Dr Oliver Mathematics
GCSE Mathematics
2007 June Paper 5H: Non-Calculator
2 hours

The total number of marks available is 100.
You must write down all the stages in your working.

1. A bag contains counters which are red or green or yellow or blue.
The table shows each of the probabilities that a counter taken at random from the bag will be red or green or blue.

Colour	Red	Green	Yellow	Blue
Probability	0.2	0.3		0.1

A counter is to be taken at random from the bag.

(a) Work out the probability that the counter will be yellow. (2)

Solution

$$\begin{aligned} \text{Probability} &= 1 - (0.2 + 0.3 + 0.1) \\ &= 1 - 0.6 \\ &= \underline{\underline{0.4}}. \end{aligned}$$

The bag contains 200 counters.

(b) Work out the number of red counters in the bag. (2)

Solution

$$200 \times 0.2 = \underline{\underline{40 \text{ counters}}}.$$

2. Kate buys 2 lollies and 5 choc ices for £6.50. (3)
Pete buys 2 lollies and 3 choc ices for £4.30.
Work out the cost of one lolly.
Give your answer in pence.

Solution

The price 2 choc ices is

$$6.50 - 4.30 = £2.20$$

which means that the cost

$$\frac{2.20}{2} = £1.10$$

each. Now, 3 choc ices are

$$3 \times 1.10 = £3.30,$$

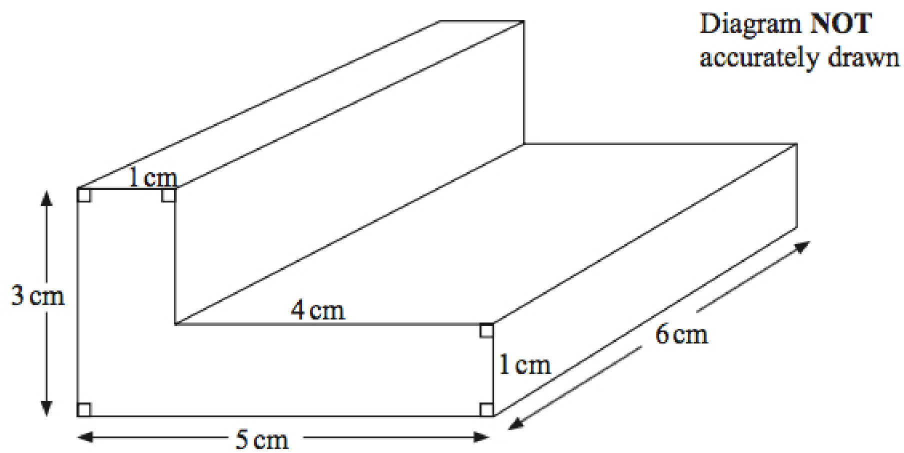
leaving £1 for 2 lollies. Therefore, one lolly costs 50 p.

3. Matthew wants to collect information about the time students take to travel to school. Design a suitable question he could use on a questionnaire. (2)

Solution

A suitable question with a time frame, e.g., "How did you get to school today/last week/last month? Tick the appropriate box."
At least three exhaustive and non-overlapping tick boxes (best defined using inequality notation): for example, $0 < \text{minutes} \leqslant 5$, $5 < \text{minutes} \leqslant 15$, $15 < \text{minutes} \leqslant 25$, $\text{minutes} > 25$.

4. Work out the total surface area of the L-shaped prism. (4)



State the units with your answer.

Solution

The area of the cross-section is

$$\begin{aligned}(3 \times 5) - (2 \times 4) &= 15 - 8 \\ &= 7 \text{ cm}^2.\end{aligned}$$

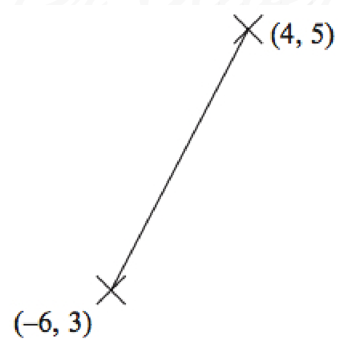
Now,

$$\begin{aligned}1 \times 6 &= 6, \\ 4 \times 6 &= 24, \\ 2 \times 6 &= 12, \\ 1 \times 6 &= 6, \\ 3 \times 6 &= 18, \\ 5 \times 6 &= 30,\end{aligned}$$

and the total surface area of the L-shaped prism is

$$7 + 7 + 6 + 24 + 12 + 6 + 18 + 30 = \underline{\underline{110 \text{ cm}^2}}.$$

5. Work out the coordinates of the midpoint of the line joining the points $(4, 5)$ and $(-6, 3)$. (2)



Solution

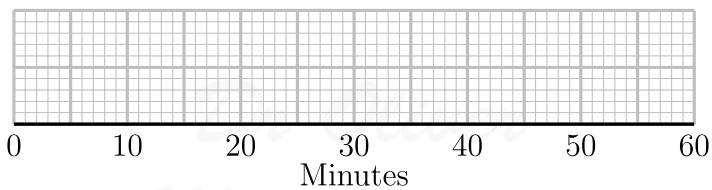
$$\left(\frac{4 + (-6)}{2}, \frac{5 + 3}{2}\right) = \underline{\underline{(-1, 4)}}.$$

