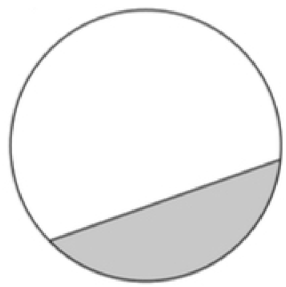
Dr Oliver Mathematics
AQA GCSE Mathematics
2018 June Paper 2: Calculator
1 hour 30 minutes

The total number of marks available is 80.
You must write down all the stages in your working.

1. Here is a circle. (1)



Circle the word that describes the shaded part.

segment chord sector arc

Solution

segment chord sector arc

2. Circle the number that is in standard form. (1)

$0.25 \times 10^4 \quad 6 \times 10^7 \quad 38 \times 10^{-3} \quad 4 \times 10^{\frac{1}{2}}$

Solution

$0.25 \times 10^4 \quad 6 \times 10^7 \quad 38 \times 10^{-3} \quad 4 \times 10^{\frac{1}{2}}$

3. y is $1\frac{1}{2}$ times x. (1)

Circle the ratio that is equivalent to $y : x$.

$$2:5 \quad 5:2 \quad 3:2 \quad 2:3$$

Solution

Well,

$$\begin{aligned} y = 1\tfrac{1}{2}x &\Rightarrow 3y = 2x \\ &\Rightarrow 2x = 3y \end{aligned}$$

so

$$2:5 \quad 5:2 \quad \underline{\underline{3:2}} \quad 2:3$$

4. Work out 40 as a percentage of 10. (1)

Circle your answer.

$$4\% \quad 25\% \quad 300\% \quad 400\%$$

Solution

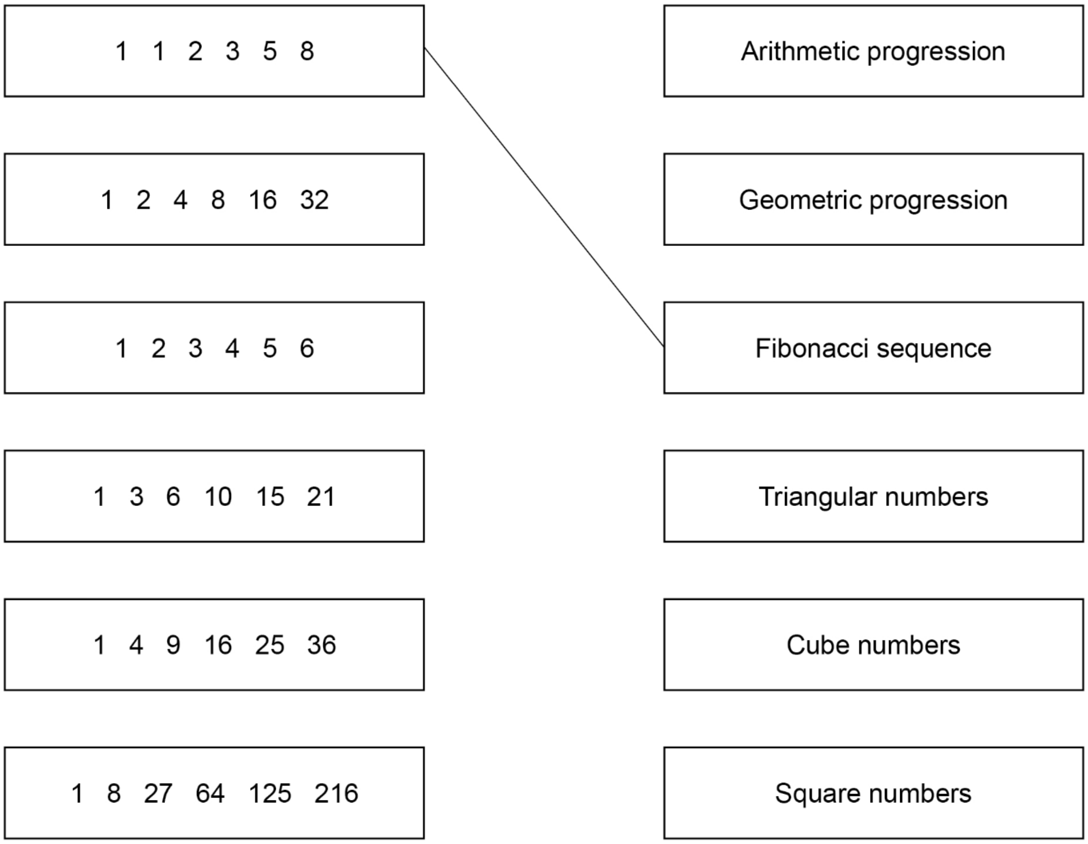
Well,

$$\begin{aligned} \frac{40}{10} \times 100\% &= 4 \times 100\% \\ &= 400\% \end{aligned}$$

