Dr Oliver Mathematics
AQA GCSE Mathematics
2016 November Paper 1: Non-Calculator
1 hour 30 minutes

The total number of marks available is 70.
You must write down all the stages in your working.

1. Solve (3)

$$5w - 3 = 3w + 15.$$

Solution

$$5w - 3 = 3w + 15 \Rightarrow 2w = 18$$
$$\Rightarrow \underline{\underline{w = 9}}.$$

2. A spinner has four sections A, B, C, and D. (2)
The table shows the probabilities of the spinner landing on A, B, or C.

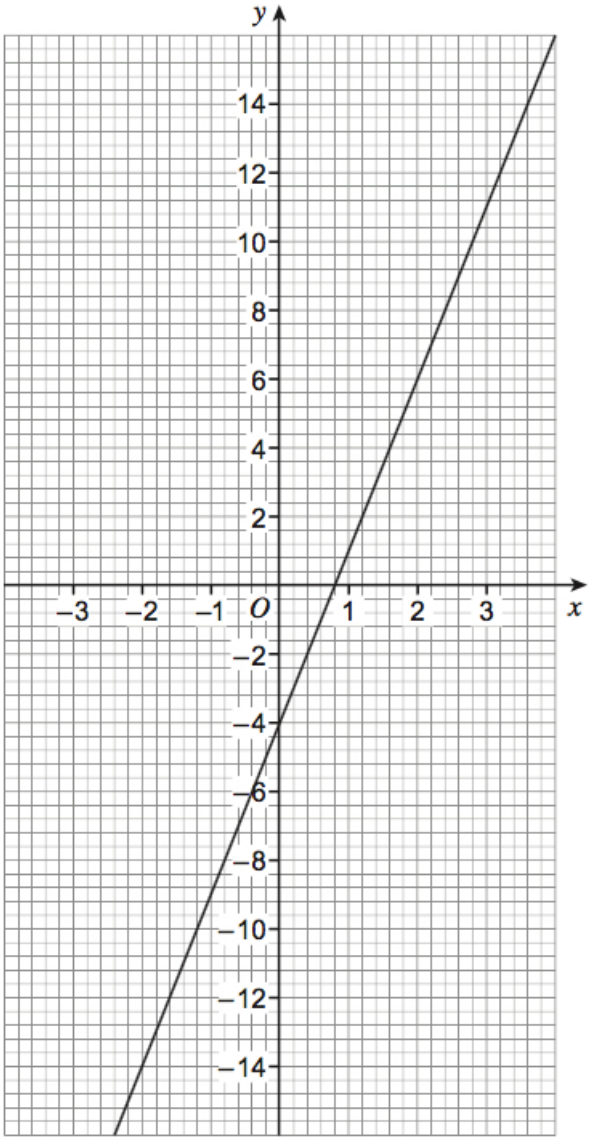
Outcome	A	B	C	D
Probability	0.2	0.3	0.15	

Work out the probability of landing on D.

Solution

$$\begin{aligned}\text{Probability} &= 1 - (0.2 + 0.3 + 0.15) \\ &= 1 - 0.65 \\ &= \underline{\underline{0.35}}.\end{aligned}$$

3. Here is a straight-line graph.



(a) Use the graph to work out the value of x when $y = 8$. (1)

Solution

Correct read-off: approximately $x = 2.4$.

(b) Work out the gradient of the line. (3)

Solution

The line goes through $(0, -4)$ and $(0.8, 0)$:

$$\begin{aligned}\text{gradient} &= \frac{-4-0}{0-0.8} \\ &= \frac{-4}{-0.8} \\ &= \underline{\underline{5}}.\end{aligned}$$

