  $(0, 1)$ .

**Solution**

$$\text{Gradient} = 3 \Rightarrow m_T = -\frac{1}{3}$$

and

$$\begin{aligned} y - 1 &= -\frac{1}{3}(x - 0) \Rightarrow y - 1 = -\frac{1}{3}x \\ &\Rightarrow \underline{\underline{y = -\frac{1}{3}x + 1.}} \end{aligned}$$

18. The owners of a car park recorded the number of cars parked at 12 noon each day. The table shows information about the number of cars parked in the car park at 12 noon each day in July and in December.

	July	December
Least number of cars	75	100
Lower quartile	90	115
Median	95	130
Upper quartile	150	150
Greatest number of cars	178	180

- (a) What type of diagram could you draw to represent the information for each month? (1)

**Solution**

A box and whisker plot or cumulative frequency diagram.

- (b) Compare the distribution of the number of cars recorded in July with the distribution of the number of cars recorded in December. (2)

**Solution**

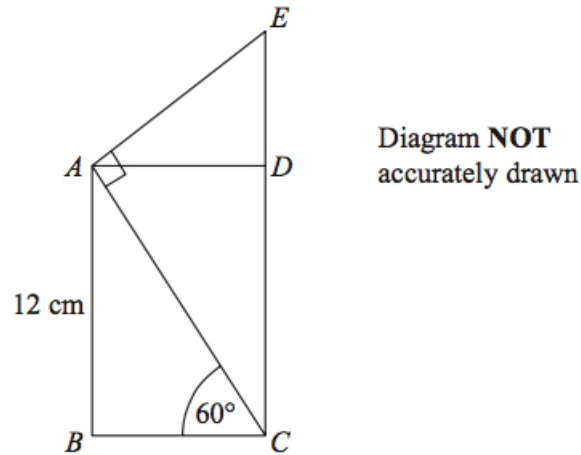
**Average:** Since the median for December (130) is higher than the median for July (95), December has more parked cars on average.

**Spread:** Since the range for December ( $180 - 100 = 80$ ) is smaller than the range for July ( $178 - 75 = 103$ ), the parking is more consistent in December.

**OR**

**Spread:** Since the IQR for December ( $150 - 115 = 35$ ) is smaller than the IQR for July ( $150 - 90 = 60$ ), the parking is more consistent in December.

19.  $ABCD$  is a rectangle. (4)



$CDE$  is a straight line.

$AB = 12$  cm.

Angle  $ACB = 60^\circ$ .

Angle  $EAC = 90^\circ$ .

Calculate the length of  $CE$ .

You must show all your working.

### Solution

Well,

$$\text{adj} = \frac{\text{opp}}{\tan} \Rightarrow AD = \frac{12}{\tan 60^\circ}.$$

Now,  $\angle EAD = 30^\circ$  (why?) and

$$\begin{aligned} \text{opp} &= \text{adj} \times \tan \Rightarrow DE = \frac{12}{\tan 60^\circ} \times \tan 30^\circ \\ &\Rightarrow DE = \frac{12 \tan 30^\circ}{\tan 60^\circ} \\ &\Rightarrow DE = 4. \end{aligned}$$

Finally,

$$\begin{aligned} CE &= CD + DE \\ &= 12 + 4 \\ &= \underline{\underline{16 \text{ cm}}}. \end{aligned}$$

20. Show that

$$(n + 3)^2 - (n - 3)^2$$

(3)



is an even number for all positive integer values of  $n$ .

**Solution**

$$\begin{aligned}(n + 3)^2 - (n - 3)^2 &= (n^2 + 6n + 9) - (n^2 - 6n + 9) \\ &= 12n \\ &= 6(2n) \\ &= \underline{6 \times \text{an even number.}}\end{aligned}$$

21. Fred is making two rectangular flower beds. (2)  
The dimensions of the larger rectangle will be three times the dimensions of the smaller rectangle.  
There is going to be the same depth of soil in each flower bed.  
Fred needs 180 kg of soil for the smaller flower bed.  
Work out how much soil Fred needs for the larger flower bed.