so

$$4\% \quad 25\% \quad 300\% \quad \underline{\underline{400\%}}$$

5. Match each sequence to its description. (4)
One has been done for you.



Solution

Well,

$$1, 2, 4, 8, 16, 32 \to \underline{\underline{\text{Geometric progression}}}$$

$$1, 2, 3, 4, 5, 6 \to \underline{\underline{\text{Arithmetic progression}}}$$

$$1, 3, 6, 10, 15, 21 \to \underline{\underline{\text{Triangular numbers}}}$$

$$1, 4, 9, 16, 25, 36 \to \underline{\underline{\text{Square numbers}}}$$

$$1, 8, 27, 64, 125, 216 \to \underline{\underline{\text{Cube numbers}}}$$

6. The table shows information about the population of a city. (3)

Population in 2001	Population in 2011
420 000	480 000

Liam claims, "From 2011 to 2021, the population of the city will increase by the same percentage as from 2001 to 2011."

He works out,

$$\begin{aligned}\text{population increase from 2001 to 2011} &= 480\,000 - 420\,000 \\ &= 60\,000\end{aligned}$$

$$\begin{aligned}\text{population in 2021} &= 480\,000 + 60\,000 \\ &= 540\,000.\end{aligned}$$

Does the population of 540 000 match his claim?
You must show your working.

Solution

Well,

$$\frac{60\,000}{420\,000} \times 100\% = 14\tfrac{2}{7}\%$$

and

$$\frac{60\,000}{480\,000} \times 100\% = 12\tfrac{1}{2}\%.$$

The percentages are not equal.
So, no: the population of 540 000 does not match his claim.

7. On three days, Ali throws darts at a target.
Here are his results.

	Number of throws	Number of hits	Number of misses
Monday	20	15	5
Tuesday	30	22	8
Wednesday	40	17	23
Overall	90	54	36

(a) Work out **two** different estimates for the probability of Ali hitting the target. (2)

Solution
Overall:
$$\frac{54}{90} = \underline{\underline{0.6}}.$$
Monday:
$$\frac{15}{20} = \underline{\underline{0.75}}.$$
(Monday is bigger than Tuesday.)

(b) Which of your two answers is the better estimate for the probability of Ali hitting the target? (1)
Give a reason for your answer.

Solution
$\underline{\underline{0.6}}$ because it is based on the $\underline{\underline{\text{overall}}}$ throws and hits.

8. Theo starts with savings of £18. (3)
James starts with no savings.

Each week from now, Theo will save £4.50 and James will save £4.

In how many weeks will Theo and James have savings in the ratio 15 : 8?

Solution
Let x be the weeks. Then
$$\begin{aligned} 18 + 4.5x : 4x = 15 : 8 &\Rightarrow \frac{18 + 4.5x}{4x} = \frac{15}{8} \\ &\Rightarrow 8(18 + 4.5x) = 15(4x) \\ &\Rightarrow 144 + 36x = 60x \\ &\Rightarrow 144 = 24x \\ &\Rightarrow x = 6; \end{aligned}$$
so, it will take $\underline{\underline{6 \text{ weeks}}}$.

9. The length of each side of a regular pentagon is 8.4 cm to 1 decimal place.

(a) Complete the error interval for the length of one side: (2)

$$\ldots\ldots\ldots \text{ cm} \leqslant \text{length} < \ldots\ldots\ldots \text{ cm}.$$

Solution

$$8.35 \text{ cm} \leqslant \text{length} < 8.45 \text{ cm}.$$

(b) Complete the error interval for the perimeter: (1)

$$\ldots\ldots\ldots \text{ cm} \leqslant \text{length} < \ldots\ldots\ldots \text{ cm}.$$

Solution

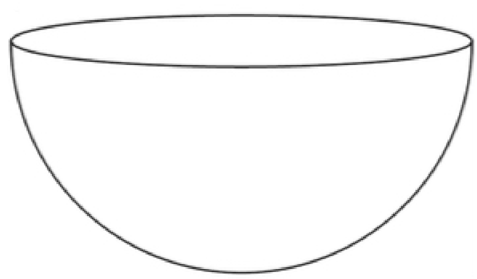
Well,

$$5 \times 8.35 = 41.75 \text{ and } 5 \times 8.45 = 42.25$$

which leads to

$$41.75 \text{ cm} \leqslant \text{length} < 42.25 \text{ cm}.$$

10. A container is a hemisphere of radius 30 cm. (3)



Sand fills the container at a rate of 4 000 cm^3 per minute.

Volume of a sphere = $\frac{4}{3}\pi r^3$ where r is the radius

Does it take **less than** a quarter of an hour to fill the container?
You **must** show your working.