6. Mrs Raja set work for the students in her class. (2)

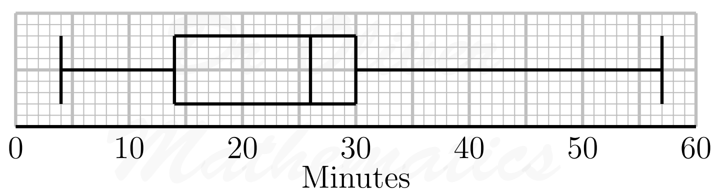
She recorded the time taken, in minutes, for each student to do the work.
She used her results to work out the information in the table.

	Minutes
Shortest time	4
Lower quartile	14
Median	26
Upper quartile	30
Longest time	57

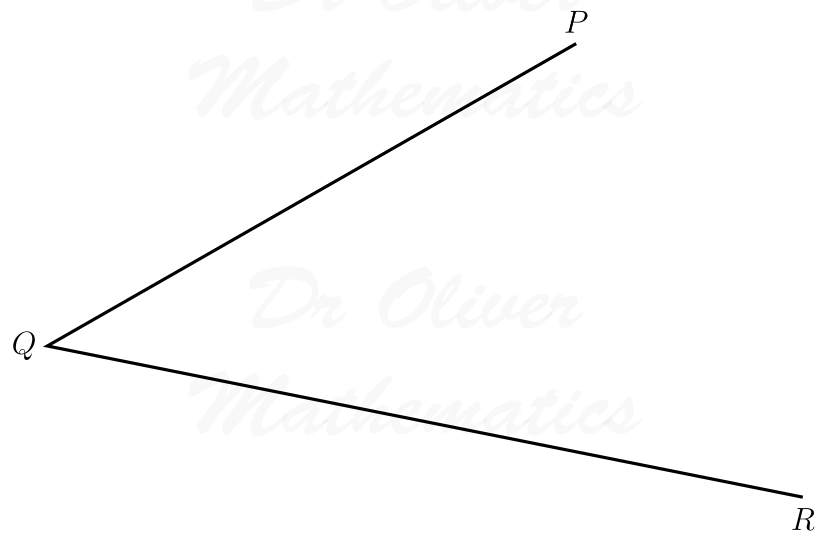
On the grid, draw a box plot to show the information in the table.



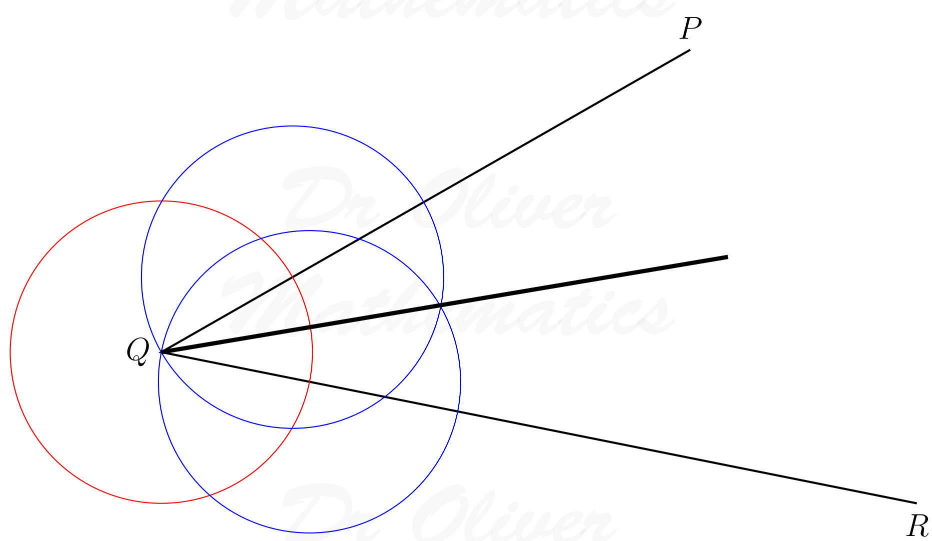
Solution



7. Use ruler and compasses to **construct** the bisector of angle PQR. (2)
You must show all your construction lines.



Solution



8. (a) Write 126 as a product of its prime factors. (2)

Solution

	126
2	63
3	21
3	7
7	1

Hence

$$\begin{aligned} 126 &= 2 \times 3 \times 3 \times 7 \\ &= \underline{\underline{\underline{2 \times 3^2 \times 7}}}. \end{aligned}$$

(b) Find the Highest Common Factor (HCF) of 84 and 126. (2)

Solution

	84
2	42
2	21
3	7
7	1

Hence

$$84 = 2^2 \times 3 \times 7.$$

Finally,

$$\text{HCF}(84, 126) = 2 \times 3 \times 7 = \underline{\underline{\underline{42}}}.$$

9. (a) m is an integer such that $-1 \leqslant m < 4$. (2)
List all the possible values of m.

