

**Dr Oliver Mathematics**  
**GCSE Mathematics**  
**2008 June Paper 4H: Calculator**  
**1 hour 45 minutes**

The total number of marks available is 100.

You must write down all the stages in your working.

1. There are 3 red pens, 4 blue pens, and 5 black pens in a box. Sameena takes a pen, at random, from the box.

(a) Write down the probability that she takes a black pen. (2)

**Solution**

$$P(\text{black pen}) = \frac{5}{3 + 4 + 5} = \frac{5}{12}$$

(b) Write down the probability that Sameena takes a pen that is **not** black. (1)

**Solution**

$$P(\text{not a black pen}) = 1 - \frac{5}{12} = \frac{7}{12}$$

2. Use your calculator to work out (2)

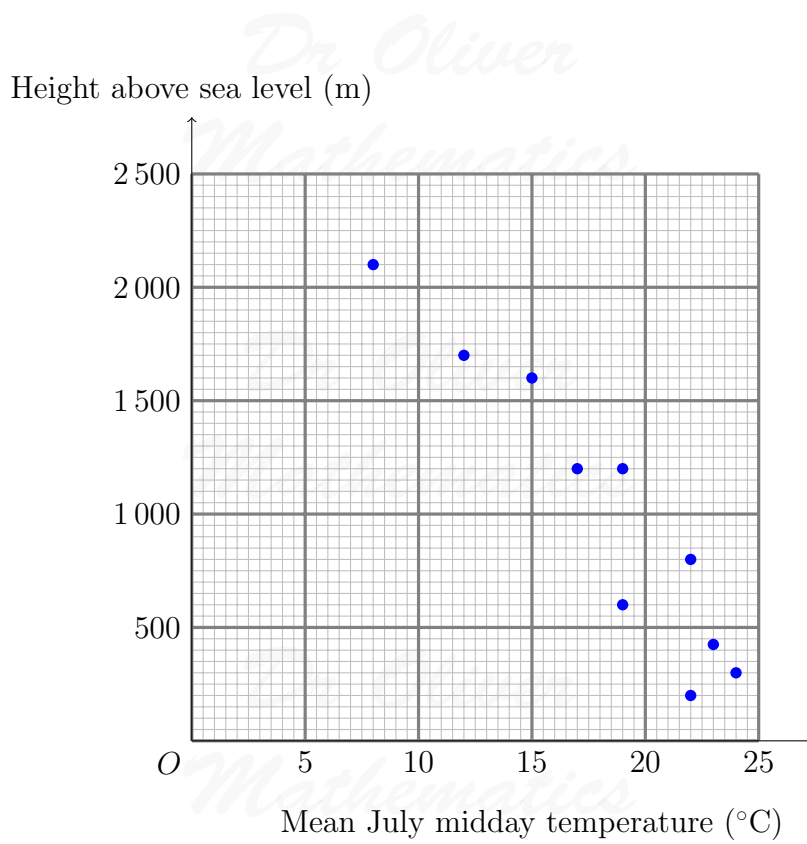
$$\frac{22.4 \times 14.5}{8.5 \times 3.2}$$

Write down all the figures on your calculator display.

**Solution**

$$\begin{aligned} \frac{22.4 \times 14.5}{8.5 \times 3.2} &= \frac{324.8}{27.2} \\ &= \underline{\underline{11.941\ 176\ 47}} \text{ (FCD)}. \end{aligned}$$

3. The scatter graph shows information for some weather stations. It shows the height of each weather station above sea level (m) and the mean July midday temperature ( $^{\circ}\text{C}$ ) for that weather station.



The table shows this information for two more weather stations.

Height of weather station above sea level (m)	1 000	500
Mean July midday temperature (°C)	20	22

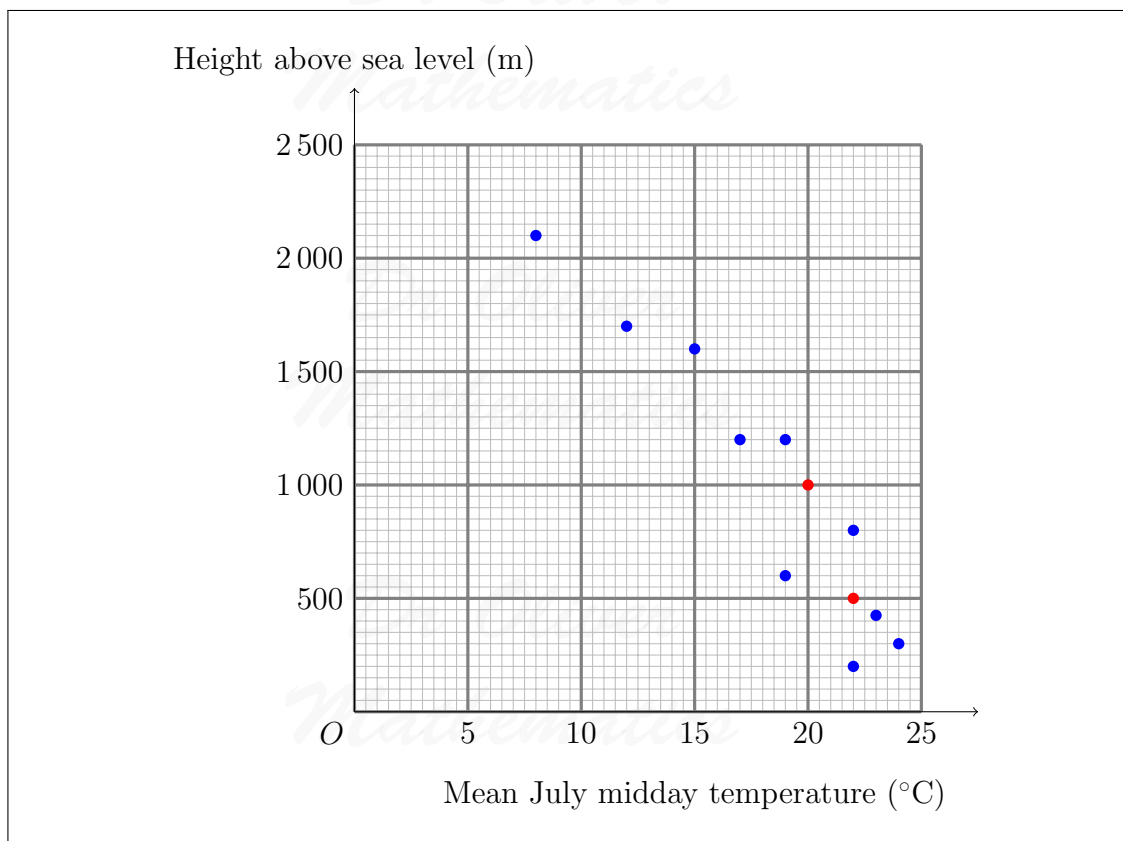
(a) Plot this information on the scatter graph.

