Dr Oliver Mathematics
Mathematics Standard Grade: Credit Level
2012 Paper 2: Calculator
1 hour 20 minutes

The total number of marks available is 53.
You must write down all the stages in your working.

1. There are 2.69 million vehicles in Scotland. (4)
   It is estimated that this number will increase at a rate of 4% each year.
   If this estimate is correct, how many vehicles will there be in 3 years' time?
   Give your answer correct to 3 significant figures.

Solution

$$\begin{aligned} 2.69 \times 1.04^3 &= 3.025\,884\,16 \text{ (FCD)} \\ &= \underline{\underline{3.03 \text{ million (3 sf)}}}. \end{aligned}$$

2. Before training, athletes were tested on how many sit-ups they could do in one minute.
   The following information was obtained:

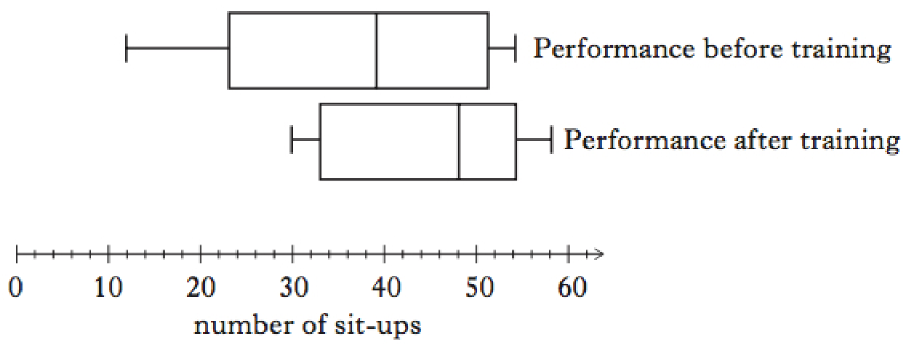
Lower quartile (Q_1)	23
Median (Q_2)	39
Upper quartile (Q_3)	51

   (a) Calculate the semi-interquartile range. (1)

Solution

$$\text{SIQR} = \tfrac{1}{2}(51 - 23) = \underline{\underline{14}}.$$

After training, the athletes were tested again.
Both sets of data are displayed as box-plots.



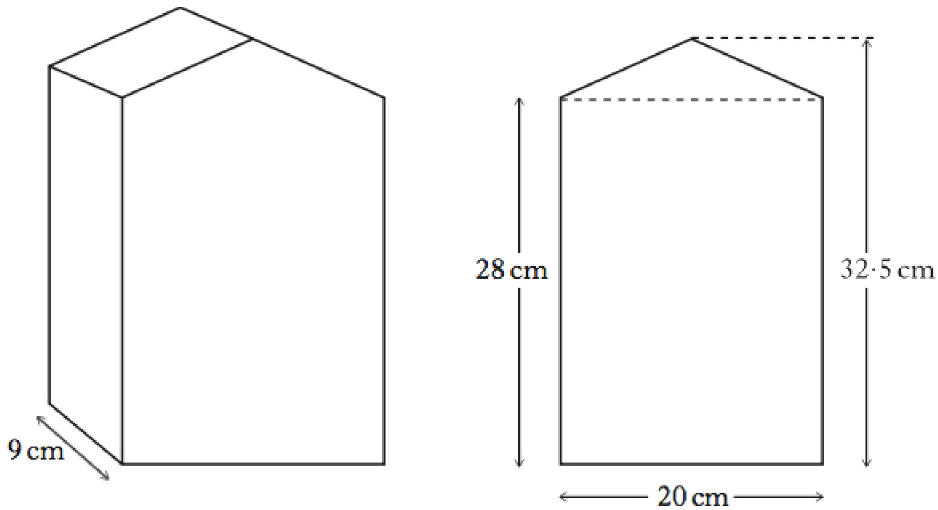
(b) Make **two** valid statements to compare the performances before and after training. (2)

Solution

Average Since the median for before training is lower than the median for after training, they did more sit-ups after training on average.

Spread Since the range/ IQR/ SIQR for after training is smaller than the range/ IQR/ SIQR for before training, the marks were more consistent in after training.

3. A container for oil is in the shape of a prism. (4)
The width of the container is 9 centimetres.
The uniform cross section of the container consists of a rectangle and a triangle with dimensions as shown.



Calculate the volume of the container, **correct to the nearest litre**.