Solution

$\underline{\underline{\underline{-1, 0, 1, 2, 3}}}.$

(b) (i) Solve the inequality (3)

$$3x \geqslant x + 7.$$

Solution

$$3x \geqslant x + 7 \Rightarrow 2x \geqslant 7$$
$$\Rightarrow \underline{\underline{x \geqslant 3\tfrac{1}{2}}}.$$

(ii) x is a whole number.
Write down the smallest value of x that satisfies

$$3x \geqslant x + 7.$$

Solution

$\underline{\underline{4}}$.

10. (a) Write as a power of 7: (3)

(i) $7^8 \div 7^3$,

Solution

$$7^8 \div 7^3 = 7^{8-3} = \underline{\underline{7^5}}.$$

(ii) $\dfrac{7^2 \times 7^3}{7}$.

Solution

$$\frac{7^2 \times 7^3}{7} = \frac{7^5}{7} = \underline{\underline{7^4}}.$$

(b) Write down the reciprocal of 2. (1)

Solution

$\underline{\underline{\tfrac{1}{2}}}$.

11. (a) Make n the subject of the formula (2)

$$m = 5n - 21.$$

Solution

$$\begin{aligned} m = 5n - 21 &\Rightarrow 5n = m + 21 \\ &\Rightarrow \underline{\underline{n = \frac{m+21}{5}}}. \end{aligned}$$

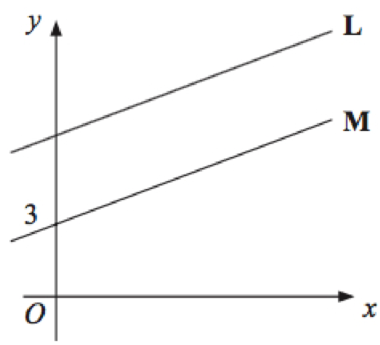
(b) Make p the subject of the formula (3)

$$4(p - 2q) = 3p + 2.$$

Solution

$$\begin{aligned} 4(p - 2q) = 3p + 2 &\Rightarrow 4p - 8q = 3p + 2 \\ &\Rightarrow \underline{\underline{p = 8q + 2}}. \end{aligned}$$

12. The straight line **L** has equation $y = \frac{1}{2}x + 7$. (2)
The straight line **M** is parallel to **L** and passes through the point (0, 3).



Write down an equation for the line **M**.

Solution

An equation for the line **M** is

$$\underline{\underline{y = \tfrac{1}{2}x + 3}}.$$

13. Work out (3)

$$2\tfrac{2}{3} \times 1\tfrac{1}{4}.$$

Give your answer in its simplest form.

Solution

$$\begin{aligned} 2\tfrac{2}{3} \times 1\tfrac{1}{4} &= \tfrac{8}{3} \times \tfrac{5}{4} \\ &= \tfrac{2}{3} \times \tfrac{5}{1} \\ &= \tfrac{10}{3} \\ &= \underline{\underline{3\tfrac{1}{3}}}. \end{aligned}$$

14. Solve the simultaneous equations (3)

$$\begin{aligned} 4x + 2y &= 8 \\ 2x - 5y &= 10. \end{aligned}$$

Solution

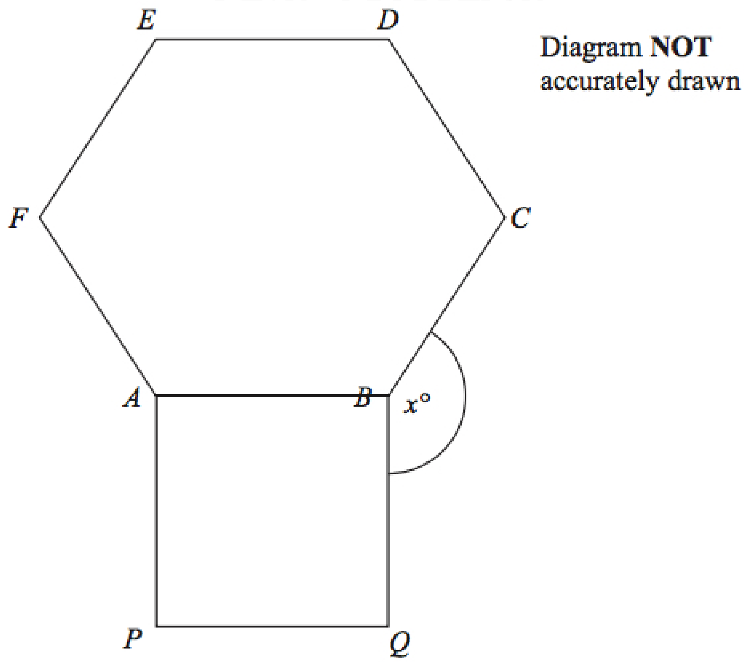
$$\begin{aligned} 4x + 2y &= 8 \quad (1) \\ 2x - 5y &= 10 \quad (2) \end{aligned}$$

Now, do $(1) - 2 \times (2)$:

$$12y = -12 \Rightarrow \underline{\underline{y = -1}}$$

$$\begin{aligned} &\Rightarrow 4x - 2 = 8 \\ &\Rightarrow 4x = 10 \\ &\Rightarrow \underline{\underline{x = 2\tfrac{1}{2}}}. \end{aligned}$$

15. $ABCDEF$ is a regular hexagon and $ABQP$ is a square. (4)
Angle $CBQ = x^\circ$.