4. Expand and simplify (3)

$$6(x-3)-4(x-5).$$

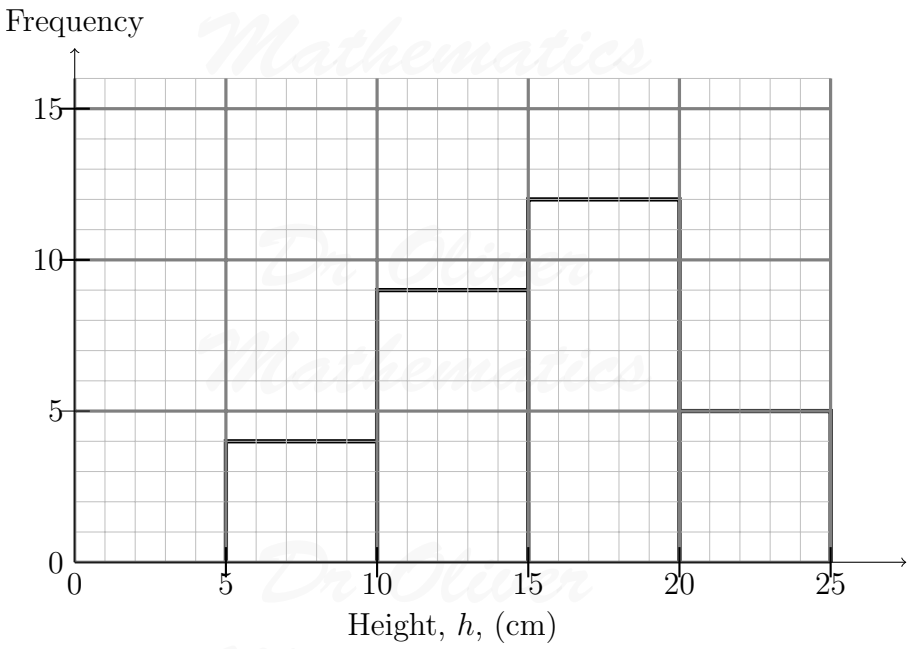
Solution

$$\begin{aligned}6(x-3)-4(x-5) &= 6x-18-4x+20 \\ &= \underline{\underline{2x+2}}.\end{aligned}$$

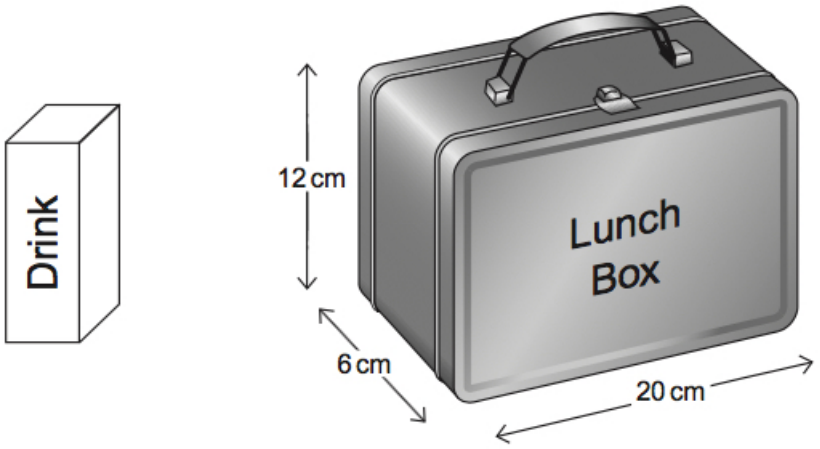
5. Draw a diagram to show this data. (3)

Height, h, (cm)	Frequency
$5 < h \leqslant 10$	4
$10 < h \leqslant 15$	9
$15 < h \leqslant 20$	12
$20 < h \leqslant 25$	5

Solution



6. Here is a drink container and a lunch box. (4)



The drink container is a cuboid with a square base.
The area of the base is 25 cm^2.
The volume of the container is 400 cm^3.

Will the container fit inside the lunch box?
You must show your working.

Solution

Well, the base is

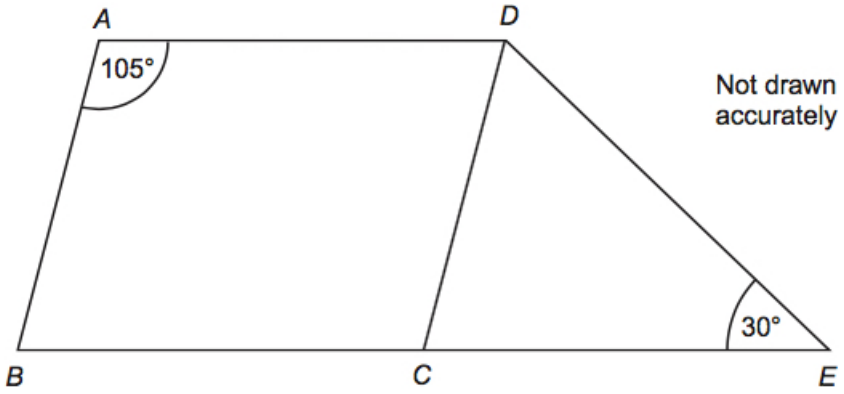
$$\sqrt{25} = 5 \text{ cm}$$

and the height, h cm, is

$$25h = 400 \Rightarrow h = 16 \text{ cm}.$$

Since $16 < 20$, $5 < 6$, and $5 < 12$, the container fit inside the lunch box

7. A parallelogram $ABCD$ and a triangle DCE are joined as shown. (4)
BCE is a straight line.



Show that DCE is an isosceles triangle.
You **must** show your working.

