

Dr Oliver Mathematics
Cambridge O Level Additional Mathematics
2009 November Paper 1: Calculator
2 hours

The total number of marks available is 80.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

You must write down all the stages in your working.

1. Given that

$$f(x) = 2x^3 - 7x^2 + 7ax + 16$$

is divisible by $(x - a)$, find

- (a) the value of the constant a , (2)
- (b) the remainder when $f(x)$ is divided by $(2x + 1)$. (2)

2. The table shows the results achieved by four teams in twelve events of an athletics match.

	1st	2nd	3rd	4th
Harriers	6	3	1	2
Strollers	3	2	4	3
Road Runners	2	5	5	0
Olympians	1	2	1	7

In each event,

- 1st place scores 5 points,
 - 2nd place scores 3 points,
 - 3rd place scores 2 points, and
 - 4th place scores 1 point.
- (a) Write down two matrices whose product shows the total number of points scored by each team. (2)
- (b) Evaluate this product of matrices. (2)
3. Find the values of k for which the equation (4)

$$x^2 - 2(2k + 1)x + (k + 2) = 0$$

has two equal roots.

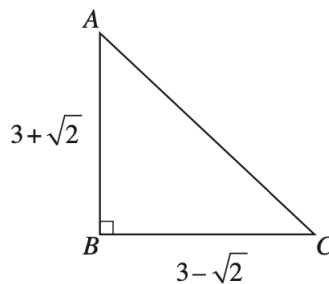
4. Solve the simultaneous equations

(5)

$$\begin{aligned}x + 3y &= 13, \\x^2 + 3y^2 &= 43.\end{aligned}$$

5. The diagram shows a triangle ABC , where

- angle ABC is a right angle,
- the length of $AB = (3 + \sqrt{2})$, and
- the length of $BC = (3 - \sqrt{2})$.



(a) Find the length of AC in the form \sqrt{k} , where k is an integer.

(2)

(b) Find $\tan BAC$ in the form

(3)

$$\frac{a + b\sqrt{2}}{c},$$

where a , b , and c are integers.

6. Set A is such that

$$A = \{x : 3x^2 - 10x - 8 \leq 0\}.$$

(a) Find the set of values of x which define the set A .

(3)

Set B is such that

$$B = \{x : 7 - 2x \leq 1\}.$$

(b) Find the set of values of x which define the set $A \cap B$.

(2)

7. A committee of 8 people is to be selected from 7 teachers and 6 students.

Find the number of different ways in which the committee can be selected if

(a) there are no restrictions,

(2)

(b) there are to be more teachers than students on the committee.

(4)

8. The number, N , of bacteria present in an experiment, t minutes after measurements begin, is given by

$$N = 1\,000e^{-kt},$$

where k is a constant.

- (a) State the number of bacteria when $t = 0$. (1)

When $t = 0$, the number of bacteria is decreasing at the rate of 20 per minute.

Find

- (b) the value of k , (3)

- (c) the time taken for the number of bacteria to decrease by 50%. (3)

9. Differentiate, with respect to x ,

- (a) $(1 - 2x)^{20}$, (2)

- (b) $x^2 \ln x$, (3)

- (c) $\frac{\tan(2x + 1)}{x}$. (3)

10. A curve has equation

$$y = 3x^3 - 2x^2 + 2x.$$

- (a) Show that the equation of the tangent to the curve at the point where $x = 1$ is (4)

$$y = 7x - 4.$$

- (b) Find the coordinates of the point where this tangent meets the curve again. (5)

11. (a) Show that (3)

$$\tan \theta + \cot \theta \equiv \operatorname{cosec} \theta \sec \theta.$$

- (b) Solve the equation

- (i) $\tan x = 3 \sin x$ for $0^\circ \leq x \leq 360^\circ$, (4)

- (ii) $2 \cot^2 y + 3 \operatorname{cosec} y = 0$ for $0 < y < 2\pi$ radians. (5)

EITHER

12. A solid circular cylinder has radius r cm and height h cm.

The volume of the cylinder is $1\,000 \text{ cm}^3$.

- (a) Find an expression for h in terms of r . (2)

- (b) Hence show that the total surface area, $A \text{ cm}^2$, of the cylinder is given by (2)

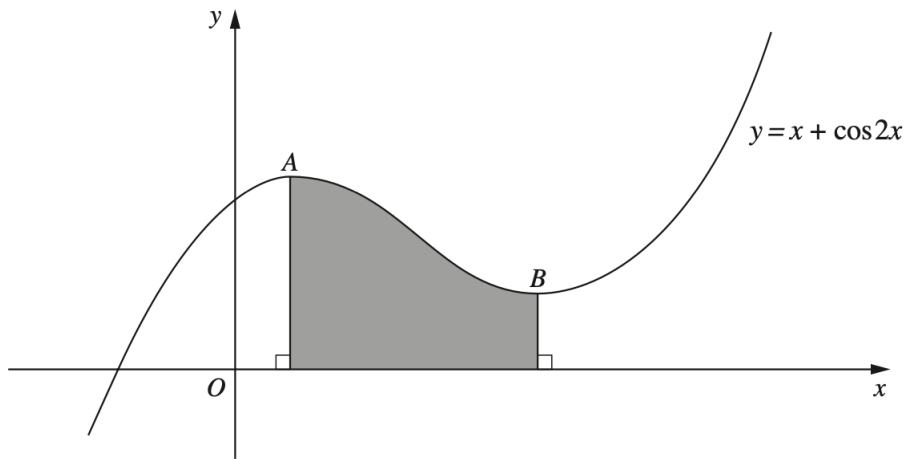
$$A = 2\pi r^2 + \frac{2\,000}{r}.$$

- (c) Given that r varies, find, correct to 2 decimal places, the value of r when A has a stationary value. (4)
- (d) Find this stationary value of A and determine its nature. (3)

OR

13. The diagram shows part of the curve

$$y = x + \cos 2x.$$



The curve has a maximum point at A and a minimum point at B .

- (a) Find the x -coordinate of the point A and of the point B . (6)
- (b) Find, in terms of π , the area of the shaded region. (5)