Work out the value of x.

Solution

$$\begin{aligned} x &= 360 - 120 - 90 \\ &= \underline{\underline{150}}. \end{aligned}$$

16. An operator took 100 calls at a call centre.
The table gives information about the time (t seconds) it took the operator to answer each call.

Time (t seconds)	Frequency
$0 < t \leqslant 10$	16
$10 < t \leqslant 20$	34
$20 < t \leqslant 30$	32
$30 < t \leqslant 40$	14
$40 < t \leqslant 50$	4

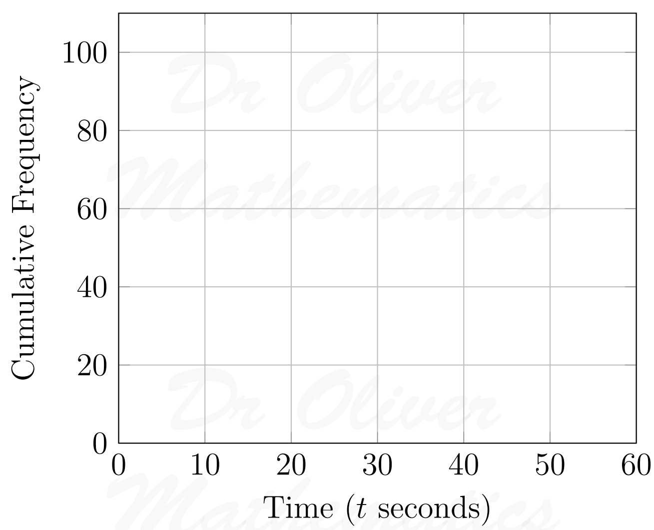
(a) Complete the cumulative frequency table. (1)

Time (t seconds)	Cumulative Frequency
$0 < t \leqslant 10$	16
$0 < t \leqslant 20$	
$0 < t \leqslant 30$	
$0 < t \leqslant 40$	
$0 < t \leqslant 50$	

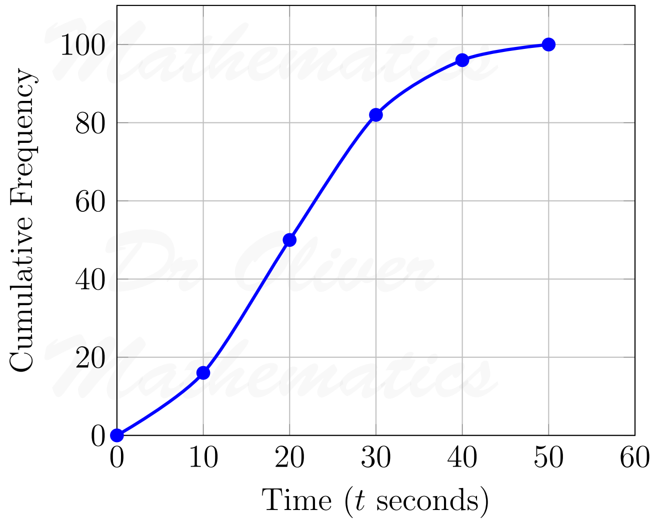
Solution

Time (t seconds)	Cumulative Frequency
$0 < t \leqslant 10$	16
$0 < t \leqslant 20$	$16 + 34 = \underline{\underline{50}}$
$0 < t \leqslant 30$	$50 + 32 = \underline{\underline{82}}$
$0 < t \leqslant 40$	$82 + 14 = \underline{\underline{96}}$
$0 < t \leqslant 50$	$96 + 4 = \underline{\underline{100}}$

(b) On the grid, draw a cumulative frequency graph for your table. (2)

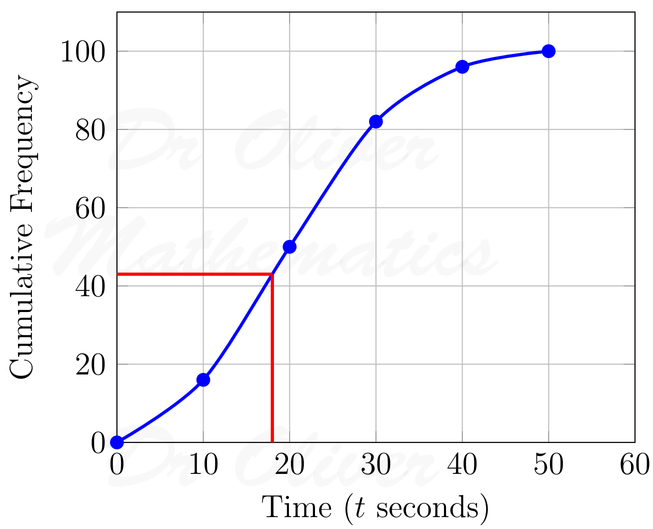


Solution



(c) Use your graph to find an estimate for the number of calls the operator took **more** than 18 seconds to answer. (2)