Solution

Well,

$$\begin{aligned}\text{volume} &= \tfrac{1}{2} \times \tfrac{4}{3} \times \pi \times 30^3 \\ &= 18\,000\pi \text{ cm}^3.\end{aligned}$$

Now, the sand fills the container at a rate of

$$\begin{aligned}4\,000 \text{ cm}^3 \text{ per minute} &= \frac{4\,000 \text{ cm}^3}{1 \text{ mins}} \\ &= \frac{4\,000 \text{ cm}^3}{60 \text{ s}} \\ &= \tfrac{200}{3} \text{ cm}^3\,\text{s}^{-1}\end{aligned}$$

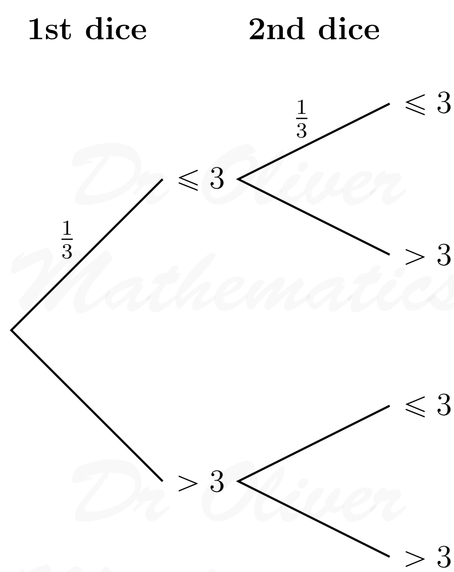
and

$$\begin{aligned}\text{fill the container} &= \frac{18\,000\pi \text{ cm}^3}{\frac{200}{3} \text{ cm}^3\,\text{s}^{-1}} \\ &= 270\pi \text{ s} \\ &= \tfrac{9}{2}\pi \text{ or } 14.137\ldots \text{ mins;}\end{aligned}$$

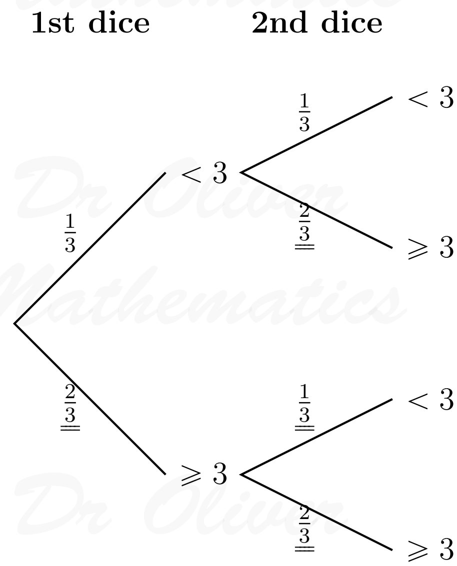
hence, it does less than a quarter of an hour to fill the container

11. Two ordinary fair dice are rolled.

(a) Complete the tree diagram. (1)



Solution



(b) Work out the probability that **both** dice land on a number less than 3 (1)

Solution

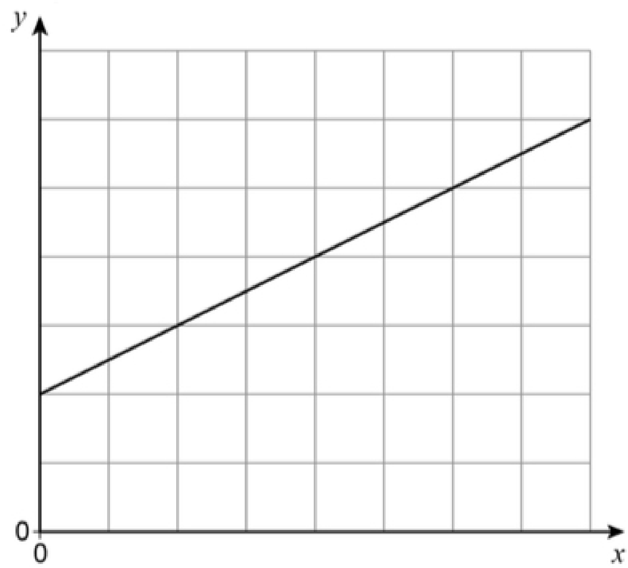
$$\tfrac{1}{3} \times \tfrac{1}{3} = \underline{\underline{\tfrac{1}{9}}}.$$

(c) Work out the probability that **exactly one** of the dice lands on a number less than 3. (2)

Solution

$$\begin{aligned}
\text{P(exactly one)} &= \text{P}(< 3, \geqslant 3) + \text{P}(\geqslant 3, < 3) \\
&= (\tfrac{1}{3} \times \tfrac{2}{3}) + (\tfrac{2}{3} \times \tfrac{1}{3}) \\
&= \tfrac{2}{9} + \tfrac{2}{9} \\
&= \underline{\underline{\tfrac{4}{9}}}.
\end{aligned}$$

12. A straight line is drawn on the centimetre grid.



Fay assumes that the scale is 1 cm represents 1 unit.