(1)

**Solution**

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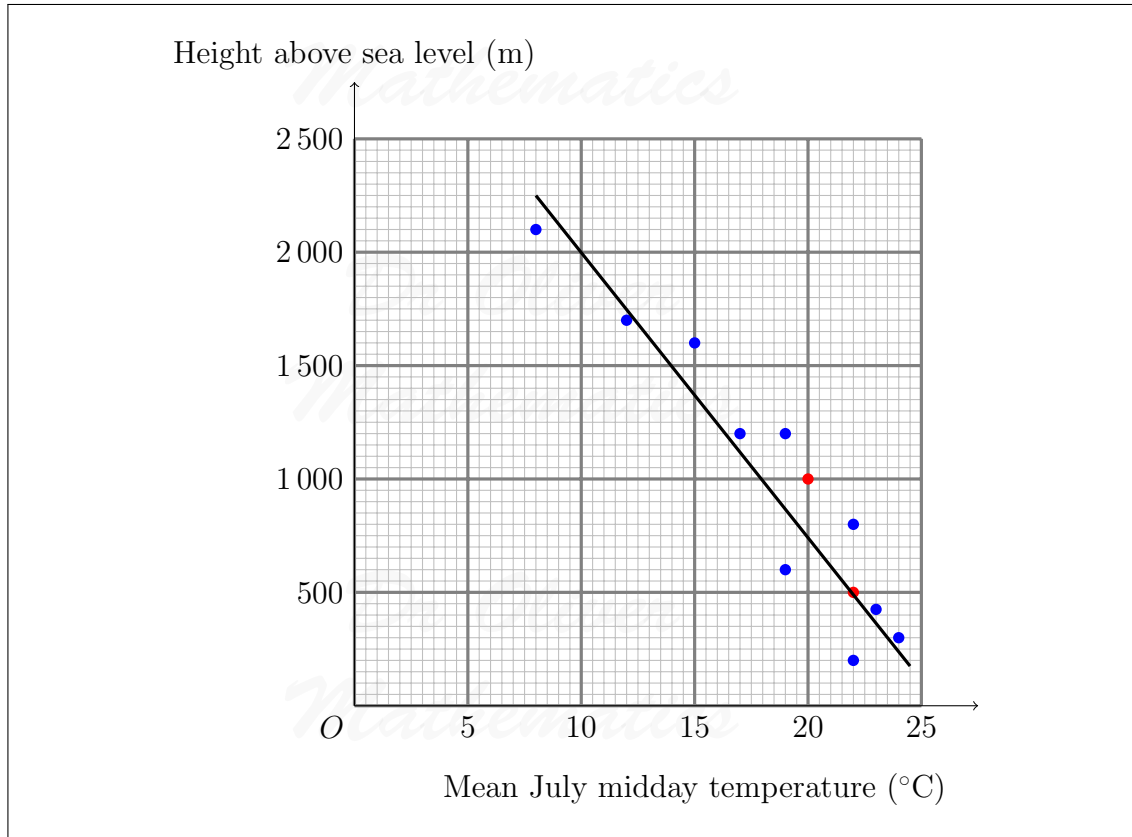


(b) What type of correlation does this scatter graph show? (1)

**Solution**  
Negative correlation, e.g., as the mean July midday temperature goes up, the height above sea level goes down.

(c) Draw a line of best fit on the scatter graph. (1)

**Solution**



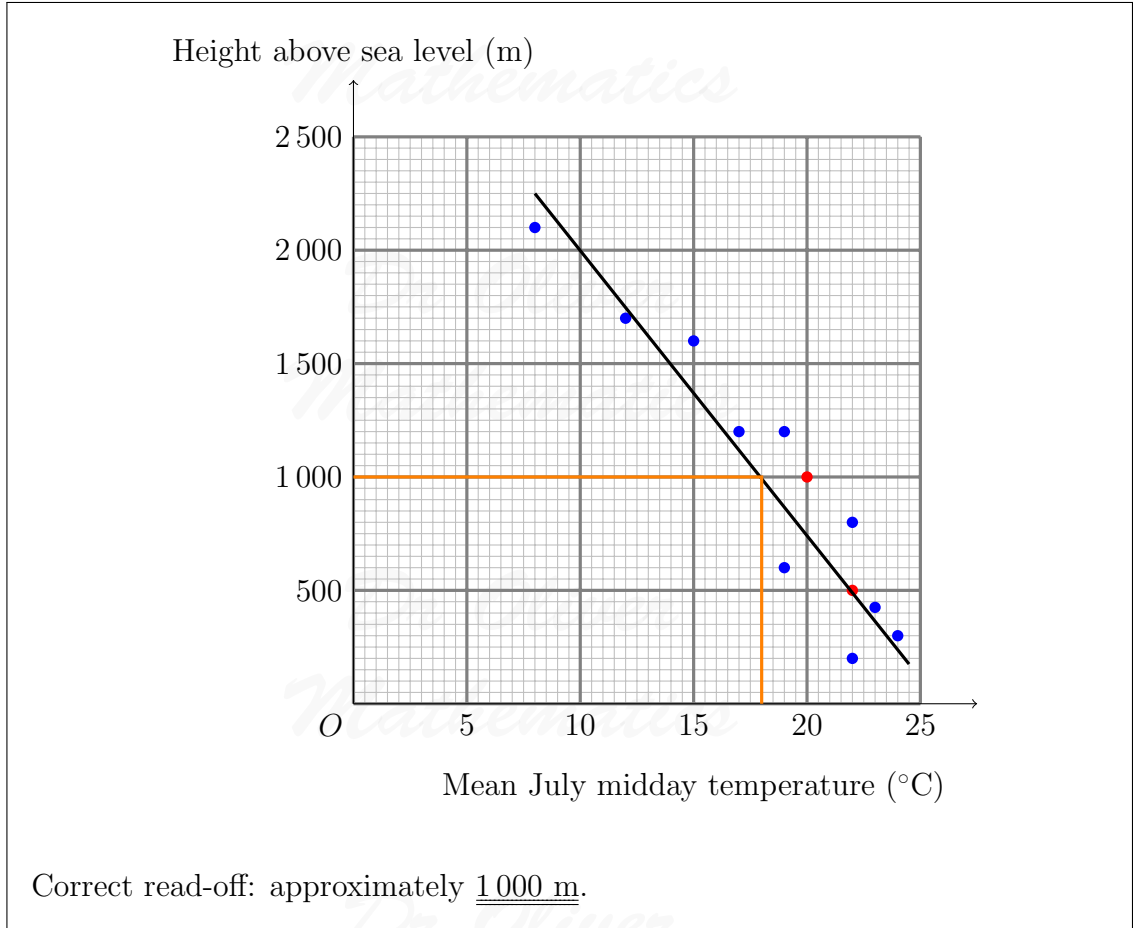
A weather station is 1800 metres above sea level.

(d) Estimate the mean July midday temperature for this weather station.

(1)

**Solution**





4.  $AB$  is parallel to  $CD$ .