Solution

Well, $\angle BCD = 105^\circ$ (opposite angles in a parallelogram)
$\angle DCE = 180 - 105 = 75^\circ$ (supplementary angles)
$\angle DCE = 180 - (30 + 75) = 180 - 105 = 75^\circ$ (completing the triangle)
Hence, DCE is an isosceles triangle (base angles are the same: $30^\circ - 75^\circ - 75^\circ$).

8. Field A is a rectangle with sides of 30 m and 70 m. (4)

Field B is a square with the same **perimeter** as Field A.

How much bigger in area is Field B than Field A?
You **must** show your working.

Solution

Field A has an area of

$$30 \times 70 = 2\,100 \text{ m}^2.$$

Field B has a perimeter of

$$2(30 + 70) = 200 \text{ m}.$$

We take a quarter of that:

$$\frac{200}{4} = 50 \text{ m}.$$

Field B has an area of

$$50 \times 50 = 2\,500 \text{ m}^2.$$

Finally, Field B has an excess area of

$$2\,500 - 2\,100 = \underline{\underline{400 \text{ m}^2}}.$$

9. Here are the first five terms of a linear sequence. (2)

$$9 \quad 15 \quad 21 \quad 27 \quad 33 \quad \ldots$$

Work out the nth term.

Solution

Let the

$$n\text{th term} = an + b.$$

$$\begin{array}{ccccc} 9 & & 15 & & 21 & & 27 \\ & 6 & & 6 & & 6 & \end{array}$$

$$\begin{array}{ccccc} a+b & & 2a+b & & 3a+b & & 4a+b \\ & a & & a & & a & \end{array}$$

We compare terms:

$$a = 6$$

and

$$\begin{aligned} a + b = 9 &\Rightarrow 6 + b = 3 \\ &\Rightarrow b = 6. \end{aligned}$$

Hence,

$$n\text{th term} = \underline{\underline{6n + 3}}.$$

10. Factorise (1)

$$x^2 - 100.$$

Solution

Difference of two squares:

$$\begin{aligned} x^2 - 100 &= x^2 - 10^2 \\ &= \underline{\underline{(x + 10)(x - 10)}}. \end{aligned}$$

11. (a) Work out (2)

$$(2 \times 10^6) \times (8 \times 10^4).$$

Give your answer in standard form.

Solution

$$\begin{aligned} (2 \times 10^6) \times (8 \times 10^4) &= (2 \times 8) \times (10^6 \times 10^4) \\ &= 16 \times 10^{10} \\ &= \underline{\underline{1.6 \times 10^{11}}}. \end{aligned}$$

(b) Work out (2)

$$\frac{2 \times 10^6}{8 \times 10^4}.$$

Give your answer as an ordinary number.

Solution

$$\begin{aligned}\frac{2 \times 10^6}{8 \times 10^4} &= (2 \div 8) \times (10^6 \div 10^4) \\ &= 0.25 \times 10^2 \\ &= 0.25 \times 100 \\ &= \underline{\underline{25}}.\end{aligned}$$

12. Solve (4)

$$2x + 3 + \frac{4x - 1}{2} = 10.$$

Do **not** use trial and improvement.

Solution

$$\begin{aligned}2x + 3 + \frac{4x - 1}{2} = 10 &\Rightarrow 2x + 3 + 2x - \tfrac{1}{2} = 10 \\ &\Rightarrow 4x = 7\tfrac{1}{2} \\ &\Rightarrow 4x = \tfrac{15}{2} \\ &\Rightarrow x = \tfrac{15}{8} \\ &\Rightarrow \underline{\underline{x = 1\tfrac{7}{8}}}.\end{aligned}$$

13. A park ranger wants to estimate the number of fish in a lake. (3)

She catches 400 fish.
She marks them with ink and puts them back in the lake.

The next day she catches 60 fish.
There are 3 marked with ink.

The ranger says, "There are about 8 000 fish in the lake."

Show that she is correct.