**Solution**

$$3^2 \times 180 = \underline{1\,620 \text{ kg.}}$$

22. Claire is making a loaf of bread. (3)  
A loaf of bread loses 12% of its weight when it is baked.  
Claire wants the baked loaf of bread to weigh 1.1 kg.  
Work out the weight of the loaf of bread before it is baked.

**Solution**

$$1 - 0.12 = 0.88$$

and

$$\text{weight} = \frac{1.1}{0.88} = \underline{1.25 \text{ kg.}}$$

23. Here is a parallelogram. (3)

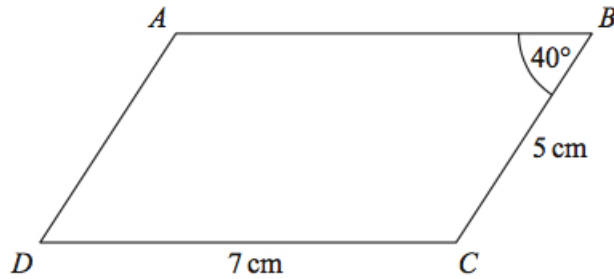


Diagram **NOT**  
accurately drawn

$DC = 7$  cm.

$CB = 5$  cm.

Angle  $ABC$  is  $40^\circ$ .

Work out the area of the parallelogram.

Give your answer correct to 1 decimal place.

**Solution**

$$\begin{aligned} \text{Area} &= 2 \times \text{area of } ABC \\ &= 2 \times \frac{1}{2} \times 7 \times 5 \times \sin 40^\circ \\ &= 22.497\ 566\ 34 \text{ (FCD)} \\ &= \underline{\underline{22.5 \text{ cm}^2}} \text{ (1 dp)}. \end{aligned}$$

24. Steve travelled from Ashton to Barnfield.

(4)

He travelled 235 miles, correct to the nearest 5 miles.

The journey took him 200 minutes, correct to the nearest 5 minutes.

Calculate the lower bound for the average speed of the journey.

Give your answer in miles per hour, correct to 3 significant figures.

You must show all your working.

**Solution**

$$232.5 \leq \text{length (km)} < 237.5$$

and

$$197.5 \leq \text{minutes (min)} < 202.5.$$

But we want km/h and so

$$\begin{aligned}\text{lower bound} &= \frac{232.5 \times 60}{202.5} \\ &= 68.888\ 888\ 889 \text{ (FCD)} \\ &= \underline{\underline{68.9 \text{ km/h (FCD)}}}.\end{aligned}$$