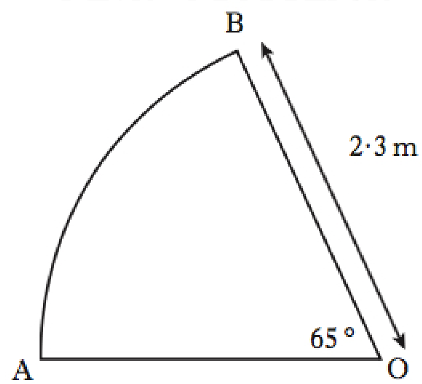
Solution

$$\begin{aligned} \text{Cross-sectional area} &= (20 \times 28) + (\tfrac{1}{2} \times 20 \times 4.5) \\ &= 560 + 45 \\ &= 605 \text{ cm}^2 \end{aligned}$$

and

$$\begin{aligned} \text{volume} &= 605 \times 9 \\ &= 5\,445 \text{ cm}^3 \\ &= 5.445 \text{ l} \\ &= \underline{\underline{5 \text{ l (nearest litre)}}}. \end{aligned}$$

4. A sector of a circle, centre O, is shown below. (3)



The radius of the circle is 2.3 metres.
Angle AOB is 65°.
Find the length of the arc AB.

Solution

$$\begin{aligned}\text{Arc} &= \frac{65}{360} \times 2 \times \pi \times 2.3 \\ &= 2.609\,267\,232 \text{ (FCD)} \\ &= \underline{\underline{2.61 \text{ m (3 sf)}}}.\end{aligned}$$

5. The depth, d, of water in a tank, varies directly as the volume, v, of water in the tank and inversely as the square of the radius, r, of the tank. (4)

When the volume of water is $60\,000$ cm^3, the depth of water is 50 cm, and the radius of the tank is 20 cm.

Calculate the depth of the water, when the volume of water is $75\,000$ cm^3 and the radius of the tank is 25 cm.

Solution

$$d \propto \frac{v}{r^2} \Rightarrow d = \frac{kv}{r^2}$$

for some k. Now,

$$50 = \frac{k \times 60\,000}{20^2} \Rightarrow k = \tfrac{1}{3}$$

and so

$$d = \frac{v}{3r^2}.$$

Finally,

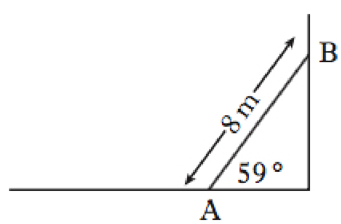
$$d = \frac{75\,000}{3 \times 25^2} = \underline{\underline{40 \text{ cm}}}.$$

6. The price for Paul's summer holiday is £894.40. (3)
The price includes a 4% booking fee.
What is the price of his holiday without the booking fee?

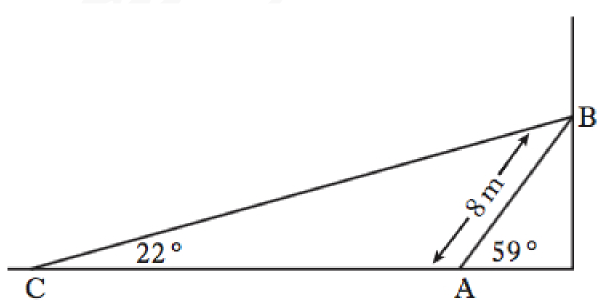
Solution

$$\begin{aligned}\text{Without the booking fee} &= \frac{894.40}{1 + 0.04} \\ &= \underline{\underline{£860}}.\end{aligned}$$

7. A heavy metal beam, AB, rests against a vertical wall as shown. (4)
The length of the beam is 8 metres and it makes an angle of 59° with the ground.



A cable, CB, is fixed to the ground at C and is attached to the top of the beam at B.
The cable makes an angle of 22° with the ground.



Calculate the length of cable CB.