Solution

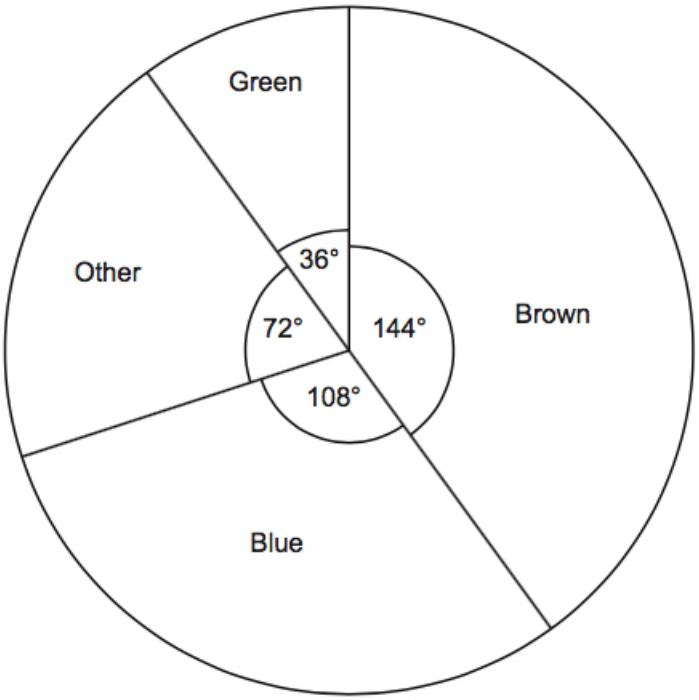
Let there be x fish in the lake. Now,

$$\frac{x}{400} = \frac{60}{3} \Rightarrow \frac{x}{400} = 20$$
$$\Rightarrow x = 20 \times 400$$
$$\Rightarrow x = 8\,000.$$

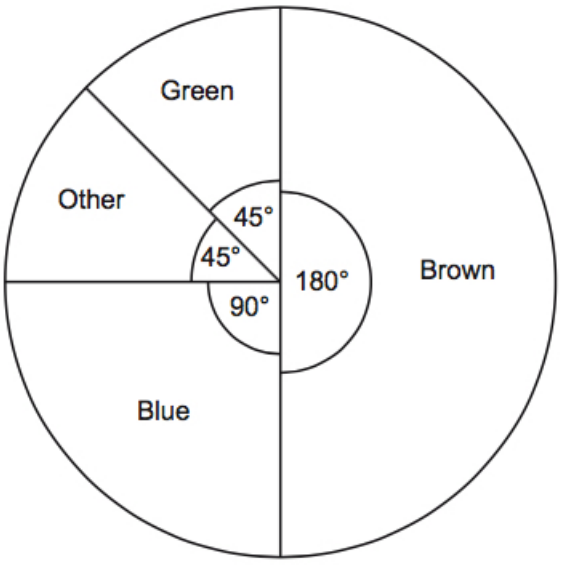
Hence, she is correct.

14. The pie charts show the eye colour of some students. (5)

Girls



Boys



The areas of the pie charts are proportional to the number of students.

The radius of the girls' pie chart is 5 cm.
The radius of the boys' pie chart is 4 cm.

5 girls have green eyes.

How many boys and girls altogether have blue eyes?

Solution

Girls': We have

$$\frac{108}{36} \times 5 = 3 \times 5 = 15 \text{ people}$$

with blue eyes and we have a total of

$$\frac{360}{36} \times 5 = 10 \times 5 = 50 \text{ people.}$$

Now, we the total number of the boys be x. We have

$$\begin{aligned} \pi \times 5^2 : \pi \times 4^2 = 50 : x &\Rightarrow 25 : 16 = 50 : x \\ &\Rightarrow \frac{16}{25} = \frac{x}{50} \\ &\Rightarrow x = \frac{16}{25} \times 50 \\ &\Rightarrow x = 16 \times 2 \\ &\Rightarrow x = 32. \end{aligned}$$

Next, we have

$$\frac{90}{360} \times 32 = \tfrac{1}{4} \times 32 = 8 \text{ people}$$

with blue eyes. Hence, the boys and girls altogether have blue eyes with

$$15 + 8 = \underline{\underline{23}}.$$

15. A headteacher wants to do a survey of students in Years 9, 10, and 11. (3)
The table shows the number of students in each year.

Year	9	10	11
Number in Year	235	215	250

She wants to take a sample of 70 students, stratified by year group.

Complete the table below to show the number of students in each year she should sample.