25. Solve the equation

$$3x^2 + 4x - 12 = 0.$$

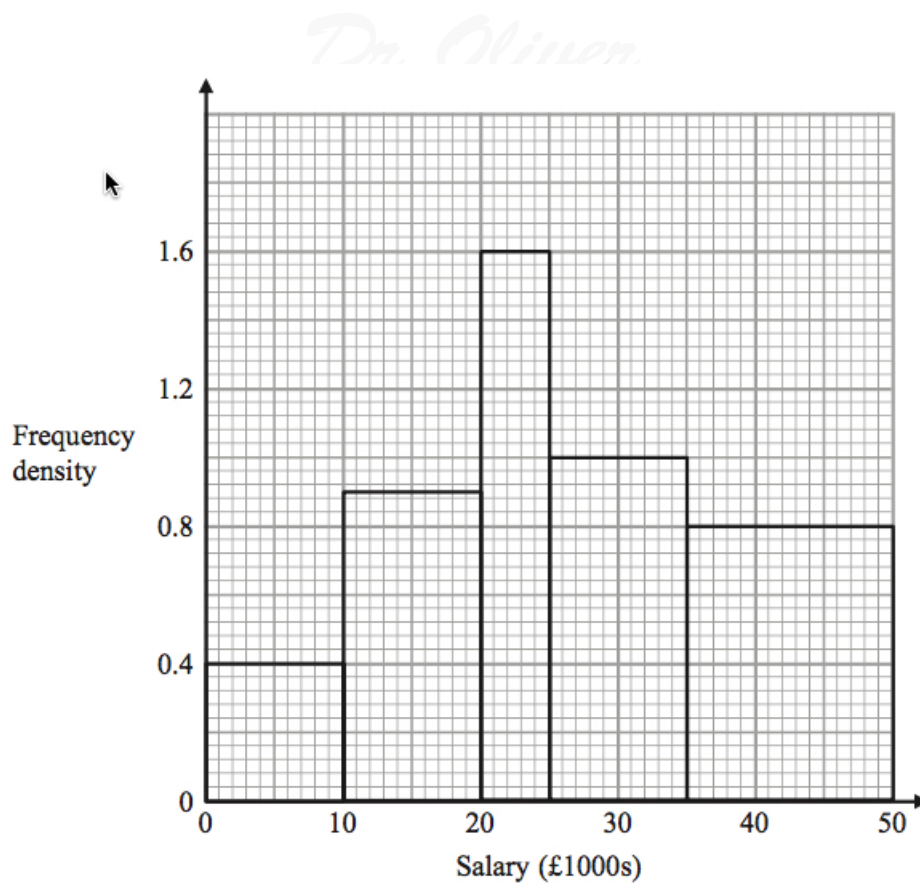
(3)

Give your solutions correct to 2 decimal places.

**Solution**

$$\begin{aligned}x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-4 \pm \sqrt{4^2 - 4 \times 3 \times (-12)}}{6} \\ &= \frac{-4 \pm \sqrt{160}}{6} \\ &= -2.774\ 851\ 773 \text{ or } 1.441\ 518\ 4 \text{ (FCD)} \\ &= \underline{\underline{-2.77 \text{ or } 1.44 \text{ (2 dp)}}}.\end{aligned}$$

26. The histogram shows some information about the salaries of a sample of people.



(a) Use the histogram to complete the frequency table.

(2)

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Salary ( $p$ ) in £1000s	Frequency
$0 < p \leq 10$	4
$10 < p \leq 20$	
$20 < p \leq 25$	
$25 < p \leq 35$	
$35 < p \leq 50$	

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**Solution**

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Salary ( $p$ ) in £1000s	Frequency
$0 < p \leq 10$	4
$10 < p \leq 20$	$10 \times 0.9 = \underline{9}$
$20 < p \leq 25$	$5 \times 1.6 = \underline{8}$
$25 < p \leq 35$	$10 \times 1 = \underline{10}$
$35 < p \leq 50$	$15 \times 0.8 = \underline{12}$

- (b) Work out the proportion of people in the sample who have a salary greater than £40 000. (2)

**Solution**

$$4 + 9 + 8 + 10 + 12 = 43.$$

Approximately, two-thirds of the people earning  $35 < p \leq 50$  are in this category and that means 8 people:  $\frac{8}{43}$ .

- (c) Find an estimate for the median salary. (2)

**Solution**

Salary ( $p$ ) in £1000s	Cumulative Frequency
$0 < p \leq 10$	4
$10 < p \leq 20$	13
$20 < p \leq 25$	21
$25 < p \leq 35$	31
$35 < p \leq 50$	43

It is the first person in the  $25 < p \leq 35$  group:

$$25 + \frac{1}{10} \times 10 = 26;$$

the amount is £26 000.

27.  $ABC$  is a straight line. (4)

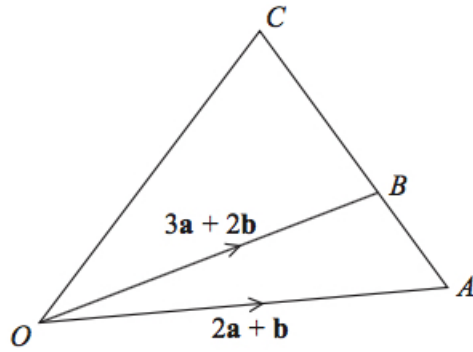


Diagram **NOT**  
accurately drawn

$$AB : BC = 2 : 5.$$

$$\vec{OA} = 2\mathbf{a} + \mathbf{b}.$$

$$\vec{OB} = 3\mathbf{a} + 2\mathbf{b}.$$

Express  $\vec{OC}$  in terms of  $\mathbf{a}$  and  $\mathbf{b}$ .