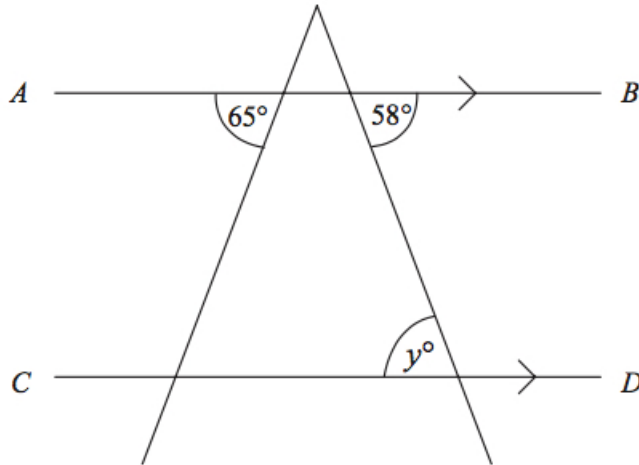


Diagram **NOT** accurately drawn

(a) Write down the value of  $y$ .

(1)

**Solution**

$$y = 180 - 65 - 57 = 180 - 122 = \underline{\underline{58^\circ}}$$

(b) Give a reason for your answer.

(1)

**Solution**

$$\text{Top angle} = 180 - 65 - 58 = 180 - 123 = 57^\circ$$

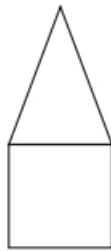
and

$$\text{left angle} = 65^\circ.$$

5. Here are the front elevation, side elevation and the plan of a 3-D shape.

(2)

Front elevation



Side elevation

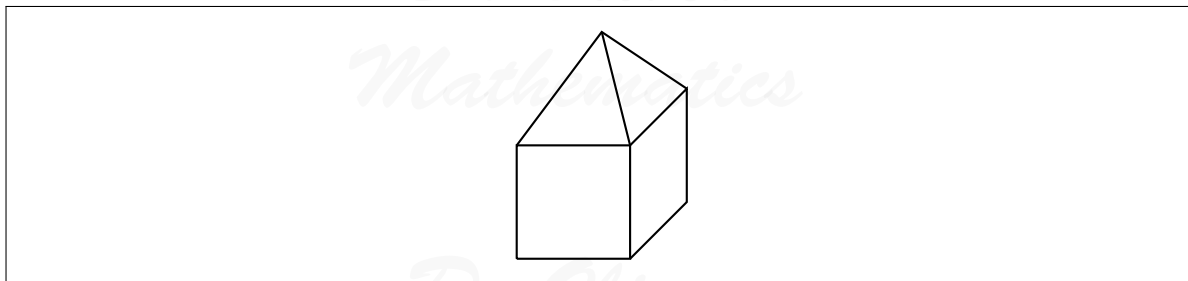


Plan



In the space below, draw a sketch of the 3-D shape.

**Solution**



6. Here are the first four terms of an arithmetic sequence. (2)

$$5 \quad 8 \quad 11 \quad 14$$

Find an expression, in terms of  $n$ , for the  $n$ th term of the sequence.

**Solution**

Let the

$$n\text{th term} = an + b.$$

5	8	11	14
3	3	3	
$a + b$	$2a + b$	$3a + b$	$4a + b$
$a$	$a$	$a$	

We compare terms:

$$a = 3$$

and

$$\begin{aligned} a + b = 5 &\Rightarrow 3 + b = 5 \\ &\Rightarrow b = 2. \end{aligned}$$

Hence,

$$n\text{th term} = \underline{\underline{3n + 2.}}$$

7. The equation (4)

$$x^3 + 2x = 26$$

has a solution between 2 and 3.

Use a trial and improvement method to find this solution.



Give your answer correct to one decimal place.  
You must show **all** your working.

**Solution**

You must be in **TABLE** mode; on my calculator (Casio fx-991) it is MODE 3.

$x$	$f(x)$	Comment
2.7	25.083	too low
2.75	26.296	too high

Clearly,

$$2.7 < x < 2.75$$

and the answer is

$$\underline{\underline{x = 2.7 \text{ (1 dp)}}}.$$

8. 60 students take a science test.

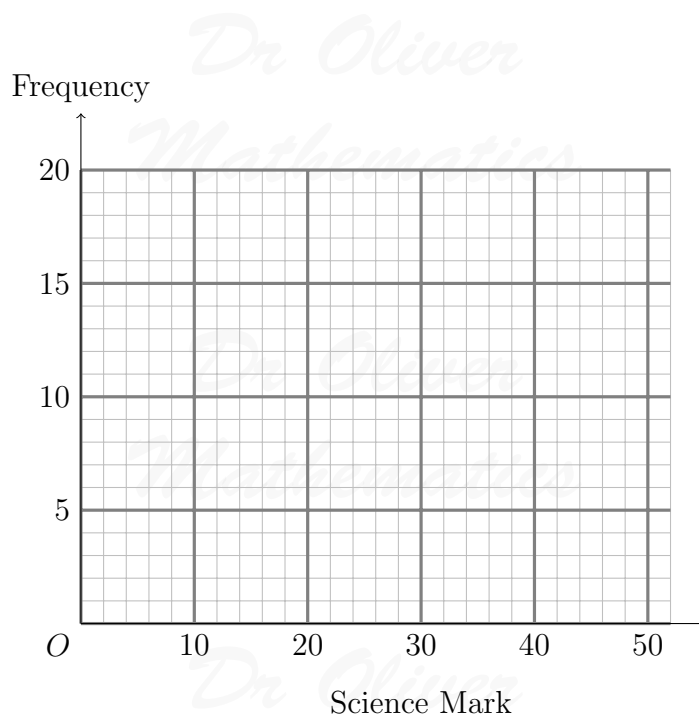
(2)

The test is marked out of 50.

This table shows information about the students' marks.

Science mark	0 – 10	11 – 20	21 – 30	31 – 40	41 – 50
Frequency	4	13	17	19	7

On the grid, draw a frequency polygon to show this information.