(a) Use her assumption to work out the gradient of the line. (1)

Solution

$$\text{Gradient} = \frac{6-2}{8-0} = \frac{4}{8} = \underline{\underline{\tfrac{1}{2}}}.$$

In fact, the scale is 1 cm represents 2 units.

(b) Which statement is correct? (1)
Tick **one** box.

☐ The answer to part (a) is too big

☐ The answer to part (a) stays the same

☐ The answer to part (a) is too small

> **Solution**
> The second box: The answer to part (a) is stays the same because we have doubled.

13. Show that, for $x \neq -1$, (3)

$$\frac{8x^2-8}{4x+4}$$

simplifies to the form

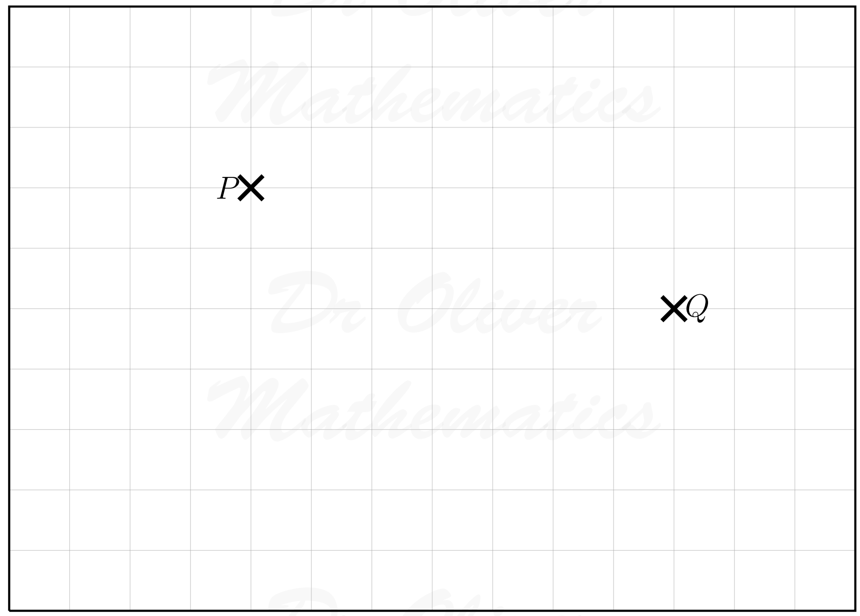
$$ax+b,$$

where a and b are integers.

> **Solution**
>
> Difference of two squares:
>
> $$\left.\begin{array}{ll}\text{add to:} & 0\\ \text{multiply to:} & -1\end{array}\right\} -1,\ +1$$
>
> so
>
> $$\begin{aligned}\frac{8x^2-8}{4x+4} &= \frac{8(x^2-1)}{4(x+1)}\\ &= \frac{8(x+1)(x-1)}{4(x+1)}\\ &= 2(x-1)\\ &= \underline{\underline{2x-2}};\end{aligned}$$
>
> hence, $\underline{\underline{a=2}}$ and $\underline{\underline{b=-2}}$.

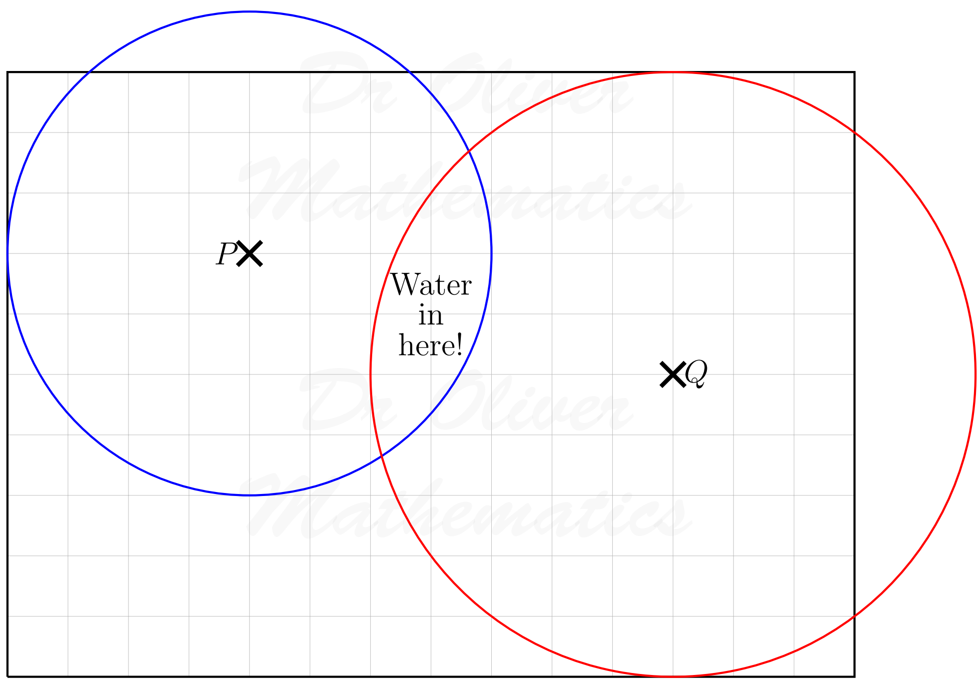
14. The scale drawing represents a garden. (2)