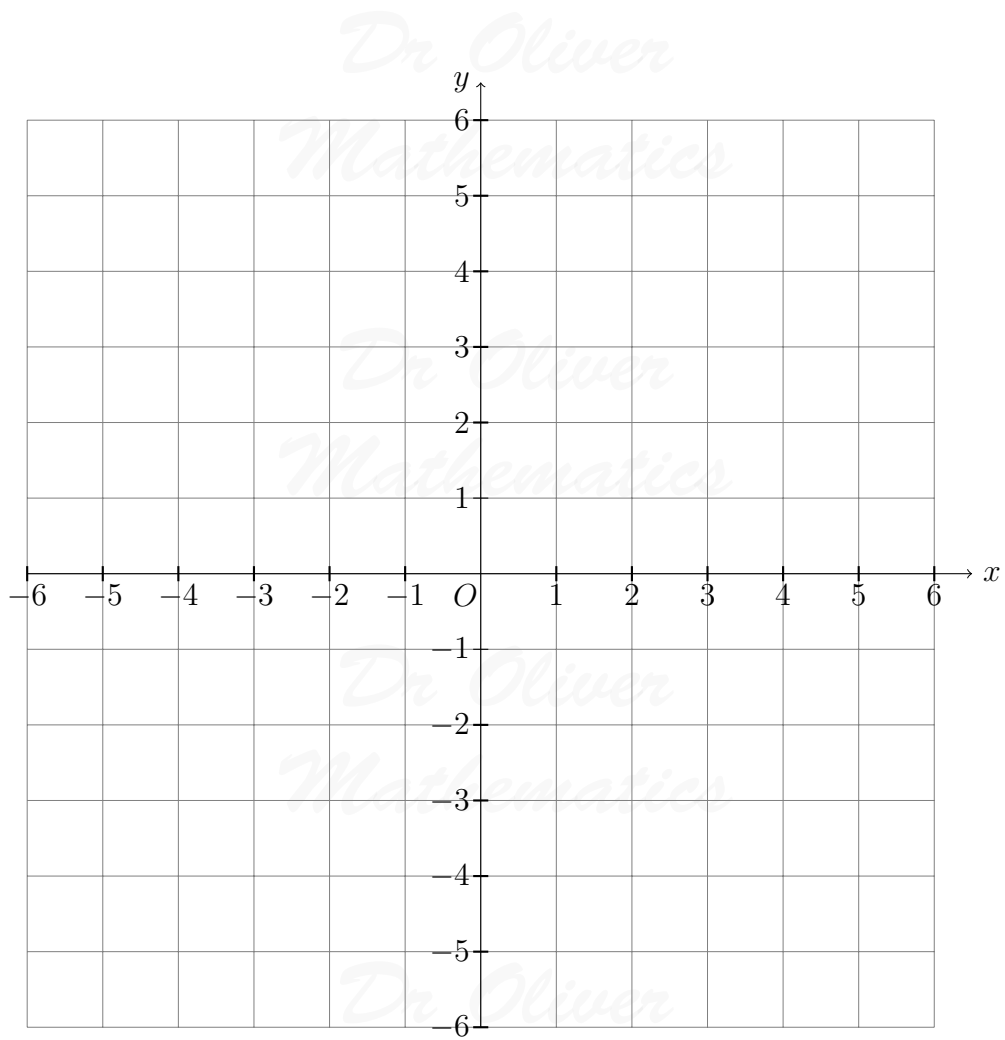
Give your answer in its simplest form

**Solution**

$$\begin{aligned} \vec{OC} &= \vec{OA} + \vec{AC} \\ &= (2\mathbf{a} + \mathbf{b}) + \frac{7}{2}\vec{AB} \\ &= (2\mathbf{a} + \mathbf{b}) + \frac{7}{2}(\mathbf{a} + \mathbf{b}) \\ &= \underline{\underline{\frac{11}{2}\mathbf{a} + \frac{9}{2}\mathbf{b}}}. \end{aligned}$$

28. (a) On the grid, construct the graph of  $x^2 + y^2 = 16$ .