Year	9	10	11
Number in Year	235	215	250
Sample size			

Solution

The total number of students is

$$235 + 215 + 250 = 700.$$

So:

$$\begin{aligned}
\text{Year 9}: \quad & \frac{70}{700} \times 235 \\
&= \frac{1}{10} \times 235 \\
&= 23.5; \\
\text{Year 10}: \quad & \frac{70}{700} \times 215 \\
&= \frac{1}{10} \times 215 \\
&= 21.5; \\
\text{Year 11}: \quad & \frac{70}{700} \times 250 \\
&= \frac{1}{10} \times 250 \\
&= 25.
\end{aligned}$$

So: a choice! Well, it is usually the case the we increase the lower one and decrease the larger one.

Year	9	10	11
Number in Year	235	215	250
Sample size	$\underline{\underline{23}}$	$\underline{\underline{22}}$	$\underline{\underline{25}}$

16. Work out (2)

$$64^{\frac{1}{3}}.$$

Solution

$$\begin{aligned} 64^{\frac{2}{3}} &= (\sqrt[3]{64})^2 \\ &= 4^2 \\ &= \underline{\underline{16}} \end{aligned}$$

17. Show that (2)

$$(\sqrt{3} + \sqrt{75})^2 = 108.$$

Solution

Now,

$$\begin{aligned} \sqrt{3} + \sqrt{75} &= \sqrt{3} + \sqrt{25 \times 3} \\ &= \sqrt{3} + (\sqrt{25} \times \sqrt{3}) \\ &= \sqrt{3} + 5\sqrt{3} \\ &= 6\sqrt{3} \end{aligned}$$

and

$$\begin{aligned} (\sqrt{3} + \sqrt{75})^2 &= (6\sqrt{3})^2 \\ &= 36 \times 3 \\ &= \underline{\underline{108}}, \end{aligned}$$

as required.

18. Show that (3)

$$\frac{2}{x-3} + \frac{1}{x-1} = 1$$

simplifies to

$$x^2 - 7x + 8 = 0.$$

Solution

Multiply by $(x-3)(x-1)$:

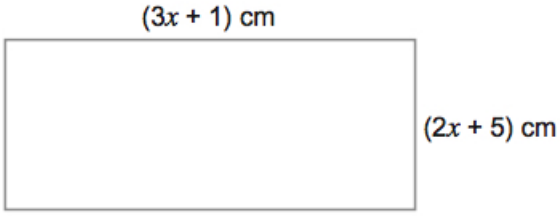
$$\frac{2}{x-3}+\frac{1}{x-1}=1 \Rightarrow 2(x-1)+(x-3)=(x-3)(x-1)$$

$\times$	x	-1
x	x^2	$-x$
-3	$-3x$	$+3$

$$\Rightarrow 2x-2+x-3=x^2-4x+3$$
$$\Rightarrow \underline{\underline{x^2-7x+8=0}},$$

as required.

19. The area of this rectangle is 8 cm^2. (4)



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Work out the value of x.

Solution

$\times$	$3x$	$+1$
$2x$	$6x^2$	$+2x$
$+5$	$+15x$	$+5$

Now,

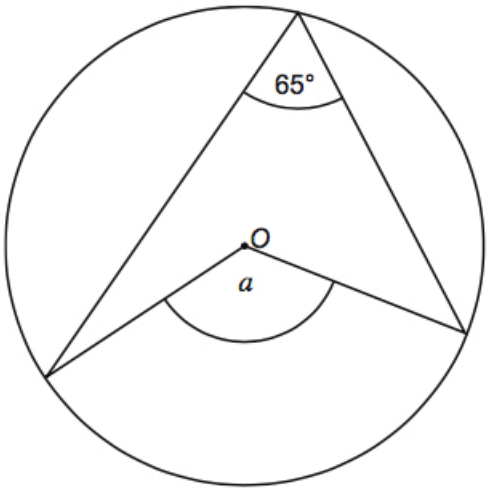
$$(3x+1)(2x+5) = 8 \Rightarrow 6x^2 + 17x + 5 = 8$$
$$\Rightarrow 6x^2 + 17x - 3 = 0$$

add to: $+17$
multiply to: $(+6) \times (-3) = -18$
$\Big\}$ $+18, -1$

$$\Rightarrow 6x^2 + 18x - x - 3 = 0$$
$$\Rightarrow 6x(x+3) - 1(x+3) = 0$$
$$\Rightarrow (6x-1)(x+3) = 0$$
$$\Rightarrow 6x - 1 = 0 \text{ or } x + 3 = 0$$
$$\Rightarrow x = \tfrac{1}{6} \text{ or } x = -3.$$

Now $x \neq -3$ (why?) and so $\underline{\underline{x = \frac{1}{6}}}$.