- Water from a sprinkler at P reaches up to 20 metres from P.
- Water from a sprinkler at Q reaches up to 25 metres from Q.
- **Scale**: 1 cm represents 5 m.



Using a pair of compasses, show the region that water from **both** sprinklers reaches.

Solution



15. 100 men and 100 women took a test. (1)

	Median	Interquartile range	Range
Men	28	7.5	31
Women	30	9	37

Using this data, which statement **must** be true?
Tick **one** box.

☐ Men had a higher average score than women

☐ Men had more consistent scores than women

☐ A woman had the highest score

☐ A man had the lowest score

Solution

Tick the second box: Men had more consistent scores than women.

16. • Some concrete has volume 3.8 m^3.
• The density of the concrete is 2400 kg/m^3.

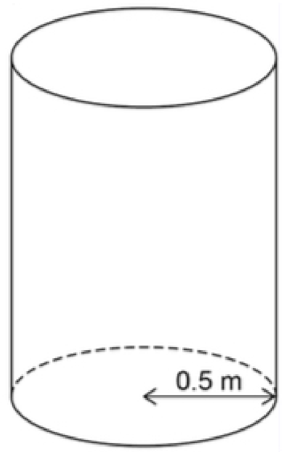
(a) Work out the mass of the concrete. (2)

Solution

Well,

$$\text{density} = \frac{\text{mass}}{\text{volume}} \Rightarrow 2\,400 = \frac{\text{mass}}{3.8}$$
$$\Rightarrow \text{mass} = 2\,400 \times 3.8$$
$$\Rightarrow \text{mass} = \underline{\underline{9\,120 \text{ kg}}}.$$

The 3.8 m^3 of concrete is made into the shape of a cylinder.
The base has radius 0.5 metres.



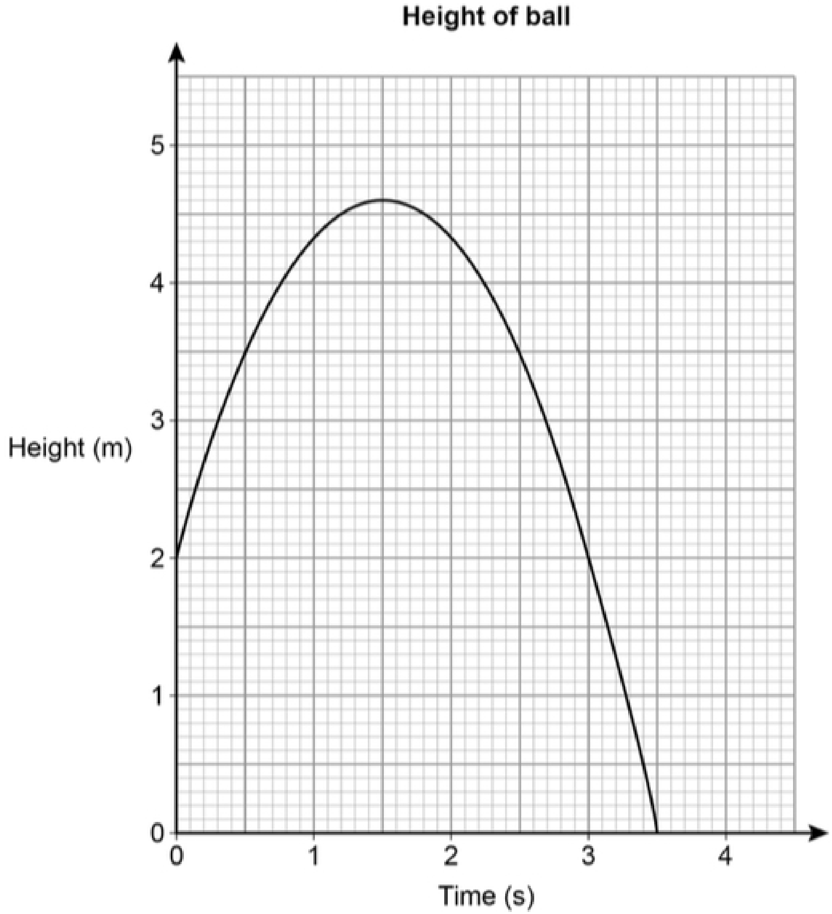
(b) Work out the height of the cylinder. (2)