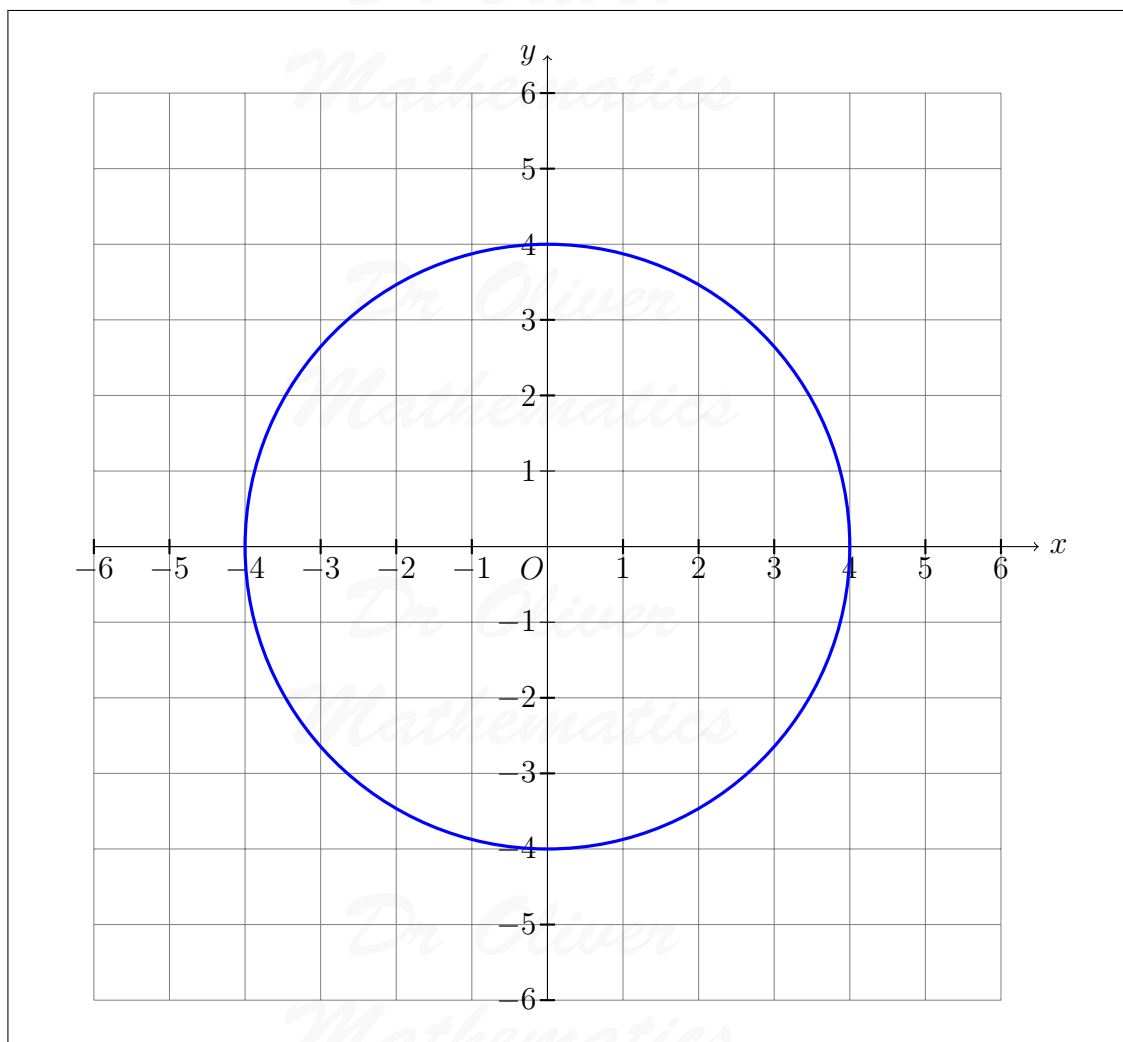
(2)