**Solution**

Now,

$$\frac{0 + 10}{2} = 5$$

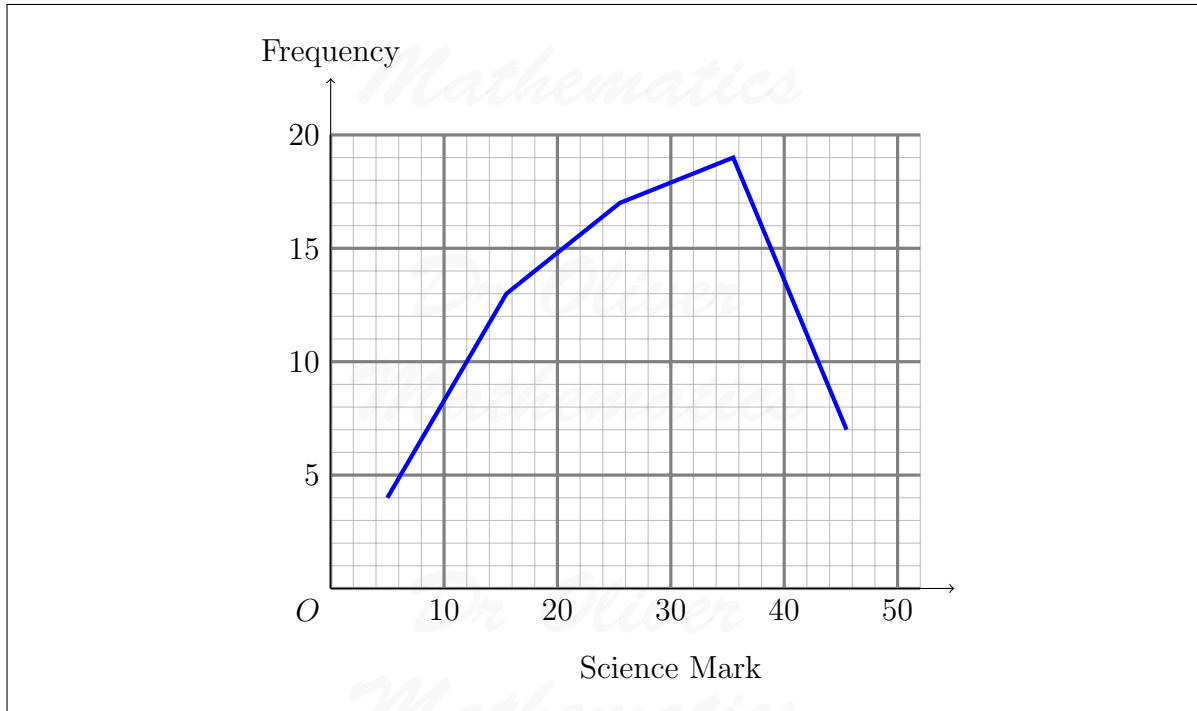
and

$$\frac{11 + 20}{2} = 15.5,$$

etc.

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9. In this quadrilateral, the sizes of the angles, in degrees, are  $x + 10$ ,  $2x$ ,  $2x$ , and  $50$ .

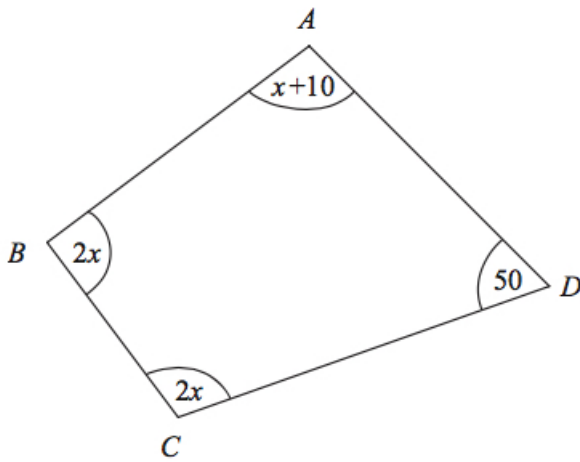


Diagram NOT accurately drawn

(a) Use this information to write down an equation in terms of  $x$ . (2)

**Solution**

$$(x + 10) + 2x + 2x + 50 = 360 \Rightarrow \underline{\underline{5x + 60 = 360}}$$

(b) Work out the value of  $x$ . (3)

**Solution**

$$5x + 60 = 360 \Rightarrow 5x = 300 \\ \Rightarrow \underline{\underline{x = 60.}}$$

10. A garage sells British cars and foreign cars.

The ratio of the number of British cars sold to the number of foreign cars sold is 2 : 7.  
The garage sells 45 cars in one week.

(a) Work out the number of British cars the garage sold that week. (2)

**Solution**

$$\frac{2}{2+7} \times 45 = \frac{2}{9} \times 45 = \underline{\underline{10 \text{ cars.}}}$$

A car tyre costs £80 plus VAT at  $17\frac{1}{2}\%$ .

(b) Work out the total cost of the tyre. (3)

**Solution**

You simply take 10% (8), 5% (4), and  $2\frac{1}{2}\%$  (2):

$$80 + 8 + 4 + 2 = \underline{\underline{£94.}}$$

The value of a new car is £12 000.

The value of the car depreciates by 20% per year.

(c) Work out the value of the car after 2 years. (3)

**Solution**

$1 - 0.2 = 0.8$  and

$$12\,000 \times 0.8^2 = 12\,000 \times 0.64.$$

×	10	2
0.6	6	1.2
0.04	0.4	0.08

Hence,

$$12\,000 \times 0.64 = 7.68 \times 10^3 = \underline{\underline{\pounds 7\,680}}.$$

11. (a) Simplify  $4a + 3c - 2a + c$ .

(1)

**Solution**

$$4a + 3c - 2a + c = \underline{\underline{2a + 4c}}.$$

- (b)  $S = \frac{1}{2}at^2$ .

(2)

Find the value of  $S$  when  $t = 3$  and  $a = \frac{1}{4}$ .

**Solution**

$$S = \frac{1}{2} \times \frac{1}{4} \times 9 = \underline{\underline{\frac{9}{8}}}.$$

- (c) Factorise  $x^2 - 5x$ .

(2)

**Solution**

$$x^2 - 5x = \underline{\underline{x(x - 5)}}.$$

- (d) Expand and simplify  $(x + 3)(x + 4)$ .

(2)

**Solution**

$\times$	$x$	$+3$
$x$	$x^2$	$+3x$
$+4$	$+4x$	$+12$

$$(x + 3)(x + 4) = \underline{\underline{x^2 + 7x + 12}}.$$

- (e) Factorise  $y^2 + 8y + 15$ .

(2)

**Solution**

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$$\left. \begin{array}{l} \text{add to:} \quad +8 \\ \text{multiply to:} \quad +15 \end{array} \right\} + 3, +5$$
$$y^2 + 8y + 15 = \underline{\underline{(y + 3)(y + 5)}}.$$

12. A shop sells mobile phones.

The table shows the number of mobile phones sold each month from January to May.

Jan	Feb	Mar	Apr	May
70	64	73	85	91

- (a) Work out the percentage increase in the number of mobile phones sold from April to May. (3)

Give your answer correct to 3 significant figures.

**Solution**

$$\frac{91 - 85}{85} \times 100\% = 7\frac{1}{17}$$
$$= \underline{\underline{7.06\% (3 \text{ sf})}}.$$

- (b) Work out the 3-month moving averages for the information in the table. (2)  
The first one has been worked out for you.

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**Solution**

Feb-Apr:

$$\frac{64 + 73 + 85}{3} = \frac{222}{3} = \underline{\underline{74}}.$$

Mar-May:

$$\frac{73 + 85 + 91}{3} = \frac{249}{3} = \underline{\underline{83}}.$$

13. A solid cylinder has a radius of 4 cm and a height of 10 cm.

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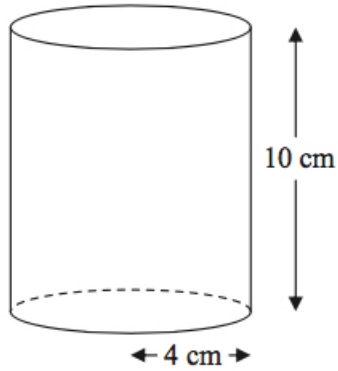


Diagram **NOT**  
accurately drawn

- (a) Work out the volume of the cylinder. (2)  
Give your answer correct to 3 significant figures.