Solution

Let h m be the height. Then

$$V = \pi r^2 h \Rightarrow 3.8 = \pi \times 0.5^2 \times h$$
$$\Rightarrow h = \frac{3.8}{0.25\pi}$$
$$\Rightarrow h = 4.838\,310\,27 \text{ (FCD)}$$
$$\Rightarrow \underline{\underline{h = 4.84 \text{ (3 sf)}}}.$$

17. A ball is thrown vertically upwards.
The graph shows the height of the ball above the ground after it is thrown.



(a) For how many seconds is the ball at a height of **more than** 2 metres? (1)

Solution

$$3 - 0 = \underline{\underline{3 \text{ seconds}}}.$$

(b) After how many seconds is the ball at instantaneous rest when it is in the air? (1)

Solution

$\underline{\underline{1.5 \text{ seconds}}}$.

(c) Work out the average speed of the ball when it is moving downwards. (2)

Solution

Moving downwards,

$$\begin{aligned}\text{average speed} &= \frac{4.6-0}{3.5-1.5}\\ &= \frac{4.6}{2}\\ &= \underline{\underline{2.3 \text{ m/s}}}.\end{aligned}$$

18. The solution of (1)

$$3^x = 300$$

lies between two consecutive integers.

Work out the two integers.

Solution

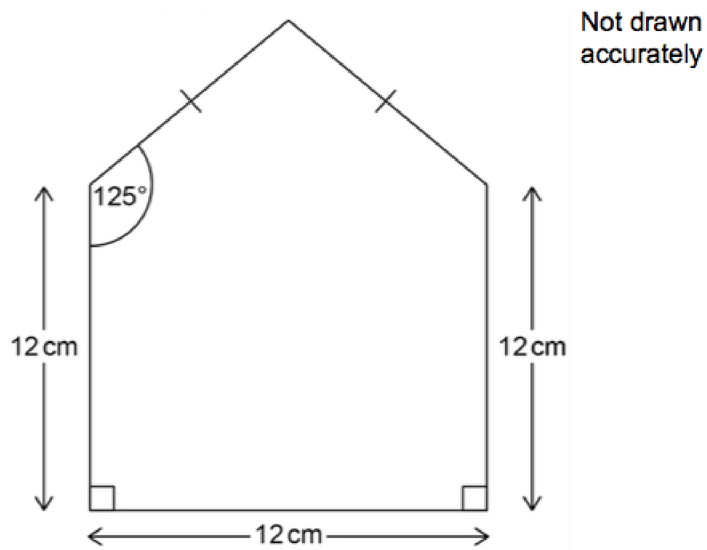
Now,

$$\begin{aligned}3^1 &= 3\\ 3^2 &= 9\\ 3^3 &= 27\\ 3^4 &= 81\\ 3^5 &= 243\\ 3^6 &= 729\end{aligned}$$

and the answer is

$$\underline{\underline{x = 5}} \text{ and } \underline{\underline{x = 6}}.$$

19. A pentagon is made from a square and an isosceles triangle. (4)



Work out the perimeter of the pentagon.

Solution

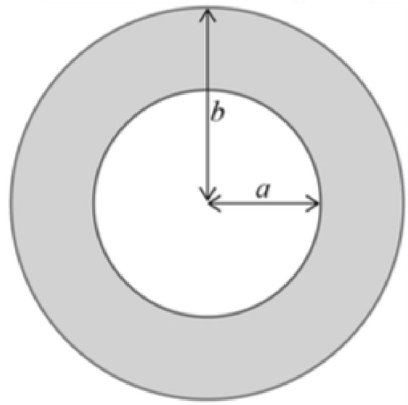
The isosceles part gives angles of 35°, 35°, and 110°.
Now, let h cm be the missing side. Then,

$$\cos = \frac{\text{adj}}{\text{hyp}} \Rightarrow \cos 35^\circ = \frac{6}{h}$$
$$\Rightarrow h = \frac{6}{\cos 35^\circ}$$

and

$$\text{perimeter} = 3(12) + 2\left(\frac{6}{\cos 35^\circ}\right)$$
$$= 50.649\,295\,07 \text{ (FCD)}$$
$$= \underline{\underline{50.6 \text{ cm (3 sf)}}}.$$