**Solution**

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(b) Find estimates for the solutions of the simultaneous equations

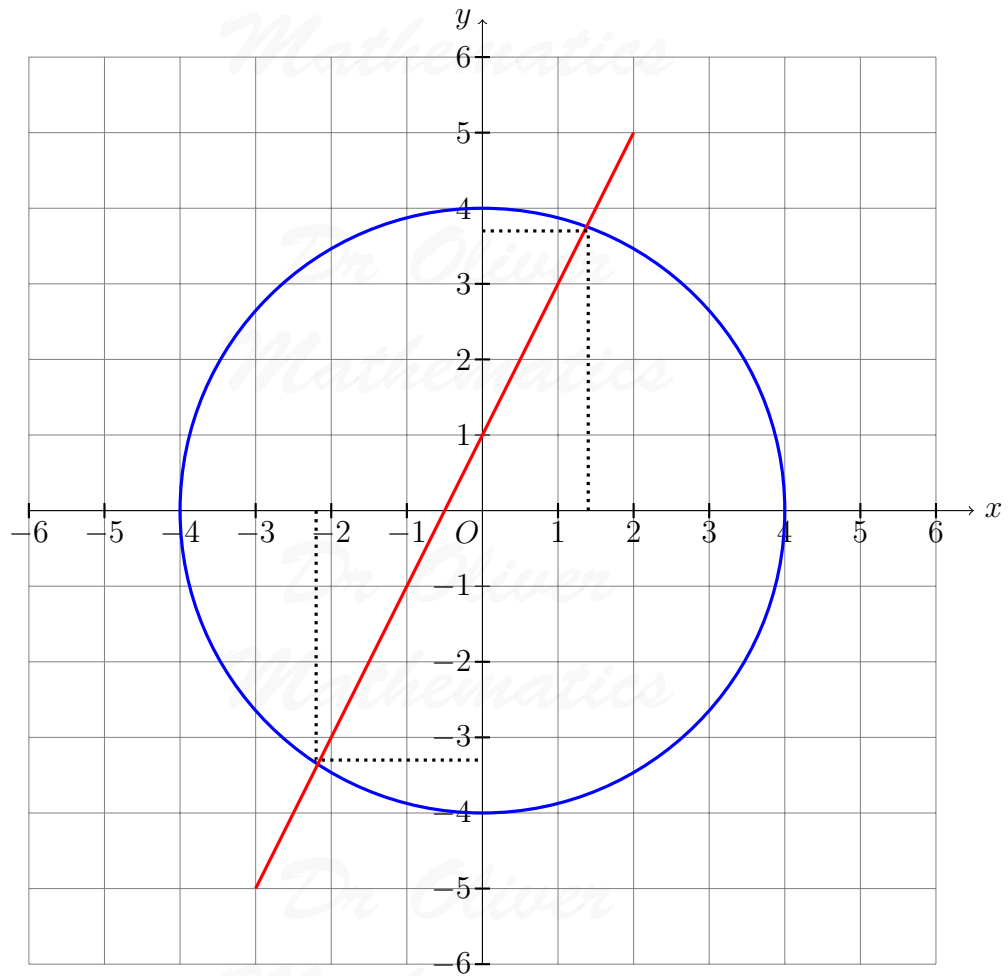
(3)

$$x^2 + y^2 = 16$$

$$y = 2x + 1.$$

**Solution**





From the graph, read-off the coordinates of the points of intersection:  
 $x = 1.4, y = 3.7$  and  $x = -2.2, y = -3.3$ .