**Solution**

$$\begin{aligned}\text{Volume} &= \pi \times 4^2 \times 10 \\ &= 502.6548246 \text{ (FCD)} \\ &= \underline{\underline{503 \text{ cm}^3}} \text{ (3 sf)}.\end{aligned}$$

The cylinder is made from wood.  
The density of the wood is 0.6 grams per  $\text{cm}^3$ .

- (b) Work out the mass of the cylinder. (2)  
Give your answer correct to 3 significant figures.

**Solution**

$$\begin{aligned}\text{Mass} &= \text{density} \times \text{volume} \\ &= 0.6 \times 502.65 \dots \\ &= 301.5928947 \text{ (FCD)} \\ &= \underline{\underline{302 \text{ g}}} \text{ (3 sf)}.\end{aligned}$$

14.  $ABC$  is a right-angled triangle.

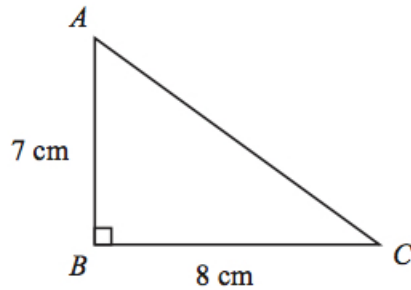


Diagram **NOT**  
accurately drawn

$$AB = 7 \text{ cm.}$$

$$BC = 8 \text{ cm.}$$

- (a) Work out the area of the triangle.

(2)

**Solution**

$$\begin{aligned} \text{Area} &= \frac{1}{2}bh \\ &= \frac{1}{2} \times 7 \times 8 \\ &= \underline{\underline{28 \text{ cm}^2}}. \end{aligned}$$

- (b) Work out the length of  $AC$ .

(3)

Give your answer correct to 2 decimal places.

**Solution**

$$\begin{aligned} AC &= \sqrt{7^2 + 8^2} \\ &= 10.630\,145\,81 \text{ (FCD)} \\ &= \underline{\underline{10.63 \text{ cm (2 dp)}}}. \end{aligned}$$



$DEF$  is another right-angled triangle.

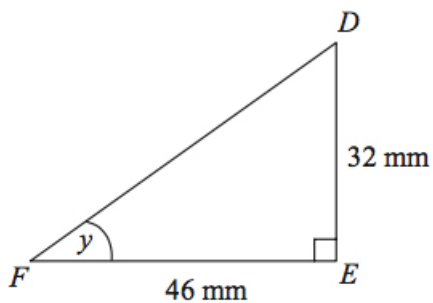


Diagram **NOT**  
accurately drawn

$$DE = 32 \text{ mm.}$$

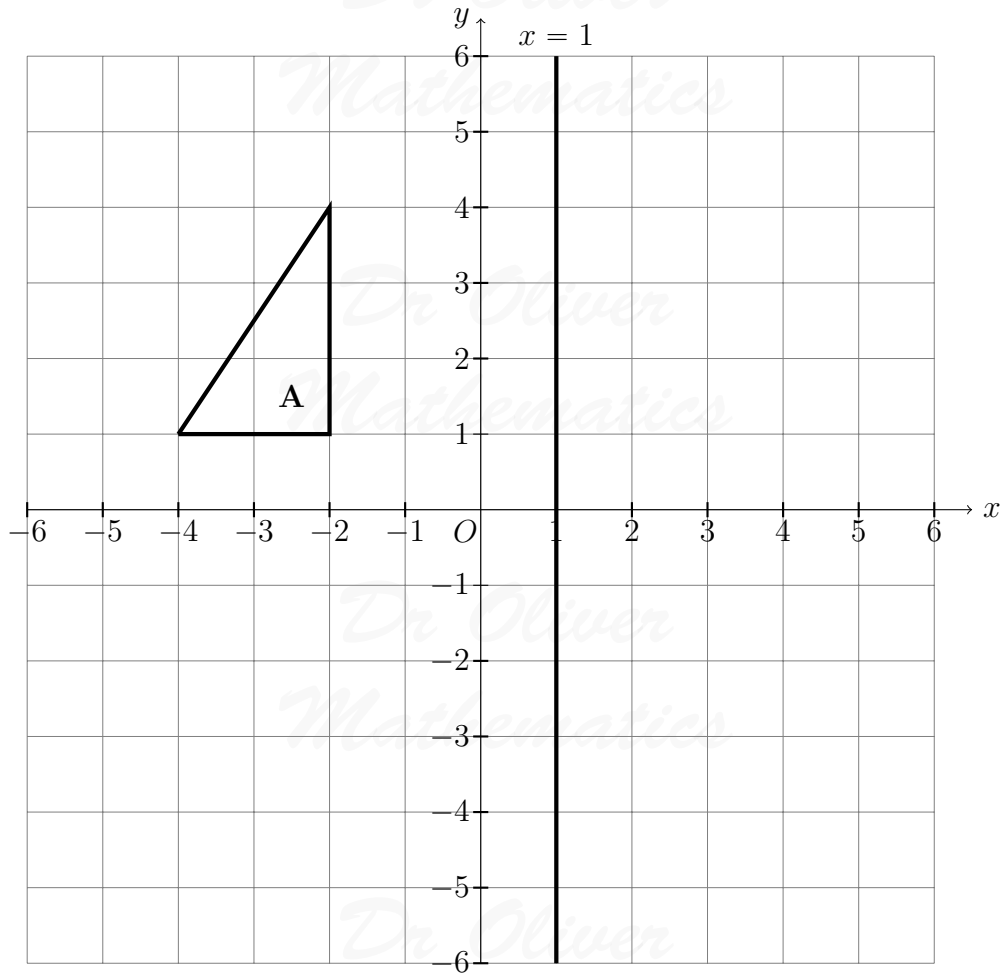
$$FE = 46 \text{ mm.}$$

- (c) Calculate the size of angle  $y$ . (3)  
Give your answer correct to 1 decimal place.