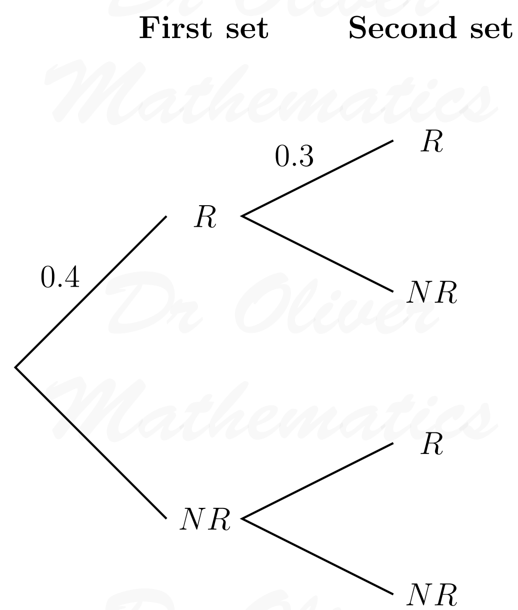
Solution



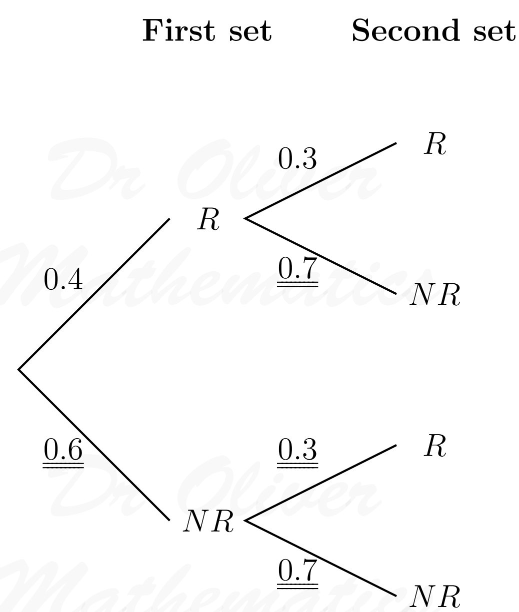
Correct read-off: approximately $100 - 43 = \underline{\underline{57 \text{ calls}}}$.

17. There are two sets of traffic lights on Georgina's route to school.
The probability that the first set of traffic lights will be red is 0.4.
The probability that the second set of traffic lights will be red is 0.3.

(a) Complete the probability tree diagram. (2)



Solution



(b) Work out the probability that both sets of traffic lights will be red. (2)

Solution

$$0.4 \times 0.3 = \underline{\underline{0.12}}.$$

(c) Work out the probability that exactly one set of traffic lights will be red. (3)

Solution

$$\begin{aligned}\text{P(exactly one set of traffic lights will be red)} &= \text{P}(R, NR) + \text{P}(NR, R)\\ &= (0.4 \times 0.7) + (0.6 \times 0.3)\\ &= 0.28 + 0.18\\ &= \underline{\underline{0.46}}.\end{aligned}$$

18. Prove that the recurring decimal (3)

$$0.\dot{4}\dot{5} = \tfrac{15}{33}.$$

Solution

Let $x = 0.\dot{4}\dot{5}$. Then,

$$\begin{aligned} x &= 0.454\,545\,\ldots \quad (1)\\ 100x &= 45.454\,545\,\ldots \quad (2)\end{aligned}$$

Now, do (2) − (1):

$$\begin{aligned}99x = 45 &\Rightarrow x = \tfrac{45}{99}\\ &\Rightarrow x = \tfrac{15\times 3}{33\times 3}\\ &\Rightarrow \underline{\underline{x = \tfrac{15}{33}}},\end{aligned}$$

as required.

19. Expand and simplify (2)

$$(\sqrt{3} - \sqrt{2})(\sqrt{3} + \sqrt{2}).$$

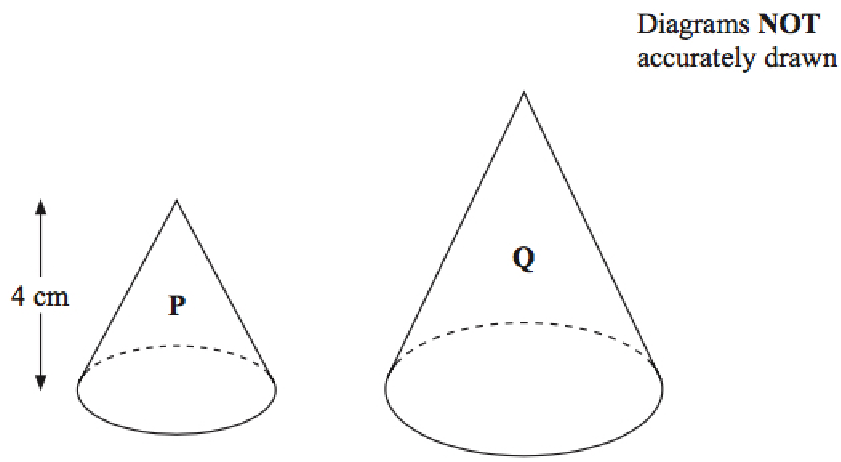
Solution

$\times$	$\sqrt{3}$	$+\sqrt{2}$
$\sqrt{3}$	3	$+\sqrt{6}$
$-\sqrt{2}$	$-\sqrt{6}$	-2