20. Here is an inflated swimming ring with dimensions in centimetres. (3)



The volume of the ring, V cm^3, is given by

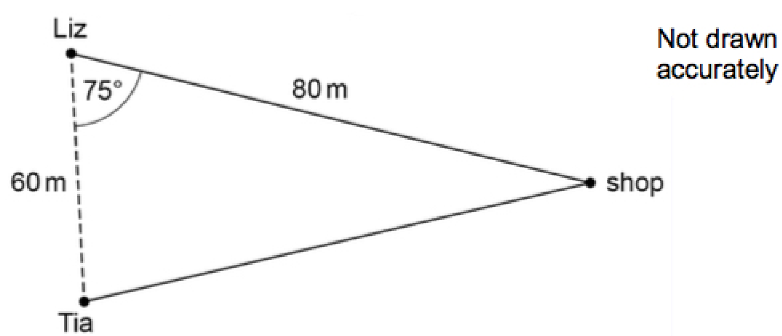
$$V = 0.25\,\pi^2(b-a)^2(b+a).$$

Work out the volume when $a = 20$ and $b = 30$.
Give your answer to 3 significant figures.

Solution

$$\begin{aligned}\text{Volume} &= 0.25\,\pi^2(30-20)^2(30+20)\\ &= 0.25\,\pi^2(10)^2(50)\\ &= 12\,337.005\,5 \text{ (FCD)}\\ &= \underline{\underline{12\,300 \text{ cm}^3 \text{ (3 sf)}}}.\end{aligned}$$

21. Liz and Tia are walking towards a shop along different straight paths.
The diagram shows their positions at 2 pm.



Assume they walk at the same speed.

(a) Who will arrive at the shop first? (3)
You **must** show your working.

Solution
We will denote L, T, and S as Liz, Tia, and the shop respectively. Now, the cosine rule:

$$\begin{aligned} & ST^2 = LT^2 + LS^2 - 2 \cdot LT \cdot LS \cdot \cos SLT \\ \Rightarrow\ & ST^2 = 60^2 + 80^2 - 2 \cdot 60 \cdot 80 \cdot \cos 75^\circ \\ \Rightarrow\ & ST^2 = 7\,515.337\,167 \text{ (FCD)} \\ \Rightarrow\ & ST = 86.691\,044\,33 \text{ (FCD)} \\ >\ & LS, \end{aligned}$$

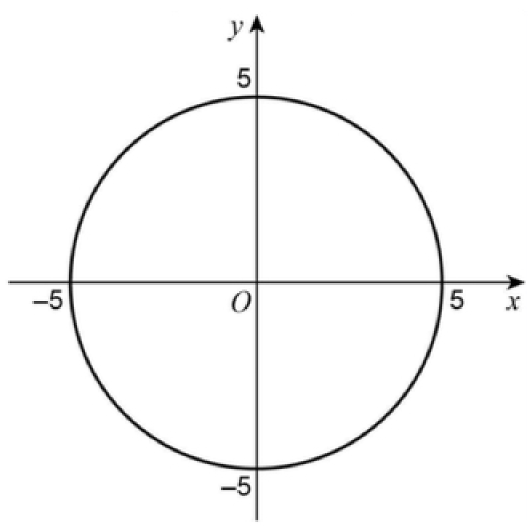
so Liz will arrive first.

In fact, Liz walks at a faster speed than Tia.

(b) How does this affect the answer to part (a)? (1)

Solution
No change: Liz will arrive first because her speed is faster.

22. A circle, centre O, passes through $(5, 0)$. (1)



What is the equation of the circle?

Circle your answer.

$$x^2 + y^2 = 25 \quad x^2 + y^2 = 5 \quad x^2 + y^2 = 10 \quad x^2 + y^2 = 100$$

Solution

$$\underline{\underline{x^2 + y^2 = 25}} \quad x^2 + y^2 = 5 \quad x^2 + y^2 = 10 \quad x^2 + y^2 = 100$$

23. Solids X and Y are similar. (3)

- X has volume 64 cm^3.
- Y has volume 343 cm^3.

The surface area of X is 176 cm^2.

Work out the surface area of Y.

Solution

The volume scale ratio (VSR) is

$$64 : 343 = 4^3 : 7^3,$$

the length scale ratio (LSR) is

$$4 : 7,$$

and the area scale ratio (ASR) is

$$4^2 : 7^2 = 16 : 49.$$

Finally,

$$\begin{aligned} \text{surface area of } Y &= 176 \times \frac{49}{16} \\ &= \underline{\underline{539 \text{ cm}^2}}. \end{aligned}$$

24. A tank is a cuboid measuring 50 cm by 35 cm by 20 cm. (4)
All lengths are to the **nearest centimetre**.

A container has a capacity of exactly 34 litres.
1 litre = 1 000 cm^3.

Which has the greater capacity?
Tick **one** box.

☐ Tank ☐ Container ☐ Cannot tell

Show working to support your answer.