**Solution**

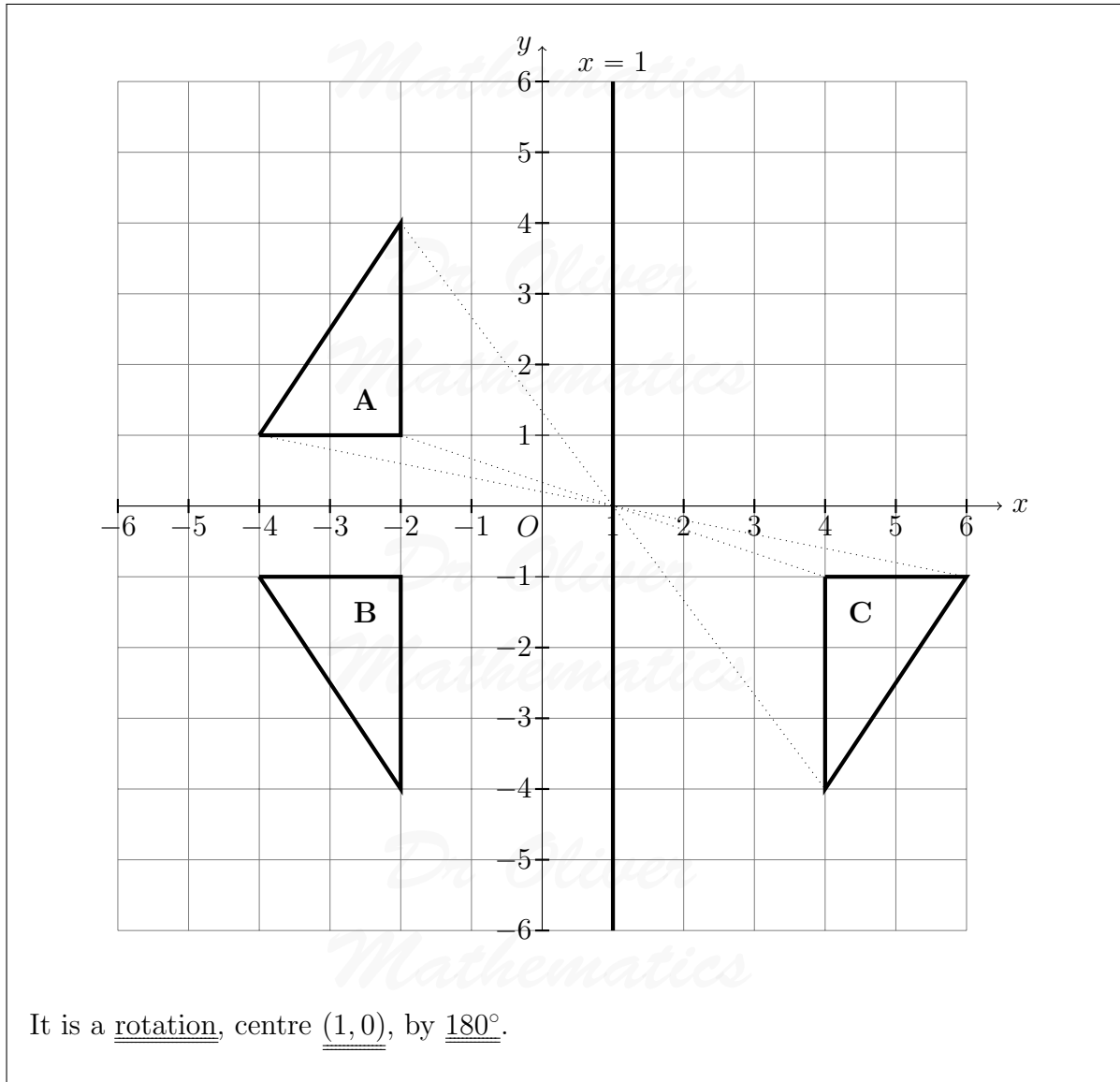
$$\begin{aligned}\tan y &= \frac{32}{46} \Rightarrow y = 34.82448916 \text{ (FCD)} \\ &\Rightarrow \underline{\underline{y = 34.8^\circ \text{ (1 dp)}}}.\end{aligned}$$

15. Triangle **A** is reflected in the  $x$ -axis to give triangle **B**. (3)  
Triangle **B** is reflected in the line  $x = 1$  to give triangle **C**.



Describe the single transformation that takes triangle **A** to triangle **C**.

**Solution**



16. (a) Express 252 as a product of its prime factors.

(3)

**Solution**

	252
2	126
2	63
3	21
3	7
7	1

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Hence,

$$252 = \underline{2^2 \times 3^2 \times 7}.$$

James thinks of two numbers.

He says, “The Highest Common Factor (HCF) of my two numbers is 3.

The Lowest Common Multiple (LCM) of my two numbers is 45.”

- (b) Write down two numbers that James could be thinking of. (3)

**Solution**

As the HCF is 3, the numbers must be  $3n$  and  $3m$  where  $n$  and  $m$  are co-prime and at most one is a multiple of 3.

Using LCM, the factors of 45 are 1, 3, 5, 9, 15, and 45.

He could have been thinking of 3 and 45 or 9 and 15.

17. The number of atoms in one kilogram of helium is  $1.51 \times 10^{26}$ . (2)  
Calculate the number of atoms in 20 kilograms of helium.  
Give your answer in standard form.

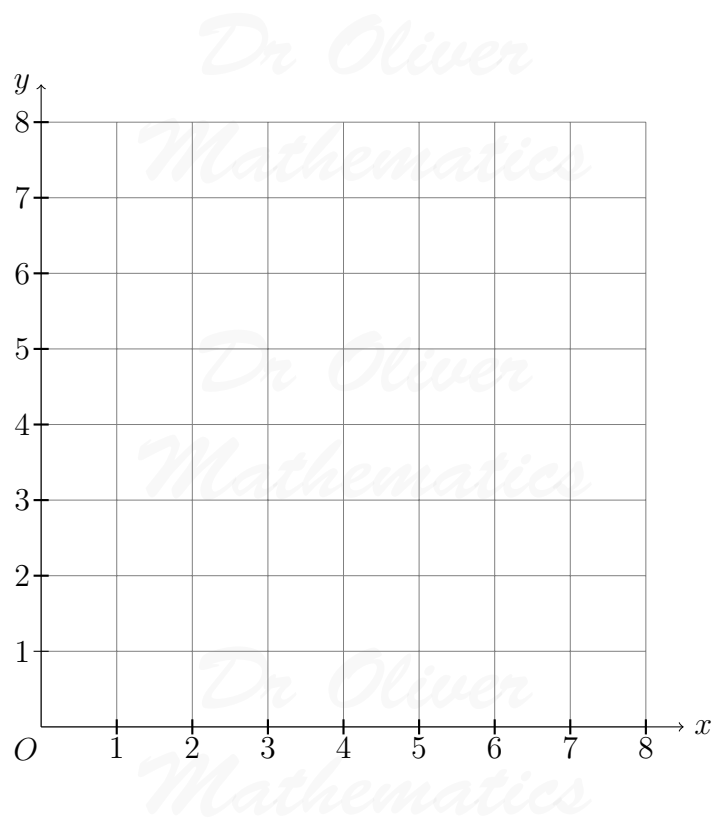
**Solution**

$$1.51 \times 10^{26} \times 20 = \underline{3.02 \times 10^{27}} \text{ atoms.}$$