Hence,

$$(\sqrt{3}-\sqrt{2})(\sqrt{3}+\sqrt{2}) = 3 - 2 = \underline{\underline{1}}.$$

20. Two cones, **P** and **Q**, are mathematically similar.



The total surface area of cone **P** is 24 cm^2.
The total surface area of cone **Q** is 96 cm^2.
The height of cone **P** is 4 cm.

(a) Work out the height of cone **Q**. (3)

Solution

The area scale ratio (ASR) is

$$\frac{96}{24} = 4 = 2^2$$

and the length scale ratio (LSR) is 2. Finally, the height of cone **Q** is

$$4 \times 2 = \underline{\underline{8 \text{ cm}}}.$$

The volume of cone **P** is 12 cm^3.

(b) Work out the volume of cone **Q**. (2)

Solution

The volume scale ratio (VSR) is $2^3 = 8$ and the volume of cone **Q** is

$$12 \times 8 = \underline{\underline{96 \text{ cm}^3}}.$$

21. (a) Expand (2)

$$x(3 - 2x^2).$$

Solution

$$x(3 - 2x^2) = \underline{\underline{3x - 2x^3}}.$$

(b) Factorise completely (2)

$$12xy + 4x^2.$$

Solution

$$12xy + 4x^2 = \underline{\underline{4x(3y + x)}}.$$

(c) Simplify (2)

$$\frac{20a^2}{4ab^2}.$$

Solution

$$\frac{20a^2}{4ab^2} = \underline{\underline{\frac{5a}{b^2}}}.$$

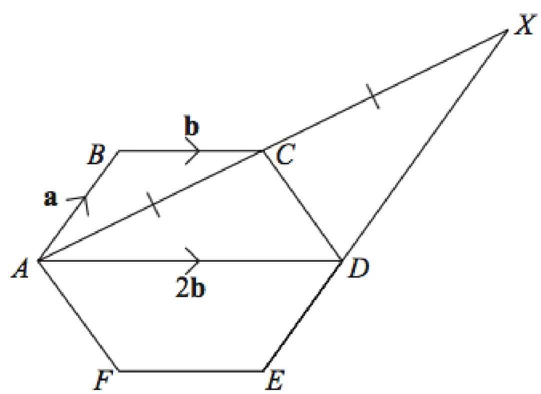
(d) Simplify (2)

$$\frac{x - 3}{x^2 - 9}.$$

Solution

$$\frac{x-3}{x^2-9} = \frac{x-3}{(x-3)(x+3)} = \underline{\underline{\frac{1}{x+3}}}.$$

22. $ABCDEF$ is a regular hexagon.



$\overrightarrow{AB} = \mathbf{a}$.
$\overrightarrow{BC} = \mathbf{b}$.
$\overrightarrow{AD} = 2\mathbf{b}$.

(a) Find the vector $\overrightarrow{AC}$ in terms of $\mathbf{a}$ and $\mathbf{b}$. (1)

Solution

$$\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC} = \underline{\underline{\mathbf{a}+\mathbf{b}}}.$$

$\overrightarrow{AC} = \overrightarrow{CX}$.

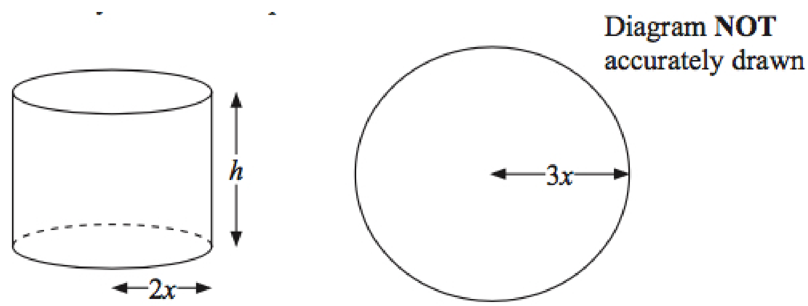
(b) Prove that AB is parallel to DX. (3)

Solution

$$\begin{aligned}\overrightarrow{DX} &= \overrightarrow{DA} + \overrightarrow{AX}\\ &= \overrightarrow{DA} + 2\overrightarrow{AC}\\ &= -2\mathbf{b} + 2(\mathbf{a}+\mathbf{b})\\ &= 2\mathbf{a}\\ &= 2\overrightarrow{AB}\end{aligned}$$

and they are parallel.