Solution

Now,

$$\begin{aligned} \text{tank}_{\text{smallest}} &= 49.5 \times 34.5 \times 19.5 \\ &= 33\,301.2 \text{ cm}^3 \end{aligned}$$

and

$$\begin{aligned} \text{tank}_{\text{largest}} &= 50.5 \times 35.5 \times 20.5 \\ &= 36\,751.6 \text{ cm}^3. \end{aligned}$$

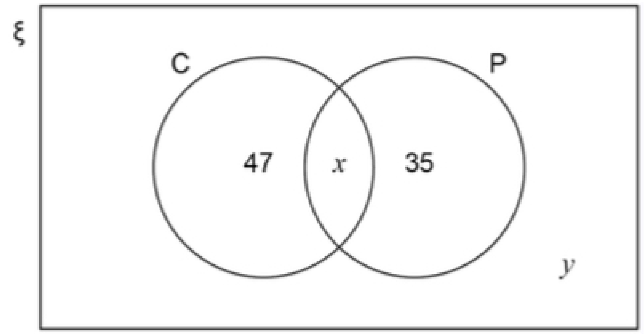
Next, the container has a capacity of

$$34 \text{ litres} = 34\,000 \text{ cm}^3.$$

Hence, we cannot tell.

25. The Venn diagram shows some information about 150 students. (4)

- $\mathscr{E}$ = 150 students.
- C = students who study Chemistry.
- P = students who study Physics.



The probability that a Physics student, chosen at random, also studies Chemistry is $\frac{5}{12}$.

One of the 150 students is chosen at random.

Work out the probability that the student does **not** study either Chemistry or Physics.

Solution

Now,

$$\frac{x}{35+x} = \tfrac{5}{12} \Rightarrow (x)(12) = 5(35+x)$$
$$\Rightarrow 12x = 175 + 5x$$
$$\Rightarrow 7x = 175$$
$$\Rightarrow x = 25.$$

Hence,

$$y = 150 - (47 + 25 + 35)$$
$$= 43$$

and

$$\text{P(does } \textbf{not} \text{ study either Chemistry or Physics)} = \underline{\underline{\tfrac{43}{150}}}.$$

26. A curve has equation (4)

$$y = 4x^2 + 5x + 3.$$

A line has equation

$$y = x + 2.$$

Show that the curve and the line have **exactly** one point of intersection.
Do **not** use a graphical method.

Solution

$$4x^2 + 5x + 3 = x + 2 \Rightarrow 4x^2 + 4x + 1 = 0$$

$$\left.\begin{array}{lr} \text{add to:} & +4 \\ \text{multiply to:} & (+4) \times (+1) = +4 \end{array}\right\} +2,\ +2$$

$$\begin{aligned} &\Rightarrow (2x+1)^2 = 0 \\ &\Rightarrow 2x + 1 = 0 \\ &\Rightarrow 2x = -1 \\ &\Rightarrow x = -\tfrac{1}{2}. \end{aligned}$$

But we have just one – but repeated – solution.
Hence, the curve and the line have exactly one point of intersection.

27. Prove algebraically that (3)

$$2.7\dot{5}$$

converts to the fraction

$$\frac{124}{45}.$$

Solution

Let $x = 2.7\dot{5}$. Then

$$\begin{aligned} 10x &= 27.\dot{5} \quad (1) \\ 100x &= 275.\dot{5} \quad (2). \end{aligned}$$

Do (2) – (1):

$$\begin{aligned} 90x = 248 \Rightarrow x &= \frac{248}{90} \\ \Rightarrow x &= \frac{124 \times 2}{45 \times 2} \\ \Rightarrow x &= \frac{124}{45}, \end{aligned}$$

as required.

28.

$$\mathrm{f}(x) = 5 - x \text{ and } \mathrm{g}(x) = 3x + 7.$$

(a) Simplify (3)

$$\mathrm{f}(2x) + \mathrm{g}(x - 1).$$

Solution

Well,

$$\begin{aligned} \mathrm{f}(2x) + \mathrm{g}(x-1) &= [5 - (2x)] + [3(x-1) + 7] \\ &= 5 - 2x + 3x - 3 + 7 \\ &= \underline{\underline{x + 9}}. \end{aligned}$$

(b) Solve (3)

$$\mathrm{g}^{-1}(x) = 2x.$$

Solution

Now,

$$\begin{aligned} y = 3x + 7 &\Rightarrow y - 7 = 3x \\ &\Rightarrow \frac{y-7}{3} = x \end{aligned}$$

and so

$$\mathrm{g}^{-1}(x) = \frac{x-7}{3}.$$

Finally,

$$\begin{aligned} \mathrm{g}^{-1}(x) = 2x &\Rightarrow \frac{x-7}{3} = 2x \\ &\Rightarrow x - 7 = 6x \\ &\Rightarrow -7 = 5x \\ &\Rightarrow \underline{\underline{x = -1\tfrac{2}{5}}}. \end{aligned}$$