Dr Oliver Mathematics
Worked Examples
Probability 4

From: Edexcel 2005 November Paper 6H (Calculator)

1. (a) Solve the equation (3)

$$19x^2 - 124x - 224 = 0.$$

Solution

E.g.,

$$\left.\begin{array}{ll} \text{add to:} & -124 \\ \text{multiply to:} & (+19) \times (-224) = -4\,256 \end{array}\right\} -152, +28$$

$$\begin{aligned} 19x^2 - 124x - 224 = 0 &\Rightarrow 19x^2 - 152x + 28x - 224 = 0 \\ &\Rightarrow 19x(x-8) + 28(x-8) = 0 \\ &\Rightarrow (19x+28)(x-8) = 0 \\ &\Rightarrow \underline{\underline{x = -\tfrac{28}{19} \text{ or } x = 8}}. \end{aligned}$$

A bag contains red counters and blue counters and white counters.
There are n red counters.
There are 2 more blue counters than red counters.
The number of white counters is equal to the total number of red counters and blue counters.

(b) Show that the number of counters in the bag is $4(n+1)$. (1)

Solution

There are

$$n + (n+2) + 2(n+2) = 4n + 4 = \underline{\underline{4(n+1)}}$$

counters in the bag.

Bob and Ann play a game.
Bob will take a counter at random from the bag.
He will record the colour and put the counter back in the bag.
Ann will then take a counter at random from the bag.
She will record its colour.
The probability that Bob's counter is red and Ann's counter is **not** red is $\frac{14}{81}$.

(c) Prove that (5)

$$19n^2 - 124n - 224 = 0.$$

Solution

$$\begin{aligned} & \frac{n}{4(n+1)} \times \frac{3n+4}{4(n+1)} = \frac{14}{81} \\ \Rightarrow\ & 81n(3n+4) = 14 \times 16(n+1)^2 \\ \Rightarrow\ & 243n^2 + 324n = 224(n^2 + 2n + 1) \\ \Rightarrow\ & 243n^2 + 324n = 224n^2 + 448n + 224 \\ \Rightarrow\ & \underline{\underline{19n^2 - 124n - 224 = 0}}, \end{aligned}$$

as required.

(d) Using your answer to part (a), or otherwise, show that the number of counters in the bag is 36. (1)

Solution

Using $x = 8$, the number of counters in the bag is

$$8 + 10 + 18 = \underline{\underline{36}}.$$

Bob and Ann play the game with all 36 counters in the bag.

(e) Calculate the probability that Bob and Ann will take counters with **different** colours. (3)

Solution

$$\begin{aligned} \text{P(same colour)} &= \text{P}(RR) + \text{P}(BB) + \text{P}(WW) \\ &= \left(\tfrac{8}{36}\right)^2 + \left(\tfrac{10}{36}\right)^2 + \left(\tfrac{18}{36}\right)^2 \\ &= \tfrac{61}{162} \end{aligned}$$

and

$$\text{P(different colour)} = 1 - \tfrac{61}{162} = \underline{\underline{\tfrac{101}{162}}}.$$