23. The diagram shows a cylinder and a sphere. (3)



The radius of the base of the cylinder is $2x$ cm and the height of the cylinder is h cm.
The radius of the sphere is $3x$ cm.
The volume of the cylinder is equal to the volume of the sphere.
Express h in terms of x.
Give your answer in its simplest form.

Solution

The volume of the cylinder is

$$\pi \times (2x)^2 \times h = 4\pi x^2 h$$

and the volume of the sphere is

$$\tfrac{4}{3} \times \pi \times (3x)^3 = \tfrac{108}{3}\pi x^3.$$

Now,

$$4\pi x^2 h = \tfrac{108}{3}\pi x^3 \Rightarrow \underline{\underline{h = 9x}}.$$

24. (a) Expand and simplify (2)

$$n^2 + (n+1)^2.$$

Solution

$$\begin{aligned} n^2 + (n+1)^2 &= n^2 + (n^2 + 2n + 1) \\ &= \underline{\underline{2n^2 + 2n + 1}}. \end{aligned}$$

n is a whole number.

(b) Prove that (2)

$$n^2 + (n+1)^2$$

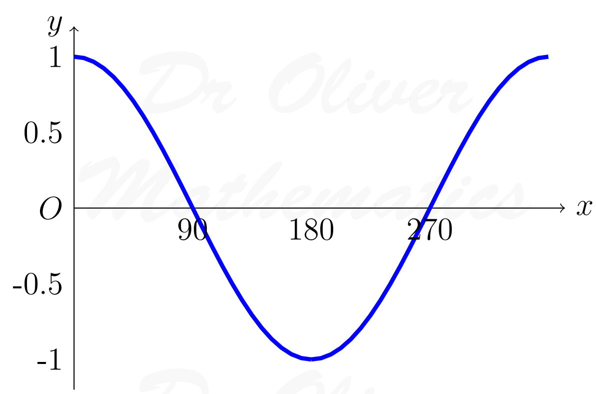
is always an odd number.

Solution

$$n^2 + (n+1)^2 = 2(n^2 + n) + 1,$$

and, since n is a whole number, $n^2 + (n+1)^2$ is always an $\underline{\underline{\text{odd number}}}$.

25. Here is a graph of the curve $y = \cos x^\circ$ for $0 \leqslant x \leqslant 360$. (2)

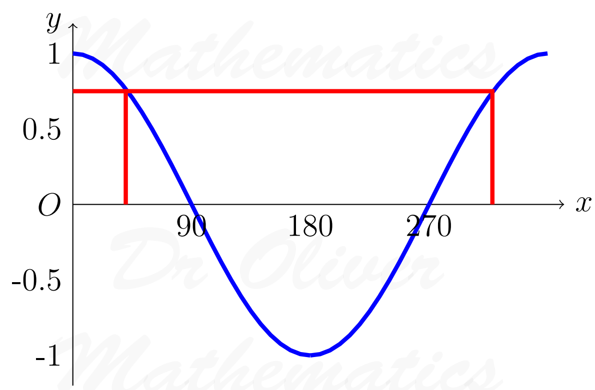


Use the graph to solve

$$\cos x^\circ = 0.75$$

for $0 \leqslant x \leqslant 360$.

Solution



Correct read-offs: approximately 41 and 319.

26. For all values of x,

$$x^2 - 6x + 15 \equiv (x - p)^2 + q.$$

(a) Find the value of p and the value of q. (2)

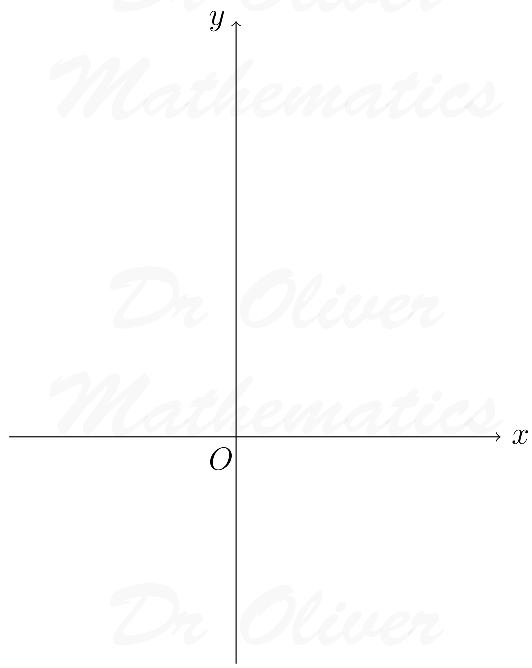
Solution

$$\begin{aligned} x^2 - 6x + 15 &\equiv (x^2 - 6x + 9) + 6 \\ &\equiv (x - 3)^2 + 6; \end{aligned}$$