20. (a) O is the centre of the circle. (1)
Work out the size of angle a.



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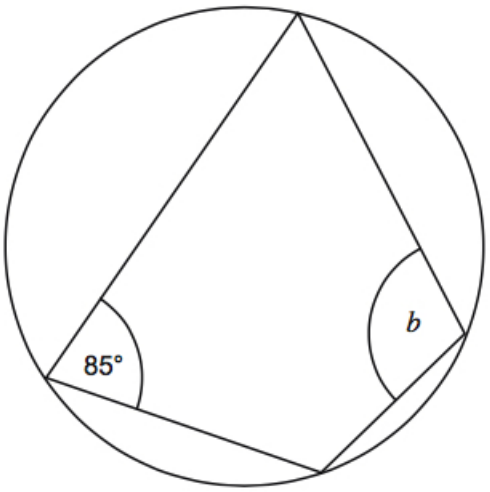
Circle your answer.

115° 130° 135° 295°

Solution

115° $\underline{130^\circ}$ 135° 295°

(b) Work out the size of angle b. (1)



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Circle your answer.

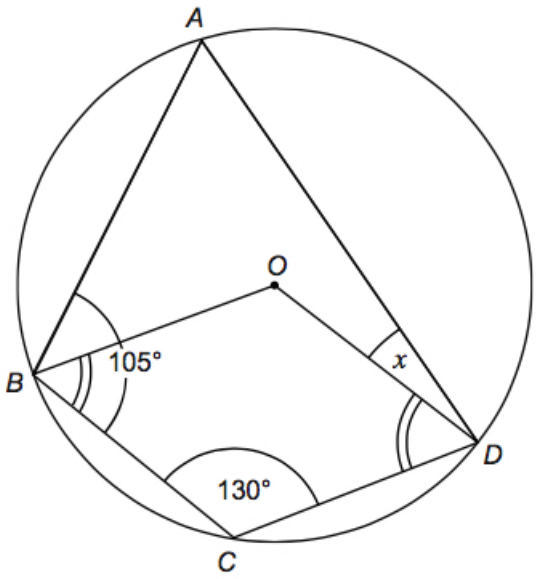
85° 90° 95° 115°

Solution

85° 90° $\underline{95^\circ}$ 115°

(c) $ABCD$ are points on the circumference of a circle centre O. (4)

- Angle $BCD = 130^\circ$,
- Angle $ABC = 105^\circ$, and
- Angle OBC = angle ODC.



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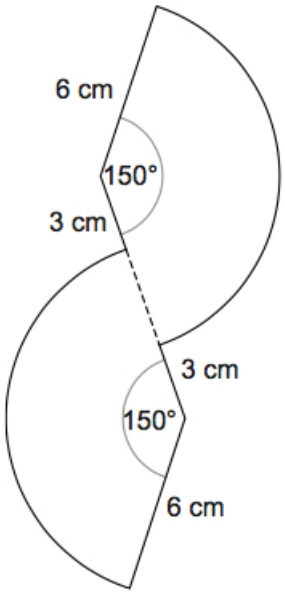
Work out the size of angle ADO, marked x on the diagram.
You **must** show your working which may be on the diagram.

Solution

$\angle BAD = 180 - 130 = 50^\circ$ (opposite angle in a cyclic quadrilateral)
$\angle BOD = 2 \times 50 = 100^\circ$ (angle in a circle is equal to twice the angle that it subtends on the remaining circumference)
$\angle OBC = \angle ODC = \frac{1}{2}(130 + 100) = 65^\circ$ (completing the quadrilateral)
$x = 360 - (130 + 105 + 50 + 65) = 360 - 350 = \underline{\underline{10^\circ}}$ (completing the quadrilateral)

21. This shape is made from two identical sectors. (4)



Not drawn accurately

Work out the perimeter of the shape.
Give your answer in terms of π.

Solution

$$\begin{aligned}\text{Perimeter} &= 2[3 + 6 + (\tfrac{150}{360} \times 2 \times \pi \times 6)] \\ &= 2[9 + (\tfrac{5}{12} \times 12\pi)] \\ &= \underline{\underline{2(9 + 5\pi)}}.\end{aligned}$$