Solution

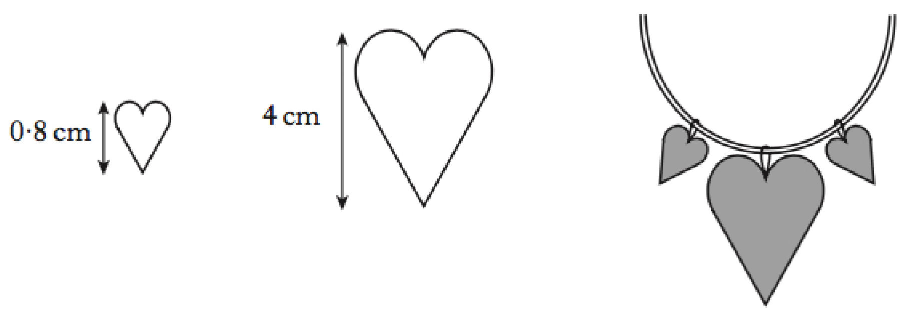
Well,

$$\angle\, BAC = 180 - 59 = 121^\circ$$

and

$$\begin{aligned}\frac{CB}{\sin BAC} = \frac{AB}{\sin ACB} &\Rightarrow \frac{CB}{\sin 121^\circ} = \frac{8}{\sin 22^\circ} \\ &\Rightarrow CB = \frac{8 \sin 121^\circ}{\sin 22^\circ} \\ &\Rightarrow CB = 18.305\,439\,7 \text{ (FCD)} \\ &\Rightarrow \underline{\underline{CB = 18.3 \text{ m (3 sf)}}}.\end{aligned}$$

8. A necklace is made of beads which are mathematically similar. (3)



The height of the smaller bead is 0.8 centimetres and its area is 0.6 square centimetres.
The height of the larger bead is 4 centimetres.
Find the area of the larger bead.

Solution

The length scale factor is

$$\frac{4}{0.8} = 5$$

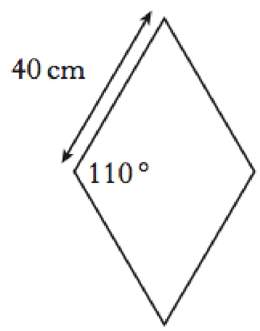
and the area scale factor is

$$5^2 = 25.$$

Hence, the area of the larger bead is

$$25 \times 0.6 = \underline{\underline{15 \text{ square centimetres}}}.$$

9. Paving stones are in the shape of a rhombus. (4)



The side of each rhombus is 40 centimetres long.
The obtuse angle is 110°.
Find the area of one paving stone.

Solution

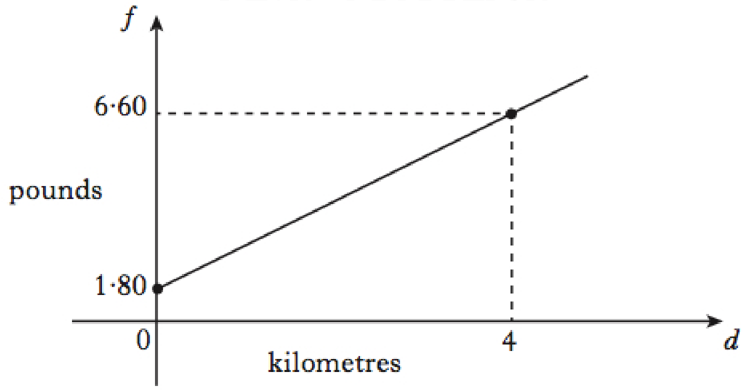
Draw a line horizontally: we can see that base angles of 55° in each of the two triangles and the makes

$$180 - 55 - 55 = 70^\circ$$

at the top and the bottom. Finally,

$$\begin{aligned} \text{area} &= 2 \times (\tfrac{1}{2} \times 40^2 \times \sin 70^\circ) \\ &= 1\,503.508\,193 \text{ (FCD)} \\ &= \underline{\underline{1\,500 \text{ cm}^2 \text{ (3 sf)}}}. \end{aligned}$$

10. A taxi fare consists of a call-out charge of £1.80 **plus** a fixed cost per kilometre.
A journey of 4 kilometres costs £6.60.
The straight line graph shows the fare, f pounds, for a journey of d kilometres.



(a) Find the equation of the straight line. (3)

Solution

$$\text{Gradient} = \frac{6.6 - 1.8}{4 - 0} = \frac{4.8}{4} = 1.2$$

and the equation of the straight line is

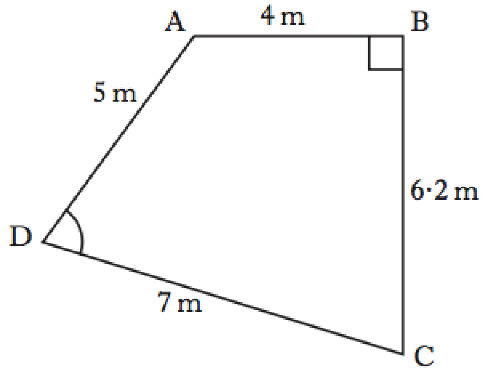
$$f - 6.6 = 1.2(d - 4) \Rightarrow f - 6.6 = 1.2d - 4.8 \Rightarrow \underline{\underline{f = 1.2d + 1.8}}.$$

(b) Calculate the fare for a journey of 7 kilometres. (2)

Solution

$$f = (1.2 \times 7) + 1.8 = \underline{\underline{£10.20}}.$$

11. Quadrilateral $ABCD$ with angle $ABC = 90^\circ$ is shown below.



$AB = 4$ metres.
$BC = 6.2$ metres.
$CD = 7$ metres.
$AD = 5$ metres.