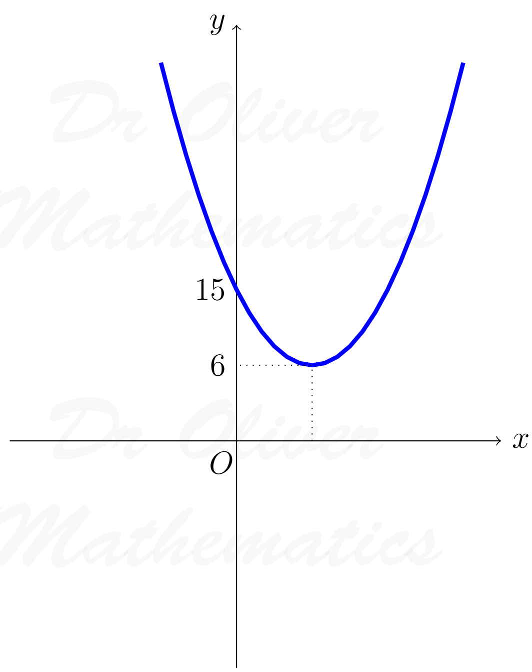
hence, $p = 3$ and $q = 6$

(b) On the axes, draw a sketch of the graph (2)

$$y = x^2 - 6x + 15.$$

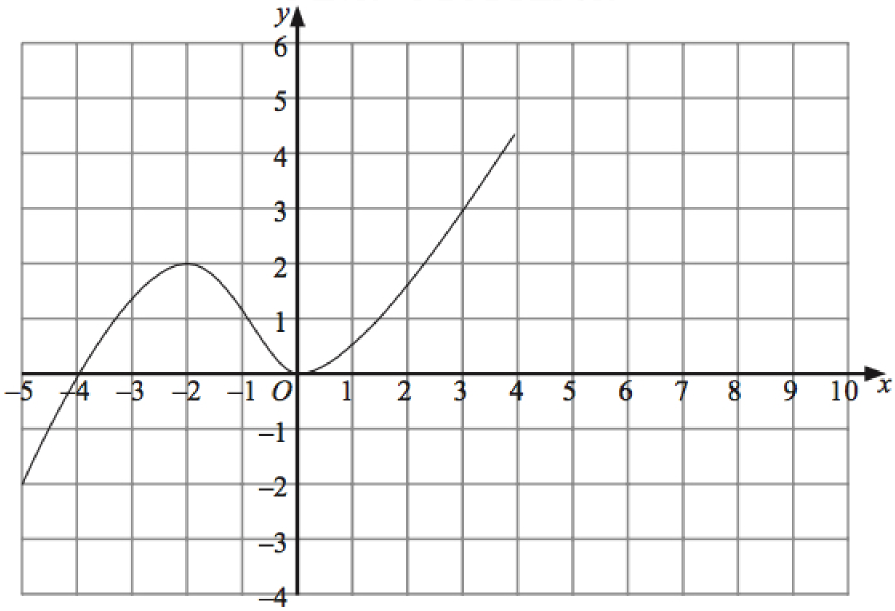


Solution



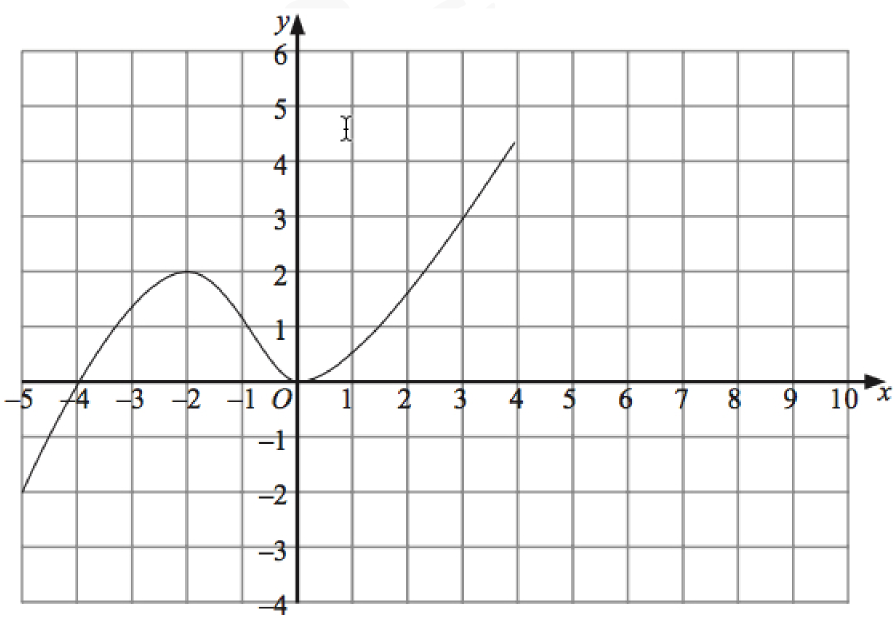
27. The graph of $y = \mathrm{f}(x)$ is shown on the grids.

  (a) On this grid, sketch the graph of $y = \mathrm{f}(x) + 2$. (2)



> **Solution**
> A "cubic graph" that goes through $(-4, 2)$, $(-2, 4)$, $(0, 2)$, and $(3, 5)$.

(b) On this grid, sketch the graph of $y = -\mathrm{f}(x)$. (2)



Solution

A "cubic graph" that goes through $(-4, 0)$, $(-2, -2)$, $(0, 0)$, and $(3, -3)$.