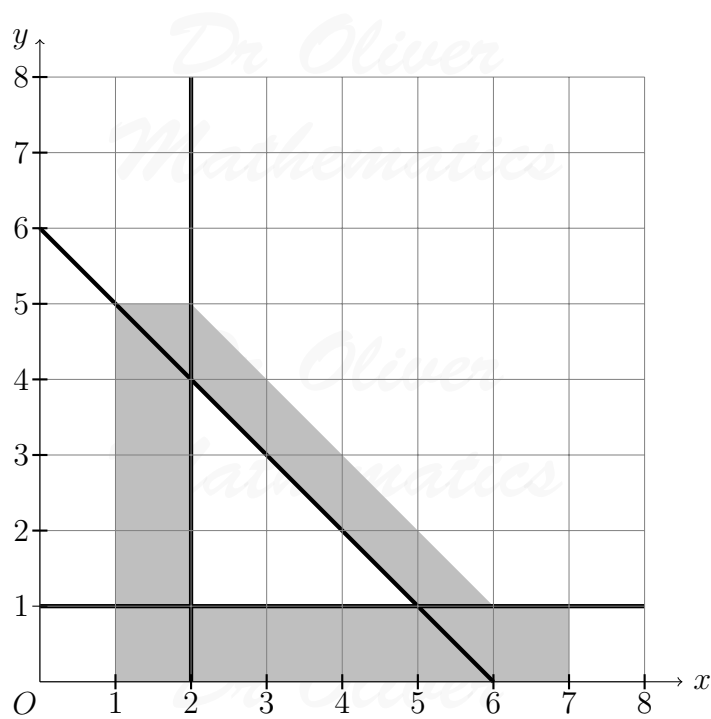
18. The region **R** satisfies the inequalities (3)

$$x \geq 2, y \geq 1, \text{ and } x + y \leq 6.$$

On the grid below, draw straight lines and use shading to show the region **R**.



**Solution**



19. The diagram shows a sector of a circle, centre  $O$ .

(2)

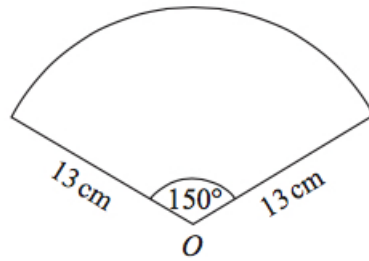


Diagram **NOT**  
accurately drawn

The radius of the circle is  $13\text{ cm}$ .  
The angle of the sector is  $150^\circ$ .  
Calculate the area of the sector.  
Give your answer correct to 3 significant figures.

**Solution**

$$\begin{aligned}\text{Area} &= \frac{150}{360} \times \pi \times 13^2 \\ &= 221.2204827 \text{ (FCD)} \\ &= \underline{\underline{221\text{ cm}^2}} \text{ (3 sf)}.\end{aligned}$$

20.  $q$  is inversely proportional to the square of  $t$ .  
When  $t = 4$ ,  $q = 8.5$ .

(a) Find a formula for  $q$  in terms of  $t$ .

(3)

**Solution**

$$q = \frac{k}{t^2}$$

for some constant  $k$ . Now,

$$8.5 = \frac{k}{4^2} \Rightarrow k = 136$$

and

$$\underline{\underline{q = \frac{136}{t^2}}}.$$

(b) Calculate the value of  $q$  when  $t = 5$ .

(1)

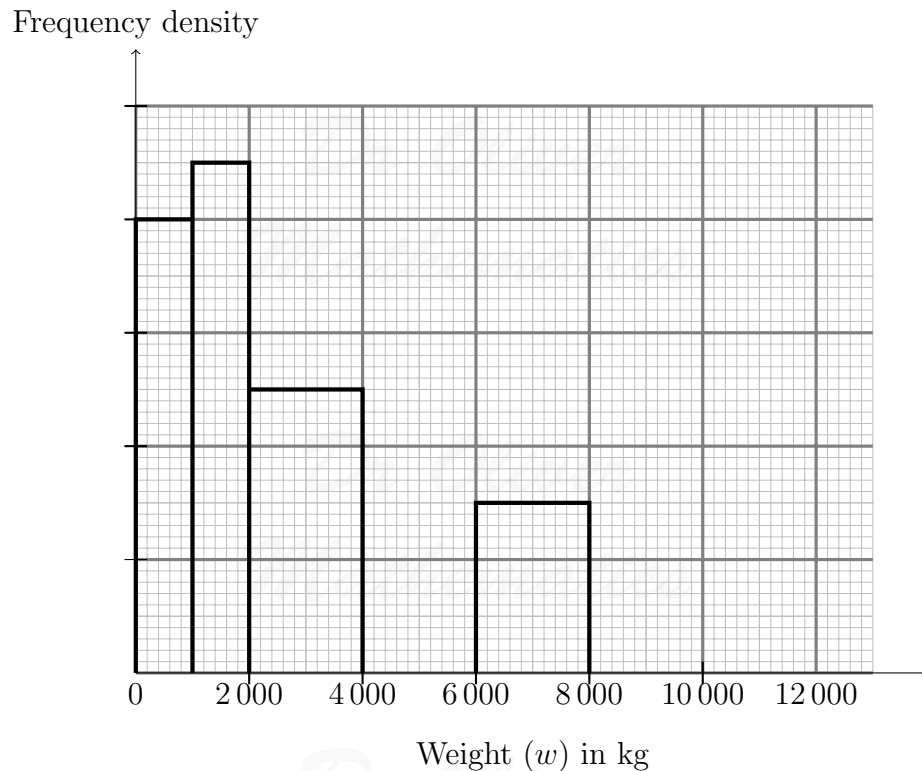
**Solution**

$$q = \frac{136}{5^2} = \underline{\underline{5.44}}.$$

21. The incomplete histogram and table show information about the weights of some containers.

Weight ( $w$ ) in kg	Frequency
$0 < w < 1\,000$	16
$1\,000 < w < 2\,000$	
$2\,000 < w < 4\,000$	
$4\,000 < w < 6\,000$	16
$6\,000 < w < 8\,000$	
$8\,000 < w < 12\,000$	8

Here is part of the histogram.



(a) Use the information in the histogram to complete the table.

(2)