(a) Calculate the length of AC. (2)

Solution

$$\begin{aligned} AC &= \sqrt{AB^2 + BC^2} \\ &= \sqrt{4^2 + 6.2^2} \\ &= 7.378\,346\,698 \text{ (FCD)} \\ &= \underline{\underline{7.38 \text{ m (3 sf)}}}. \end{aligned}$$

(b) Calculate the size of angle ADC. (4)

Solution

$$\begin{aligned} & AC^2 = AD^2 + CD^2 - 2 \cdot AD \cdot CD \cdot \cos ADC \\ \Rightarrow \quad & \cos ADC = \frac{AD^2 + CD^2 - AC^2}{2 \cdot AD \cdot CD} \\ \Rightarrow \quad & \cos ADC = \frac{5^2 + 7^2 - 7.378\ldots^2}{2 \cdot 5 \cdot 7} \\ \Rightarrow \quad & \angle ADC = 73.773\,896\,96 \text{ (FCD)} \\ \Rightarrow \quad & \underline{\underline{\angle ADC = 73.8^\circ \text{ (1 dp)}}}. \end{aligned}$$

12.

$$f(x) = 3\sin x^\circ,\ 0 \leqslant x < 360.$$

(a) Find f(270). (1)

Solution

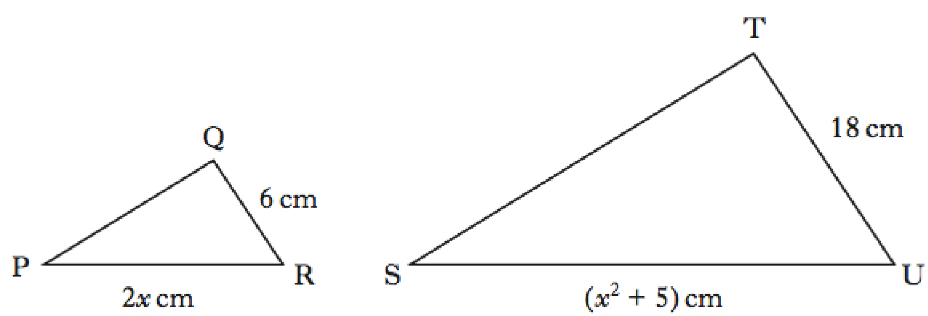
$f(270) = \underline{\underline{-3}}.$

(b) $f(t) = 0.6$. (4)
Find the two possible values of t.

Solution

$$\begin{aligned} f(t) = 0.6 &\Rightarrow \sin t^\circ = 0.2 \\ &\Rightarrow t = 11.536\,959\,03, 168.463\,041 \text{ (FCD)} \\ &\Rightarrow \underline{\underline{t = 11.5, 168.5 \text{ (1 dp)}}}. \end{aligned}$$

13. Triangles PQR and STU are mathematically similar.
The scale factor is 3 and PR corresponds to SU.



(a) Show that (2)

$$x^2 - 6x + 5 = 0.$$

Solution

$$\begin{aligned} \frac{x^2+5}{2x} = \frac{18}{6} &\Rightarrow \frac{x^2+5}{2x} = 3 \\ &\Rightarrow x^2 + 5 = 6x \\ &\Rightarrow \underline{\underline{x^2 - 6x + 5 = 0}}, \end{aligned}$$

as required.

(b) Given QR is the shortest side of triangle PQR, find the value of x. (3)

Solution

$$\left.\begin{array}{ll}\text{add to:} & -6 \\ \text{multiply to:} & +5\end{array}\right\} -5,\ -1$$

$$\begin{aligned} x^2 - 6x + 5 = 0 &\Rightarrow (x-5)(x-1) = 0 \\ &\Rightarrow x - 5 = 0 \text{ or } x - 1 = 0 \\ &\Rightarrow x = 5 \text{ or } x = 1. \end{aligned}$$

$\underline{x = 1}$:

$$2x = 2 \Rightarrow x = 1$$

and this is not right because QR is *not* the shortest side of triangle PQR.

Hence, $\underline{\underline{x = 5}}$.