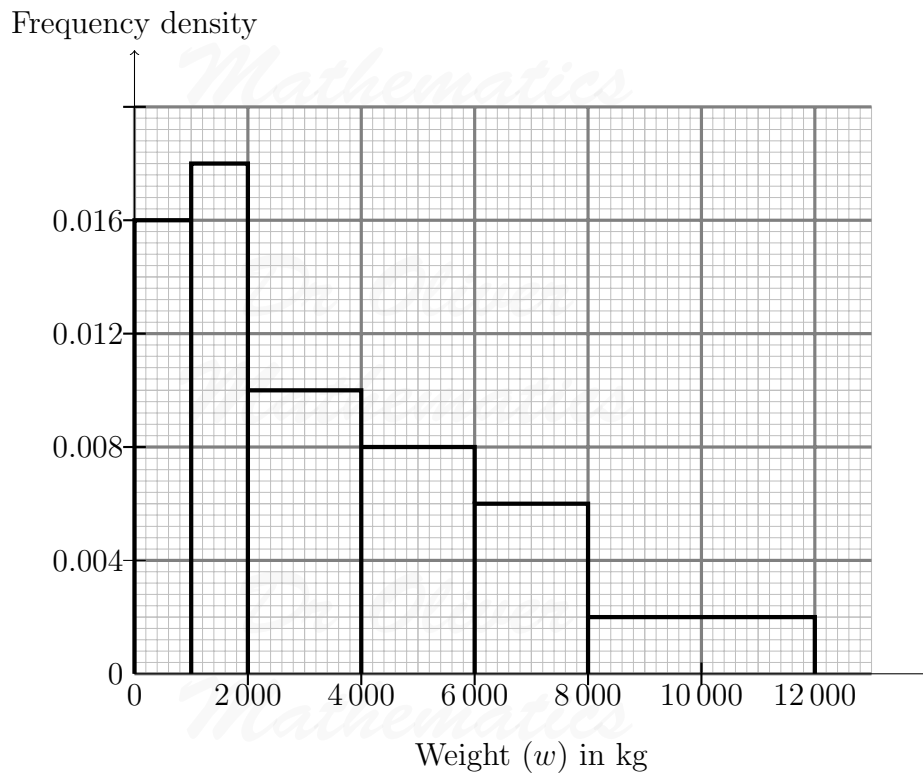
**Solution**

Weight ( $w$ ) in kg	Frequency	Width	Frequency density
$0 < w < 1\,000$	16	1 000	$\frac{16}{1\,000} = 0.016$
$1\,000 < w < 2\,000$	<u>18</u>	1 000	$\frac{18}{1\,000} = 0.018$
$2\,000 < w < 4\,000$	<u>20</u>	2 000	$\frac{20}{2\,000} = 0.01$
$4\,000 < w < 6\,000$	16	2 000	$\frac{16}{2\,000} = 0.008$
$6\,000 < w < 8\,000$	<u>12</u>	2 000	$\frac{12}{2\,000} = 0.006$
$8\,000 < w < 12\,000$	8	4 000	$\frac{8}{4\,000} = 0.002$

(b) Use the information in the table to complete the histogram.

(2)

**Solution**



22. Katy drove for 238 miles, correct to the nearest mile.

She used 27.3 litres of petrol, to the nearest tenth of a litre.

(3)

$$\text{Petrol consumption} = \frac{\text{Number of miles travelled}}{\text{Number of litres of petrol used}}$$



Work out the upper bound for the petrol consumption for Katy's journey.  
Give your answer correct to 2 decimal places.

**Solution**

Now,

$$237.5 \leq \text{distance} < 238.5$$

and

$$27.25 \leq \text{litres} < 27.35.$$

Finally,

$$\begin{aligned} \text{upper bound} &= \frac{238.5}{27.25} \\ &= 8.752\,293\,578 \text{ (FCD)} \\ &= \underline{\underline{8.75 \text{ mile/litre (2 dp)}}}. \end{aligned}$$

23. (a) Show that the equation

$$\frac{5}{x+2} = \frac{4-3x}{x-1}$$

(3)

can be rearranged to give

$$3x^2 + 7x - 13 = 0.$$

**Solution**

$$\begin{aligned} \frac{5}{x+2} = \frac{4-3x}{x-1} &\Rightarrow 5(x-1) = (4-3x)(x+2) \\ &\Rightarrow 5x - 5 = 8 - 2x - 3x^2 \\ &\Rightarrow \underline{\underline{3x^2 + 7x - 13 = 0}}, \end{aligned}$$

as required.

(b) Solve

$$3x^2 + 7x - 13 = 0.$$

(3)

Give your solutions correct to 2 decimal places.

**Solution**

$a = 3$ ,  $b = 7$ , and  $c = -13$ :

$$\begin{aligned}x &= \frac{-7 \pm \sqrt{7^2 - 4 \times 3 \times (-13)}}{6} \\&= \frac{-7 \pm \sqrt{205}}{6} \\&= -3.552970177 \text{ or } 1.219636844 \text{ (FCD)} \\&= \underline{\underline{-3.55 \text{ or } 1.22 \text{ (2 dp)}}}.\end{aligned}$$

24.  $ABC$  is a triangle.

(3)

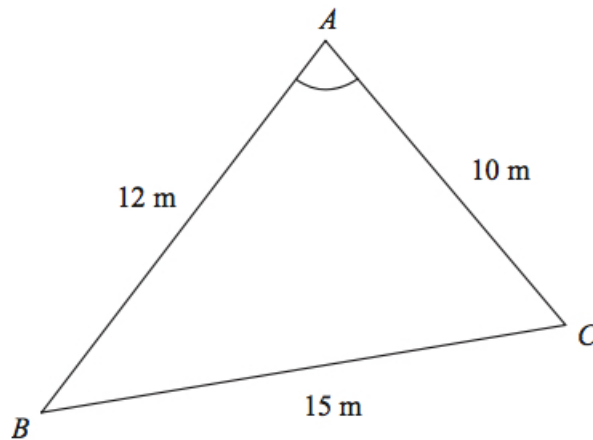


Diagram **NOT**  
accurately drawn

$AB = 12 \text{ m.}$

$AC = 10 \text{ m.}$

$BC = 15 \text{ m.}$

Calculate the size of angle  $BAC$ .

Give your answer correct to one decimal place.

**Solution**

$$\begin{aligned}\cos BAC &= \frac{10^2 + 12^2 - 15^2}{2 \times 10 \times 12} \Rightarrow \cos BAC = \frac{19}{240} \\&\Rightarrow BAC = 85.45933267 \text{ (FCD)} \\&\Rightarrow \underline{\underline{BAC = 85.5^\circ \text{ (1 dp)}}}.\end{aligned}$$

25. The sketch shows a curve with equation

(3)

$$y = ka^x$$

where  $k$  and  $a$  are constants, and  $a > 0$ .

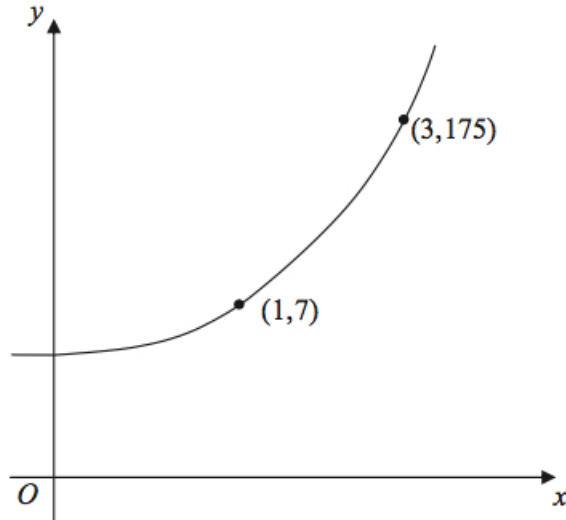


Diagram **NOT**  
accurately drawn

The curve passes through the points  $(1, 7)$  and  $(3, 175)$ .  
Calculate the value of  $k$  and the value of  $a$ .

**Solution**

$(1, 7)$ :

$$7 = ka \quad (1).$$

$(3, 175)$ :

$$175 = ka^3 \quad (2).$$

Divide (2) by (1):

$$\frac{ka^3}{ka} = \frac{175}{7} \Rightarrow a^2 = 25$$

$$\Rightarrow \underline{\underline{a = 5}}$$

$$\Rightarrow \underline{\underline{k = 1.4}}.$$